Write $F = \varkappa^{-1}(\Gamma)$, $A' = A \cap F$ and let us consider the metric spaces (A', ϱ). We shall show that this is an absolutely Borel space [2]. We adopt the notation of the proof of Lemma 3. Let $F_m = \bigcup\{G_{sm}: s \in S\}$. Since G_{sm} is an F_σ-set and $\varrho(G_{sm}, G_{tm}) \geqslant 1/m$ for distinct s, t, we infer that $F = X \setminus \bigcup F_m$ is a G_δ-set in (X, ϱ). Thus (F, ϱ) is an absolutely Borel space and so is (A', ϱ), as A' is an F_σ-set in (F, ϱ). By Lemma 1 the space (A', ϱ) is not σ-discrete and thus by a Theorem of A. H. Stone ([6], Theorem 1) it must contain a Cantor set. This gives the contradiction, because separable subspaces of (A', ϱ) are countable (compare with [6], Sec. 5).

Remark 4. Let E be the space considered in the Example (Sec. 1). One can prove (see R. Pol, Comment. Math. 22 (1977)) that the product $E^{\aleph_0}$ is perfectly normal, while E is not paracompact.

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(2) A metrizable space is absolutely Borel if it can be embedded as a Borel subspace in a completely metrizable space.

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A hereditarily normal strongly zero-dimensional space with a subspace of positive dimension and an N-compact space of positive dimension

by

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Abstract. In this paper we give a solution of an old Čech's problem on dimension by constructing a hereditarily normal strongly zero-dimensional space containing a subspace of positive dimension. We give also an example of an N-compact space of positive dimension.

The aim of this paper is to construct spaces with the properties mentioned in the title.

The problem of existence of a hereditarily normal space X containing a subspace with the covering dimension greater than the covering dimension of X is an old problem of Čech (see [2]; compare also [7] Appendix, [3], [11] Problem 11-14, [1] VII, Introduction). Recently, V. V. Filippov [6] showed that the existence of a Souslin Tree yields a space of this kind. Further examples, with many additional properties, were constructed by V. V. Fedorčuk [5]; he used, however, some additional set theoretic assumptions, too. The example we shall construct needs only the usual axioms for the set theory. It solves at the same time a problem on the local dimension raised by C. H. Dowker in [3].

The problem of existence of a closed subspace with the positive covering dimension in a product of countable discrete spaces appears in the natural way in the theory of N-compactness (see [12]). It was solved recently by S. Mrówka [10] (see also [13]). We give another example of this kind (it seems to us that it is simpler than the Mrówka's one).

1. Notation and terminology. Our terminology will follow [4]. We shall use the following notation: I denotes the closed real unit interval, Q stands for rationals of I, P — for irrationals of I and N — for natural numbers. For an ordinal α we shall denote by $D(\alpha)$ the set of all ordinals less than α with the discrete topology and by $W(\alpha)$ the same set with the order topology. The word "dimension" will denote the covering dimension dim (see [4], § 7.1); a space X with $\dim X = 0$ is called *strongly zero-dimensional*. We say that the *local dimension* of a space X is at most n (abbreviated $\operatorname{locdim} X \leqslant n$) if each point $x \in X$ has an open neighbourhood U with $\dim \overline{U} \leqslant n$ (see [3] and [11] Definition 11-6). All spaces under discussion are assumed completely regular.

2. Auxiliary construction. The construction of the Broom due to Knaster and Kuratowski (see [8] and [4] P. 6.3.23) is a source of the following observation which play the key role in the sequel.

Let X be a topological space, A a subspace of X and let $Q_0 \supset \{0, 1\}$ be a subset of Q such that the set $Q_1 = Q \setminus Q_0$ is dense in Q. Let

$$B(X, A) = (X \times Q_0) \cup (A \times P) \cup [(X \setminus A) \times Q_1]$$

be the subspace of the Cartesian product $X \times I$. For $Y \subset X$ put

$$C(Y) = (Y \times I) \cap B(X, A) = B(Y, A \cap Y).$$

We have the following

LEMMA 1. *If A is not an F_σ-set in X, then for arbitrary $q, q' \in Q_0$ the sets $X \times \{q\}$ and $X \times \{q'\}$ cannot be separated in $B(X, A)$ by the empty set. In particular,* $\dim B(X, A) > 0$.

