Added in proof. Regarding Problem 3.11 Dr. Z. Grande has shown in a communication to the author that for a real-valued, relatively proper, connected function f on a connected, locally connected, hereditarily normal space X, it can be deduced from Theorems 3.3 and 3.6 with some work that f is weakly monotone relative to the set $\overline{S_c(f)}$. With a little modification In his argument the following result is obtained which may be compared with Theorem 3.10: if f is a real-valued, relatively proper, connected function on a connected, locally connected Hausdorff space X, then its restriction to the set $\overline{S_c(f)}$ is continuous, Morrey monotone and proper.

The following simplified version of an example communicated by Grande shows that Problem 5.10 does not have an affirmative answer when X is not complete. Let A be the set of rational numbers and $B = R-A$. Then $X = (A\times R)\cup(R\times B)$ is locally connected relative to the induced metric of R^2. The projection $f\big((x, y)\big) = x$, $(x, y)\in X$, is continuous and nowhere monotone, but no point (x, y) in the residual subset $R\times B$ of X is a limit point of the level $f^{-1}\{f\big((x, y)\big)\}$ along the arc $R\times\{y\}$ that is contained in X.

References

[1] G. Choquet, *Lectures on Analysis*, Vol. I, New York 1969.
[2] K. M. Garg, *On nowhere monotone functions*. I. *Derivates at a residual set*, Ann. Univ. Sci. Budapest. Eötvös Sect. Math. 5 (1962), pp. 173–177.
[3] — *On level sets of a continuous nowhere monotone function*, Fund. Math. 52 (1963), pp. 59–68.
[4] — *Monotonicity, continuity and levels of Darboux functions*, Colloq. Math. 28 (1973), pp. 91–103.
[5] — *On bilateral derivates and the derivative*, Trans. Amer. Math. Soc. 210 (1975), pp. 295–329.
[6] R. Hrycay, *Noncontinuous multifunctions*, Pacific J. Math. 35 (1970), pp. 141–154.
[7] — *Weakly Connected Functions*, Ph. D. Thesis, Univ. of Alberta 1971.
[8] K. Kuratowski, *Topology*, Vol. I, New York–London–Warszawa 1966.
[9] — *Topology*, Vol. II, New York–London–Warszawa 1968.
[10] J. S. Lipiński, *Une remarque sur la continuité et la connexité*, Colloq. Math. 19 (1968), pp. 251–253.
[11] — *On level sets of Darboux functions*, Fund. Math. 86 (1974), pp. 193–199.
[12] C. B. Morrey, Jr., *The topology of (Path) surfaces*, Amer. J. Math. 57 (1935), pp. 17–50.
[13] W. J. Pervin and N. Levine, *Connected mappings of Hausdorff spaces*, Proc. Amer. Math. Soc. 9 (1958), pp. 488–496.
[14] D. E. Sanderson, *Relations among some basic properties of non-continuous functions*, Duke Math. J. 35 (1968), pp. 407–414.
[15] W. Sierpiński, *Sur un problème concernant les ensembles mesurables superficiellement*, Fund. Math. 1 (1920), pp. 112–115.
[16] — *Sur l'ensemble des valeurs qu'une fonction continue prend une infinité non dénombrable de fois*, Fund. Math. 8 (1926), pp. 370–373.
[17] J. R. Walker, *Monotone Mappings and Decompositions*, Ph. D. Thesis, Syracuse Univ. 1970.
[18] G. T. Whyburn, *Analytic Topology*, Amer. Math. Soc. Colloq. Publ., Vol. 28, Amer. Math. Soc., New York 1942.
[19] — *Dynamic topology*, Amer. Math. Monthly 77 (1970), pp. 556–570.

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A perfectly normal locally metrizable non-paracompact space

by

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Abstract. We construct an example of a perfectly normal locally second — countable and non — paracompact space by a modification of a metrizable space.

The aim of this paper is to describe a construction which by a modification of a metric space yields a locally metrizable, perfect, collectionwise normal and non-paracompact space containing a locally countable non F_σ-set. An application in the dimension theory, given in [4], has been the motivation for such a construction.