Proof. Suppose that $B(X, A)$ is the union of two disjoint open-and-closed subsets U and U' such that $U \supset X \times \{q\}$ and $U' \supset X \times \{q'\}$. The set $F = \overline{U} \cap \overline{U'}$, where bar denotes the closure in $X \times I$, separates the sets $X \times \{q\}$ and $X \times \{q'\}$ in $X \times I$ and $F \cap B(X, A) = \emptyset$. For each $s \in Q_1$ the set $F(s) = \{x \in X: (x, s) \in F\}$ is closed in X. We shall show that $A = \bigcup_{s \in Q_1} F(s)$, i.e. that A is an F_σ-set in X. Indeed, if $x \in F(s)$ for some $s \in Q_1$ then $(x, s) \in F = F \setminus B(X, A)$, hence $x \in A$; for every $x \in X$ there exists a $t \in I$ such that $(x, t) \in F = F \setminus B(X, A)$ and if $x \in A$, then $t \in Q_1$ so that $x \in F(t)$.

3. A hereditarily normal strongly zero-dimensional space with a subspace of positive dimension. C. H. Dowker [3] showed that the existence of such a space is equivalent to the existence of a hereditarily normal space L with $\operatorname{locdim} L = 0 < \dim L$ (see [11] Remark 11-18); for the construction of L we shall need the following

LEMMA 2. *There exists a perfectly normal and locally second-countable space K with* $\operatorname{locdim} K = 0$ *which contains a locally countable subset A which is not an F_σ-set in K.*

We take the space X defined in Example of [14] as the space K; we recall the construction below. Let $B(\aleph_1) = D(\omega_1)^N$ be the Baire space of weight $\aleph_1$ (see [4] Example 4.2.12). For each $x \in B(\aleph_1)$ let $\varkappa(x) = \min\{\alpha: x(i) < \alpha \text{ for } i \in N\}$ and let K be the graph $\{(x, \varkappa(x)): x \in B(\aleph_1)\} \subset B(\aleph_1) \times W(\omega_1)$ of the function $\varkappa$. The space K is perfectly normal (see [14] Proposition 1) and, since $K \cap (B(\aleph_1) \times W(\xi)) = K \cap (D(\xi)^N \times W(\xi))$ for every $\xi < \omega_1$, K is locally second-countable and $\operatorname{locdim} K$

$= 0$. Finally, if we choose for each $\xi < \omega_1$ a point $x_\xi \in \varkappa^{-1}(\xi)$ then the set $A = \{(x_\xi, \xi): \xi < \omega_1\}$ has the required property, by [14] Remark 3, Proposition 2'.

EXAMPLE 1. *There exists a perfectly normal locally second-countable space L such that* $\operatorname{locdim} L = 0 < \dim L$.

Let us put $Q_0 = \{0, 1\}$ and let $L = B(K, A)$, where K and A are as in Lemma 2. By Morita's theorem L is perfectly normal (see for example [4] P. 4.5.16) and it is locally second-countable. By Lemma 1 we have $\dim L > 0$. It remains to show that $\operatorname{locdim} L = 0$. Take an arbitrary point $(x, t) \in L$, where $x \in K$, $t \in I$. There exists an open-and-closed neighbourhood U of x such that $\dim U = 0$ and $|U \cap A| \leqslant \aleph_0$. The set

$$C(U) = (U \times Q_0) \cup [(U \cap A) \times (P \cup Q_0)] \cup [(U \setminus A) \times Q_1]$$

is the countable union of its closed strongly zero-dimensional subsets $U \times \{t\}$ for $t \in Q_0$, $(U \setminus A) \times \{t\}$ for $t \in Q_1$ and $\{y\} \times (P \cup Q_0)$ for $y \in U \cap A$. Hence by the Sum Theorem $\dim C(U) = 0$. It follows that the point (x, t) has an open-and-closed strongly zero-dimensional neighbourhood.

We shall use Dowker's construction (see [11] Theorem 11-17) to obtain the following

EXAMPLE 2. *There exists a hereditarily normal strongly zero-dimensional Lindelöf space containing a subspace of positive dimension.*

Let $L^* = L \cup \{p\}$ where L is the space from Example 1 and p is a point which does not belong to L. The topology of L^* consists of all open subsets of L and the sets V such that $p \in V$ and $L \setminus V$ is a second-countable closed subspace of L. It is easy to see that L^* is Lindelöf. By the construction it follows that each separable subset of K is contained in an open-and-closed second-countable and strongly zero-dimensional subspace of K; the same holds in L. Thus the space L^* is hereditarily normal, because for any separated sets A, B either $A \cup B \subset L$ or one of the sets is second-countable. Finally, it is not hard to verify that $\dim L^* = 0 < \dim L$.

Remark 1. The space L we have constructed is collectionwise normal. Indeed, the space K is perfect and collectionwise normal (see [14] Remark 2) and thus the same is true for the product $K \times I$ which contains L (cf. [4], P. 4.5.16, P. 5.5.19 and P. 5.5.1). Let us notice that L cannot be paracompact because in the class of paracompact spaces $\operatorname{locdim} = \dim$ (see [11], Corollary 11-8).

Remark 2. As proved by C. H. Dowker [3], for the function locdim the Finite Sum Theorem holds in the class of normal spaces. It is easy to show that the space $Z = (N \times L) \cup \{a\}$, where the sets of the form $\{a\} \cup \bigcup_{k \geqslant n} [\{k\} \times L]$ form a base of neighbourhoods at the point a, is the union of countably many closed subsets with $\operatorname{locdim} = 0$, whereas $\operatorname{locdim} Z > 0$. Thus the Countable Sum Theorem fails for locdim in the class of perfectly normal spaces.

4. An N-compact space of positive dimension. A space X is N-compact if it can be embedded as a closed subspace in a product of copies of N (see [16]).

Let S be a set of cardinality $\aleph_1$. For every $T\subset S$ by $p_T\colon N^S\to N^T$ we shall denote the projection; if $|T|\leqslant\aleph_0$ then the set $p_T^{-1}(x)$, where $x\in N^T$, will be called an $\aleph_0$-cube.

The following lemma was proved by the authors in [15] (Example 2).

LEMMA 3. *There exists a subset E of N^S which has the following properties:*
(i) *E is locally an F_σ-set in N^S* (1),
(ii) *E is not an F_σ-set in N^S,*
(iii) *E is the union of $\aleph_0$-cubes.*

Notice that $N^S\setminus E$ is also the union of $\aleph_0$-cubes, because by (i) it is the union of G_δ-sets and every G_δ-set in N^S is the union of $\aleph_0$-cubes.

EXAMPLE 3 (cf. [10]). *An N-compact space M which is not strongly zero-dimensional.*

Let Q_0 be a dense subset of Q such that $Q_1 = Q\setminus Q_0$ is also dense in Q. Define $M = \boldsymbol{B}(N^S, E)$, where E is as in Lemma 3. By Lemma 1 and (ii) it follows that $\dim M>0$. It remains to show that M is N-compact.

First we shall prove that

(1) M is realcompact.

Indeed, the space $N^S\times I$ is realcompact and the complement $(N^S\times I)\setminus M = (E\times Q_1)\cup[(N^S\setminus E)\times P]$ is the union of $\aleph_0$-cubes and thus, as each $\aleph_0$-cube is a G_δ-set in $N^S\times I$, (1) follows by Mrówka's theorem (see [4] P. 3.12.25).

Let U be an open-and-closed subset of N^S. Then

(2) $\dim \boldsymbol{C}(U) = 0$ if and only if $U\cap E$ is an F_σ-set in U.