1. Terminology and notation. We shall use the terminology of [1]. For an ordinal α we shall denote by $D(\alpha)$ the set of all ordinals less than α with the discrete topology and by $W(\alpha)$ the same set with the order topology. The symbol Lim stands for limit countable ordinals. A set $\Sigma\subset W(\omega_1)$ is called *stationary* if it intersects each closed, cofinal set in $W(\omega_1)$; equivalently (cf. [3], Appendix 1.5), if for each function $\varphi\colon \Sigma\to W(\omega_1)$ with $\varphi(\alpha)<\alpha$ there exists $\xi<\omega_1$ such that $|\varphi^{-1}(\xi)| = \aleph_1$. If M is a set and ϱ a metric on M, then $w(M,\varrho)$ denotes the weight of the metric space (M,ϱ) and $\bar{A}^\varrho = \{x\in M\colon \varrho(x, A) = 0\}$ denotes the closure of $A\subset M$ with respect to ϱ. The set of natural numbers is denoted by N, I denotes the unit real interval and $|M|$ stands for the cardinality of a set M.

2. The definition of X. Let X be a set and ϱ a metric on X such that $w(X,\varrho) = \aleph_1$. Suppose that for $\xi<\omega_1$ we have given sets X_ξ satisfying the following conditions (cf. [5], (3), (4)):

$$(1)\qquad X_1\subset\ldots\subset X_\xi\subset\ldots\subset X,\quad \bar{X}_\xi^\varrho = X_\xi,\quad w(X_\xi,\varrho)\leqslant\aleph_0,$$

$$(2)\qquad X = \bigcup_{\xi<\omega_1} X_\xi \quad\text{and, for } \xi\in\mathrm{Lim},\quad X_\xi = \overline{\bigcup_{\alpha<\xi} X_\alpha}^\varrho.$$

We can obtain such sets by taking $X_\xi = \overline{\{x_\alpha\colon \alpha<\xi\}}^\varrho$ for a set $\{x_\alpha\colon \alpha<\omega_1\}$ dense in the space (X,ϱ).

Let us introduce a topology in the set X taking as a base the sets $U\cap X_\xi$ where U is open with respect to ϱ and $\xi<\omega_1$. By open and closed sets in X we shall understand sets which are open or closed with respect to that topology.

Let us put for $x \in X$

(3) $$\varkappa(x) = \min\{\xi : x \in X_\xi\}.$$

It is easy to see that X is first-countable and that

(4) $$(x_n \to x) \equiv (\varrho(x_n, x) \to 0 \text{ and } \varkappa(x_n) \leqslant \varkappa(x) \text{ for almost all } n).$$

The topology of X is, by (4) and (1), the weak topology introduced by the functions $\mathrm{id}\colon X \to (X, \varrho)$ and $\varkappa\colon X \to W(\omega_1)$; in other words the function $x \to (x, \varkappa(x))$ maps the space X homeomorphically onto the graph of the function $\varkappa$ considered as a subspace of the product $(X, \varrho) \times W(\omega_1)$. It follows that

(5) $$X_\xi \underset{\mathrm{top}}{\subset} (X_\xi, \varrho) \times W(\xi+1)$$

and hence an open-and-closed set X_ξ is metrizable and separable.

EXAMPLE. Let $X = D(\omega_1)^N$ and let ϱ be the standard metric on X, i.e. $(X, \varrho) = B(\aleph_1)$ be the Baire space of weight $\aleph_1$ (see [1], Example 4.2.2). The sets $X_\xi = D(\xi)^N$ satisfy (1) and (2) and for $x \in X$ we have $\varkappa(x) = \min\{\alpha : \alpha > x(i), \text{ for } i \in N\}$. We shall consider X with the topology defined as above. For each $\xi \in \mathrm{Lim}$ let us choose a point $x_\xi \in X$ with $\varkappa(x_\xi) = \xi$ and put $E = \{x_\xi : \xi \in \mathrm{Lim}\}$ ([1]). The space E is homeomorphic to the graph of $\varkappa$ restricted to the set E, i.e.

$$E \underset{\mathrm{top}}{=} \{(x_\xi, \xi) : \xi \in \mathrm{Lim}\} \subset B(\aleph_1) \times W(\omega_1).$$

Notice that the topology of E is the supremum of the metric topology introduced by ϱ and the order topology induced by the relation $(x_\xi \prec x_\eta) \equiv (\xi < \eta)$ (compare with [1] Problem 3. F(c), or [3] Example 6.3).

3. Auxiliary lemmas. The following lemma can be derived easily from the Theorem 1 of [5] (cf. also [5] Remark 5). We shall give however a simple proof of it for the sake of completness.