If $U\cap E$ is not an F_δ-set in U then by Lemma 1 $\dim\boldsymbol{C}(U)>0$. Conversely, let $U\cap E$ be an F_σ-set in N^S. Since U is open-and-closed it depends on countably many coordinates, i.e. $U = p_{T_1}(U)\times N^{S\setminus T_1}$ for some countable set $T_1\subset S$ (see [4], P. 2.7.12). Because $U\cap E$ is an F_σ-set which is the union of $\aleph_0$-cubes,

$$U\cap E = p_{T_2}(U\cap E)\times N^{S\setminus T_2}, \quad \text{where } T_2\subset S \text{ is countable},$$

by Theorem 2 of [15]. By Remark 2 of [15] there exists $T\subset S$, $T\supset T_1\cup T_2$, $|T|\leqslant\aleph_0$ such that $p_T(U\cap E)$ is an F_σ-set in N^T. Thus we have

$$U = U'\times N^{S\setminus T}, \quad \text{where } U' \text{ is open in } N^T,$$

and

$$U\cap E = E'\times N^{S\setminus T}, \quad \text{where} \quad E' = p_T(U\cap E) = \bigcup_{i=1}^{\infty} F_i,$$

(1) This means that for each $x\in N^S$ there exists a neighbourhood V of x such that $V\cap E$ is an F_σ-set in V.

where F_i are closed in N^T. It follows that

$$\begin{aligned}\boldsymbol{C}(U) &= (U\times Q_0)\cup[(U\cap E)\times P]\cup[(U\setminus E)\times Q_1]\\ &= (U'\times N^{S\setminus T}\times Q_0)\cup(E'\times N^{S\setminus T}\times P)\cup[(U'\setminus E')\times N^{S\setminus T}\times Q_1]\\ &\overset{\text{top}}{=} \{(U'\times Q_0)\cup(E'\times P)\cup[(U'\setminus E')\times Q_1]\}\times N^{S\setminus T}.\end{aligned}$$

The space $Z = (U'\times Q_0)\cup(E'\times P)\cup[(U'\setminus E')\times Q_1]\subset N^T\times I$ is a metrizable separable space and it is the union of its closed zero-dimensional subsets $U'\times\{t\}$, for $t\in Q_0$, $(U'\setminus E')\times\{t\}$, for $t\in Q_1$ and $F_i\times(P\cup Q_0)$, for $i = 1, 2, \ldots$ Hence $\dim Z = 0$ by the Sum Theorem. Thus $\boldsymbol{C}(U)$ is the product of zero-dimensional second-countable spaces and hence $\dim\boldsymbol{C}(U) = 0$ by Morita's theorem ([9], Theorem 3). The proof of (2) is completed.

Let U be an open-and-closed subset of M. For $q\in Q_0$ put

$$U(q) = \{x\in N^S\colon (x, q)\in U\};$$

clearly $U(q)$ is open-and-closed in N^S.

We shall verify that

(3) $(U(q)\setminus U(q'))\cap E$ is an F_σ-set in $U(q)\setminus U(q')$ for every $q, q'\in Q_0$.

Indeed, the set $V = U\cap\boldsymbol{C}(U(q)\setminus U(q'))$ is open-and-closed in $\boldsymbol{C}(U(q)\setminus U(q'))$ and $(U(q)\setminus U(q'))\times\{q\}\subset V\subset\boldsymbol{C}(U(q)\setminus U(q'))\setminus(U(q)\setminus U(q'))\times\{q'\}$. Hence from Lemma 1 it follows that $(U(q)\setminus U(q'))\cap E$ is an F_σ-set in $U(q)\setminus U(q')$.

For an open-and-closed set $U\subset M$ we define

$$J_U = \bigcap_{q\in Q_0} U(q)\subset N^S.$$

The sets J_U satisfy

(4) $$\boldsymbol{C}(J_U)\subset U$$

and

(5) $U\setminus\boldsymbol{C}(J_U)$ is the countable union of strongly zero-dimensional open-and-closed subsets of M.

Consider an $(x, t)\in\boldsymbol{C}(J_U)$. Then $x\in\bigcap_{q\in Q_0} U(q)$ and because Q_0 is dense in Q and U is closed, we have $U\supset\boldsymbol{C}(\{x\})\ni(x, t)$. Thus (4) holds. To establish (5) let us assume that $(x, t)\in U\setminus\boldsymbol{C}(J_U)$. Then $(x, q')\notin U$ for some $q'\in Q_0$. Since U is open and Q_0 is dense in Q there exists $q\in Q_0$ such that $(x, q)\in U$. Thus $x\in U(q)\setminus U(q')$ and $(x, t)\in\boldsymbol{C}(U(q)\setminus U(q'))$. We have obtained the equality

$$U\setminus\boldsymbol{C}(J_U) = \bigcup_{q, q'\in Q_0}\boldsymbol{C}(U(q)\setminus U(q'))\cap U$$

which proves (5) by (3) and (2).

We shall prove now that M is N-compact. Since by (i) and (2) it follows that

(6) $$\operatorname{loc}\dim M = 0\,,$$

it suffices only to verify that every open-and-closed ultrafilter in M with the countable intersection property has nonempty intersection (see [16], p. 478). Let $\mathcal{U}$ be such an ultrafilter. We shall show that

(7) there exists $U \in \mathcal{U}$ with $\dim U = 0$.

Suppose on the contrary that $\dim U > 0$ for each $U \in \mathcal{U}$. Fix an arbitrary $q_0 \in Q_0$ and let

$$\mathcal{V} = \{U(q_0)\colon U \in \mathcal{U}\}\,.$$

We shall prove that $\mathcal{V}$ is an open-and-closed ultrafilter in N^S and has the countable intersection property. $\mathcal{V}$ is a filter because for $U_1, U_2 \in \mathcal{U}$ the intersection $U_1(q_0) \cap U_2(q_0) = (U_1 \cap U_2)(q_0)$ belongs to $\mathcal{V}$ and if an open-and-closed set $A \subset N^S$ contains $U_1(q_0)$ then $A = (C(A) \cup U_1)(q_0)$ also belongs to $\mathcal{V}$ (because $\mathcal{U}$ is a filter). Now let U be an open-and-closed subset of N^S. Then either $C(U) \in \mathcal{U}$ or $M \setminus C(U) = C(N^S \setminus U) \in \mathcal{U}$, hence either $U \in \mathcal{V}$ or $N^S \setminus U \in \mathcal{V}$; thus $\mathcal{V}$ is an ultrafilter. Let $U_i \in \mathcal{U}$ for $i = 1, 2, \ldots$, we shall show that $\bigcap_{i=1}^{\infty} U_i(q_0) \neq \emptyset$. As shown in (5), we have $\bigcup_{i=1}^{\infty} (U_i \setminus C(J_{U_i})) = \bigcup_{j=1}^{\infty} V_j$, where the sets V_j are strongly zero-dimensional and open-and-closed in M. Since $\mathcal{U}$ is an ultrafilter it follows from the negation of (7) that $M \setminus V_j \in \mathcal{U}$ for $j = 1, 2, \ldots$ By the countable intersection property of $\mathcal{U}$ there exists a point

$$(x_0, t_0) \in \bigcap_{i=1}^{\infty} U_i \cap \bigcap_{j=1}^{\infty} (M \setminus V_j) = \bigcap_{i=1}^{\infty} U_i \setminus \bigcup_{j=1}^{\infty} V_j = \bigcap_{i=1}^{\infty} U_i \setminus \bigcup_{i=1}^{\infty} (U_i \setminus C(J_{U_i})) = \bigcap_{i=1}^{\infty} C(J_{U_i})\,.$$

We obtain

$$x_0 \in \bigcap_{i=1}^{\infty} J_{U_i} \subset \bigcap_{i=1}^{\infty} U_i(q_0)\,.$$

Now $\mathcal{V}$, being an open-and-closed ultrafilter in N^S with the countable intersection property, has the nonempty intersection, and thus there exists an $x \in \bigcap \mathcal{U}$. By (6) there exists an open-and-closed strongly zero-dimensional neighbourhood U of x. We have $U \in \mathcal{U}$ contrary to our assumption that $\mathcal{U}$ does not contain strongly zero-dimensional sets. This completes the proof of (7).