LEMMA 1. *Let A be a subspace of X such that the set $\varkappa(A)$ is stationary. Then the space A is not discrete.*

Proof. Write $\Lambda = \varkappa(A) \cap \mathrm{Lim}$, choose for each $\lambda \in \Lambda$ a point $a_\lambda \in A$ with $\varkappa(a_\lambda) = \lambda$ and put $A_\lambda = \{a_\alpha : \alpha \in \Lambda, \alpha < \lambda\}$. First we shall prove that there exists $\lambda \in \Lambda$ with $\varrho(a_\lambda, A_\lambda) = 0$. Otherwise, we have $\varrho(a_\lambda, A_\lambda) > 0$, for $\lambda \in \Lambda$, i.e. $\Lambda = \bigcup_n \Lambda_n$, where $\Lambda_n = \{\lambda \in \Lambda : \varrho(a_\lambda, A_\lambda) \geqslant 1/n\}$. There exists $n \in N$ such that Λ_n is stationary. Since, by (2), $a_\lambda \in \overline{\bigcup_{\alpha<\lambda} X_\alpha}^{\varrho}$ we can choose for each $\lambda \in \Lambda_n$ an ordinal $\varphi(\lambda) < \lambda$ and a point $b_\lambda \in X_{\varphi(\lambda)}$ such that $\varrho(a_\lambda, b_\lambda) < 1/3n$. There exists $\xi < \omega_1$ with $|\varphi^{-1}(\xi)| = \aleph_1$, because Λ_n is stationary. Thus $\{b_\lambda : \lambda \in \varphi^{-1}(\xi)\} \subset X_\xi$ and, by (1), there exist $\alpha, \lambda \in \varphi^{-1}(\xi)$ such that $\alpha < \lambda$ and $\varrho(b_\alpha, b_\lambda) < 1/3n$. We have obtained $\varrho(a_\alpha, a_\lambda) < 1/n$ which is impossible, as $a_\alpha \in A_\lambda$ and $\lambda \in \Lambda_n$.

([1]) The space (E, ϱ) was investigated by A. H. Stone in [6] Section 5 as an example in Borel Theory.

It follows that for some $\lambda \in \Lambda$ there exist $\lambda_n < \lambda$ with $\varrho(a_{\lambda_n}, a_\lambda) \to 0$ which gives $a_{\lambda_n} \to a_\lambda$, by (4). The proof is completed.

Let us put for $A \subset X$

(6) $$R(A) = \bar{A}^{\varrho} \setminus \bar{A}.$$

In the sequel the key role will be played by the following

LEMMA 2. *For each $A \subset X$ the set $\varkappa(R(A))$ is not stationary.*

Proof. Suppose to the contrary that the set $\varkappa(R(A)) = \Sigma$ is stationary. For each $\lambda \in \Sigma$ let us choose

(7) $$x_\xi \in R(A) \quad \text{with} \quad \varkappa(x_\xi) = \xi,$$

and for $m \in N$

(8) $$a_\xi^m \in A \quad \text{with} \quad \varrho(a_\xi^m, x_\xi) < 1/m.$$

Let us put for $\xi \in \Sigma$

(9) $$\varphi(\xi) = \sup\{\varkappa(a_\xi^m) : m \in N\}.$$

We can easily define by the transfinite induction a closed, cofinal set $\Gamma \subset W(\omega_1)$ such that

(10) $$\text{if} \quad \xi \in \Gamma \cap \Sigma \text{ and } \xi < \lambda \in \Gamma \quad \text{then} \quad \varphi(\xi) \leqslant \lambda.$$

The set $\Lambda = \Gamma \cap \Sigma$ is stationary and hence, by Lemma 1 and (4), there exist $\lambda \in \Lambda$ and a sequence $(\lambda_m) \subset \Lambda$ such that $\lambda_m < \lambda$ and $\varrho(x_{\lambda_m}, x_\lambda) \to 0$. We have, by (8),

$$\varrho(a_{\lambda_m}^m, x_\lambda) \leqslant \varrho(x_{\lambda_m}, x_\lambda) + 1/m$$

and, by (9) and (10),

$$\varkappa(a_{\lambda_m}^m) \leqslant \varphi(\lambda_m) \leqslant \lambda = \varkappa(x_\lambda).$$

We have obtained, by (4), $a_{\lambda_m}^m \to x_\lambda$ and hence the contradiction $x_\lambda \in \bar{A} \cap R(A) = \emptyset$.

LEMMA 3. *Let $\Gamma \subset W(\omega_1)$ be a closed and cofinal set. Let us write $F = \varkappa^{-1}(\Gamma)$ and $G = X \setminus F$. Then*

(11) *G has a base σ-discrete in X;*
(12) *$F = f^{-1}(0)$ for a continuous function $f\colon X \to I$;*
(13) *we can assign to each set $L \subset G$ an open set $G(L) \supset L$ in such a way that*
 (i) *if $L' \subset L''$ then $G(L') \subset G(L'')$,*
 (ii) *$\overline{G(L)} \cap F = \bar{L} \cap F$.*

Proof. Let $\{\Sigma_s : s \in S\}$ be the family of all order components of the set $W(\omega_1) \setminus \Gamma$. Let us write $G_s = \varkappa^{-1}(\Sigma_s)$ and take for each $s \in S$ the ordinal μ_s such that $\mu_s + 1 = \min \Sigma_s$ (we assume that $0 \in \Gamma$). Let us put

$$G_{sm} = \{x \in G_s : \varrho(x, X_{\mu_s}) > 1/m\}.$$

Since for different $s, t \in S$ we have either $G_s \subset X_{\mu_t}$ or $G_t \subset X_{\mu_s}$ it follows that $\varrho(G_{sm}, G_{tm}) \geqslant 1/m$. Thus each family $\mathscr{G}_m = \{G_{sm} : s \in S\}$ is discrete in X. Since each

G_{sm} has a countable base, by (5), and $G = \bigcup_m \bigcup \mathscr{G}_m$ ($G_s = \bigcup_m G_{sm}$, because $G_s \cap X_{\mu_s} = \emptyset$) the proof of (11) is finished.

Let $\mathscr{B} = \bigcup_m \mathscr{B}_m$ be a base of G such that $\mathscr{B}_m$ is discrete in X and $\overline{U} \subset G$ fo each $U \in \mathscr{B}$. Let us put $U_k = \bigcup_{m \leqslant k} \bigcup \mathscr{B}_k$. Then $U_1 \subset U_2 \subset \ldots$, $G = \bigcup_k U_k$ and $\overline{U}_k \subset G$. Since, by (11) and Nagata-Smirnov Theorem, the space G is metrizable we can choose continuous functions f_k: $G \to I$ with $f_k^{-1}(0) = G \setminus U_k$. Since $\overline{U}_k \subset G$ we can extend f_k continuously over the whole of X assuming $f_k(x) = 0$ for $x \notin G$. The function $f = \sum_k 2^{-k} f_k$ satisfies (12).

For the proof of (13) let us put, for $x, y \in G$, $\sigma(x, y) = 0$ if x and y belong to the same set G_s and $\sigma(x, y) = 1$ otherwise. Since G_s is an open-and-closed subspace of G the pseudometric σ is continuous. Let us put $d(x, y) = \varrho(x, y) + \sigma(x, y)$ and assume

$$G(L) = \{x \in G\colon\ d(x, L) < f(x)\},$$

where f is such as in (12). It is obvious that (i) is satisfied. Let $x \in \overline{G(L)} \cap F$. Then there exists, by (4), a sequence $(x_m) \subset G(L)$ such that $\varrho(x_m, x) \to 0$ and $\varkappa(x_m) \leqslant \varkappa(x)$. For each $m \in N$ let us choose a point $y_m \in L$ with $d(x_m, y_m) < f(x_m)$. Since $\varrho(x_m, y_m) \leqslant d(x_m, y_m) < f(x_m)$ we have $\varrho(y_m, x) \to 0$. Let $\varkappa(x_m) \in \Sigma_s$. Since $\sigma(x_m, y_m) \leqslant d(x_m, y_m) < 1$ we have $\varkappa(y_m) \in \Sigma_s$ and hence $\varkappa(y_m) \leqslant \varkappa(x)$, because $\varkappa(x_m) \leqslant \varkappa(x) \in \Gamma$. From (4) we infer that $y_m \to x$ and thus $x \in \overline{L}$.

3. The properties of X. We shall prove in this section that X is perfectly normal and, for sufficiently complicated metric ϱ, it is not paracompact.