Let us take an open-and-closed set $U_0 \in \mathcal{U}$ with $\dim U_0 = 0$. The set U_0 is realcompact by (1) and thus it is N-compact (see [16], p. 478). The family $\mathcal{W} = \{U_0 \cap U\colon U \in \mathcal{U}\}$ is an open-and-closed ultrafilter in U_0 with the countable intersection property and hence $\emptyset \neq \bigcap \mathcal{W} \subset \bigcap \mathcal{U}$. This completes the proof that M is N-compact.

Remark 3. If we take in the above construction K instead of $N^{\aleph_1}$ and A instead of E, where K and A are as in Lemma 2, then we obtain a space M' which is perfectly normal, locally second-countable, N-compact, and satisfies $\dim M' > \operatorname{loc}\dim M' = 0$ (the space M' is a slight modification of Example 1). The proof of N-compactness of M' is analogous to the proof in Example 3 and reduces to the proof of N-compactness of K(²) (which follows by Mrówka's result [10] from the fact that K can be mapped continuously in a one-to-one way into the metrizable strongly zero-dimensional space $B(\aleph_1)$) and of realcompactness of M' (which follows from the fact that M' can be mapped in a one-to-one way into the space $B(\aleph_1) \times I$ (see [4] Exercise 3.11.B)). The remaining properties of M' can be proved in the same way as the properties of the space L in Example 1.

We are grateful to Professor R. Engelking for valuable discussions about the subject of this paper.

Added in proof.

(a) In the paper *A hereditarily normal strongly zero-dimensional space containing subspaces of arbitrarily large dimensional*, Fund. Math. (to appear) the authors have developed essentially the idea described in Section 3.

(b) E. Pol, Bull. Acad. Polon. Sci. 24 (1976), pp. 749–752 gave under CH an example of a locally compact perfectly normal space X_n with $\operatorname{loc}\dim X_n = 0$ and $\dim X_n > n$, where $n = 1, 2, \ldots$; some very strong examples of this kind, also under CH, were constructed recently by V. V. Fedorčuk, *On the dimension of hereditarily normal spaces* (to appear).

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(²) In fact, one can prove that K is strongly zero-dimensional.

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Addition and correction to the paper "On stability and products"

Fund. Math. 93 (1976), pp. 81–95

by

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In the paper quoted in the title the second part of Corollary 5.5 was formulated wrongly. Namely it should have the following form:

The class of all ω-stable theories for which α_T is finite is closed under finite droducts.

Now we shall show that "ω-stable" cannot be omitted. The notation and terminology are taken from [1].

Namely, let $\mathfrak{B} = \langle Q \cup (Q\times Q\times Q), W, C, D, R, \sim_n\rangle_{n\in\omega}$, where

Q is the set of rationals numbers,

W is a unary relation and $W(a)$ iff $a \in Q$,

C is a unary relation and $C(a)$ iff $a \notin Q$,

D is a ternary relation and $D(a,b,c)$ iff $W(a)$, $W(b)$ and $\exists q \in Q$, $c = \langle a, b, q\rangle$,

R is a ternary relation and $R(a,b,c)$ iff $D(a,b,c)$ and

$$\begin{cases} a<b \to \exists q \in Q\ c = \langle a,b,q\rangle \text{ and } q \text{ is a natural number,} \\ a\geqslant b \to \exists q \in Q\ c = \langle a,b,q\rangle \text{ and } q \neq 0, \end{cases}$$

$\sim_n$ are equivalence relations on Q with infinitely many classes and $\sim_{n+1}$ divides every equivalence class of $\sim_n$ into infinitely many equivalence classes of $\sim_{n+1}$. Moreover every equivalence class of $\sim_n$ is a dense linear ordering without endpoints (with the ordering taken from Q).

Note that we can define a formula which linearly orders W into type η.

Fact 1. For every $\mathfrak{A} \models \mathrm{Th}(\mathfrak{B})$ and every $p \in SA$ either $\operatorname{rank}(p) = 0$, either $\operatorname{rank}(p) = 1$, or $\operatorname{rank}(p) = \infty$.

Indeed, every type contains one of the following sets of formulas:

1) $\{x_0 = a\}$ for some $a \in A$,