PROPOSITION 1. *The space X is perfectly normal.*

Proof. Let A_0 and A_1 be disjoint, closed subsets of X. There exists, by Lemma 2, a closed, cofinal set $\Gamma \subset W(\omega_1)$ such that

$$\Gamma \cap \varkappa(\boldsymbol{R}(A_0) \cup \boldsymbol{R}(A_1)) = \emptyset. \tag{14}$$

Let us put $F = \varkappa^{-1}(\Gamma)$, $G = X \setminus F$, $K_i = A_i \cap F$, $L_i = A_i \cap G$. Since, by Lemma 3 and Nagata-Smirnov Theorem, the space G is metrizable there exist open sets U_i, $i = 0, 1$, such that (see (13))

$$G(L_i) \supset U_i \supset L_i \quad \text{and} \quad U_0 \cap U_1 = \emptyset.$$

By (14) we have $(\overline{K}_i^{\varrho} \cap F) \setminus K_i \subset \boldsymbol{R}(A_i) \cap F = \emptyset$, thus K_0 and K_1 are separated with respect to the metric ϱ and hence we can find open sets W_i, $i = 0, 1$, such that

$$W_i \supset K_i \quad \text{and} \quad W_0 \cap W_1 = \emptyset,$$

We have (see (13), (ii))

$$V_0 = (U_0 \cup W_0) \setminus \overline{U}_1 \supset A_0,$$
$$V_1 = (U_1 \cup W_1) \setminus \overline{U}_0 \supset A_1,$$

and since $V_0 \cap V_1 = \emptyset$ the proof of normality of X is completed.

We shall prove that X is perfect. From (14) we infer that

$$X \setminus A_0 = (X \setminus \overline{A}_0^{\varrho}) \cup (G \setminus A_0).$$

The first member of the union is an F_σ-set with respect to ϱ and thus it is an F_σ-set in X; the second is an F_σ-set in X by (11) and (12) of Lemma 3. Hence $X \setminus A_0$ is an F_σ-set in X.

PROPOSITION 2. *If the set $\varkappa(X)$ is stationary (this is satisfied in the case considered in Example) then the space X is not paracompact.*

Proof. Let us choose for each $\xi \in \varkappa(X)$ a point $x_\xi \in X$ with $\varkappa(x_\xi) = \xi$. The open set X_α contains only countably many points of the set $A = \{x_\xi\colon\ \xi \in \varkappa(X)\}$ and thus A is locally countable in X. But, by Lemma 1, the space A is not σ-discrete and thus the space X cannot be paracompact.

4. **Remarks.** We shall establish some further properties of our construction.

Remark 1. By Theorem 1 of [5] the stationarity of $\varkappa(X)$ depends on the metric ϱ only; namely, it is equivalent to the property that the metric space (X, ϱ) cannot be expressed as the union of countably many locally separable subspaces. This is the case if (X, ϱ) is a complete space each nonempty open subspace of which has the weight $\aleph_1$ (see [7] Section 2).

Remark 2. *The space X is collectionwise normal.*

We sketch the proof. First notice that the following strengthening of Lemma 2 holds.

LEMMA 2′. *Let $\mathscr{F}$ be a discrete family of closed sets in X. Then the union $\bigcup \{\varkappa(\boldsymbol{R}(A))\colon\ A \in \mathscr{F}\}$ is not stationary.*

Let us put $\Sigma_A = \varkappa(\boldsymbol{R}(A))$ and let $x_A \in \boldsymbol{R}(A)$ satisfies $\varkappa(x_A) = \min \Sigma_A$. Using reasonings analogous to those in the proof of Lemma 2 we can prove that the set $\{\varkappa(x_A)\colon\ A \in \mathscr{F}\}$ is not stationary. Since, by Lemma 2, each set Σ_A is not stationary we conclude by Fodor's theorem ([2] Hilfssatz) that the union $\bigcup \{\Sigma_A\colon\ A \in \mathscr{F}\}$ is not stationary.

Our remark can be derived now from Lemma 2′ in the same way as Proposition 1 from Lemma 2 (we must use in addition the property (13), (i)).

Remark 3. Let ϱ be a complete metric on a set X and assume that each nonempty open set in (X, ϱ) has the weight $\aleph_1$ (cf. Remark 1). Let us choose for each $\xi \in \varkappa(X)$ a point $x_\xi \in X$ with $\varkappa(x_\xi) = \xi$ and put $A = \{x_\xi\colon\ \xi \in \varkappa(X)\}$. Then Proposition 2 can be strengthened in the following way.

PROPOSITION 2′. *The set A is not an F_σ-set in X (being locally countable in X).*

Suppose the contrary. Then $A = \bigcup_m A_m$, where A_m are closed subsets of X. By Lemma 2 we can find a closed, cofinal set $\Gamma \subset W(\omega_1)$ such that

$$\Gamma \cap \bigcup \{\varkappa(\boldsymbol{R}(A_m))\colon\ m \in N\} = \emptyset.$$

Write $F = \varkappa^{-1}(\Gamma)$, $A' = A \cap F$ and let us consider the metric spaces (A', ϱ). We shall show that this is an absolutely Borel space (2). We adopt the notation of the proof of Lemma 3. Let $F_m = \bigcup \{G_{sm}: s \in S\}$. Since G_{sm} is an F_σ-set and $\varrho(G_{sm}, G_{tm}) \geqslant 1/m$ for distinct s, t, we infer that $F = X \setminus \bigcup F_m$ is a G_δ-set in (X, ϱ). Thus (F, ϱ) is an absolutely Borel space and so is (A', ϱ), as A' is an F_σ-set in (F, ϱ). By Lemma 1 the space (A', ϱ) is not σ-discrete and thus by a Theorem of A. H. Stone ([6], Theorem 1) it must contain a Cantor set. This gives the contradiction, because separable subspaces of (A', ϱ) are countable (compare with [6], Sec. 5).

Remark 4. Let E be the space considered in the Example (Sec. 1). One can prove (see R. Pol, Comment. Math. 22 (1977)) that the product $E^{\aleph_0}$ is perfectly normal, while E is not paracompact.

(2) A metrizable space is absolutely Borel if it can be embedded as a Borel subspace in a completely metrizable space.

References

[1] R. Engelking, *General Topology*, Warszawa 1977.
[2] G. Fodor, *Eine Bemerkung zur Theorie der regressiven Funktionen*, Acta Sci. Math. (Szeged) 17 (1956), pp. 139–142.
[3] I. Juhász, *Cardinal functions in topology*, Math. Centre Tracts 34, Amsterdam 1971.
[4] E. Pol and R. Pol, *A hereditarily normal strongly zero-dimensional space with a subspace of positive dimension and an N-compact space of positive dimension*, Fund. Math. 97 (1977), pp. 43–50.
[5] R. Pol, *Note on decompositions of metrizable spaces I*, Fund. Math. 95 (1977), pp. 95–103.
[6] A. H. Stone, *On σ-discretness and Borel isomorphism*, Amer. J. Math. 85 (1963), pp. 655–666.
[7] — *Non-separable Borel sets II*, Gen. Top. and Appl. 2 (1972), pp. 249–270.

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A hereditarily normal strongly zero-dimensional space with a subspace of positive dimension and an N-compact space of positive dimension

by

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Abstract. In this paper we give a solution of an old Čech's problem on dimension by constructing a hereditarily normal strongly zero-dimensional space containing a subspace of positive dimension. We give also an example of an N-compact space of positive dimension.

The aim of this paper is to construct spaces with the properties mentioned in the title.

The problem of existence of a hereditarily normal space X containing a subspace with the covering dimension greater than the covering dimension of X is an old problem of Čech (see [2]; compare also [7] Appendix, [3], [11] Problem 11-14, [1] VII, Introduction). Recently, V. V. Filippov [6] showed that the existence of a Souslin Tree yields a space of this kind. Further examples, with many additional properties, were constructed by V. V. Fedorčuk [5]; he used, however, some additional set theoretic assumptions, too. The example we shall construct needs only the usual axioms for the set theory. It solves at the same time a problem on the local dimension raised by C. H. Dowker in [3].

The problem of existence of a closed subspace with the positive covering dimension in a product of countable discrete spaces appears in the natural way in the theory of N-compactness (see [12]). It was solved recently by S. Mrówka [10] (see also [13]). We give another example of this kind (it seems to us that it is simpler than the Mrówka's one).

1. Notation and terminology. Our terminology will follow [4]. We shall use the following notation: I denotes the closed real unit interval, Q stands for rationals of I, P — for irrationals of I and N — for natural numbers. For an ordinal α we shall denote by $D(\alpha)$ the set of all ordinals less than α with the discrete topology and by $W(\alpha)$ the same set with the order topology. The word "dimension" will denote the covering dimension dim (see [4], § 7.1); a space X with $\dim X = 0$ is called *strongly zero-dimensional.* We say that the *local dimension* of a space X is at most n (abbreviated $\operatorname{locdim} X \leqslant n$) if each point $x \in X$ has an open neighbour-