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Metrization of Moore spaces and generalized manifolds

by

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Abstract. Through the investigation of new mapping conditions, the authors are able to establish metrization theorems for certain locally compact, locally connected spaces. In particular, it is shown that: (1) each normal locally compact, locally connected Moore space is metrizable; and (2) each perfectly normal, subparacompact generalized manifold is metrizable.

The authors would like to dedicate these results to their teachers, Ben Fitzpatrick and D. R. Traylor.

1. Introduction. In this paper, the authors introduce new mapping conditions and investigate spaces which are the preimages of metric spaces under maps satisfying these conditions. As a consequence of this investigation, significant progress is made on two long outstanding questions in general topology concerning the metrization of locally compact, locally connected spaces.

In 1937, F. B. Jones showed in [13] that under the assumption of the continuum hypothesis each normal, separable Moore space (1) is metrizable. Since that time, Jones' "normal Moore space conjecture", i.e., the conjecture that each normal Moore space is metrizable, has been one of the most tantalizing open questions in general topology. Furthermore, other than R. H. Bing's result of 1951 in [6] that each collectionwise normal Moore space is metrizable, the only positive results on this particular problem have depended on various set theoretic assumptions. In fact, the work of Bing in [6] and [7], R. W. Heath in [12], J. H. Silver, and F. D. Tall in [24] and [25] has shown that the metrizability of normal, separable Moore spaces, as well as several related conjectures, are actually independent of set theory.

In Section 3, a positive result concerning the metrization of normal, locally compact Moore spaces is given which requires no set theoretic assumptions. B. Fitzpatrick and D. R. Traylor showed in [10] that if there exists a normal, separable, nonmetrizable Moore space, then there exists one that is also locally compact. Also, W. G. Fleissner has recently shown in [11] that it is consistent that each normal, locally compact Moore space be metrizable. Thus, it is now known that

(1) A Moore space is a developable T_3-space.

the metrizability of normal, locally compact Moore spaces is independent of set theory. However, the question has often been raised (Jones in [14] and Traylor in [26], for example) as to whether each normal, locally compact, locally connected Moore space is metrizable. This question seems to be a good candidate for yet another independence result since it follows from known results that it is consistent to claim the existence of a normal, complete, locally connected, nonmetrizable Moore space. But this is not the case, for in Theorem 3.4, the authors show, without any set theoretic assumptions other than the axiom of choice, that each normal, locally compact, locally connected Moore space is metrizable.

In [5] by Alexandroff and again in [27] by Wilder, the question has been raised as to whether each perfectly normal generalized manifold [2] is metrizable. To the authors' knowledge the only two substantial results pertaining to this question are: (1) each paracompact generalized manifold is metrizable (Smirnov's Metrization Theorem); and (2) each generalized manifold M such that M^2 is perfectly normal is metrizable (Zenor in [29]). Recently, the seemingly unrelated question as to whether each perfectly normal space is subparacompact has arisen in the study of abstract spaces. In Theorem 3.8 it is shown that each perfectly normal, subparacompact generalized manifold is metrizable.

2. Preliminary results and definitions. Throughout this paper, our spaces are Hausdorff. $\mathscr{M}$ will denote the class of metric spaces and $\mathscr{SM}$ will be the class of separable metric spaces. If $\mathscr{E}$ is a class of spaces, then $C(X, \mathscr{E})$ will denote the class of continuous functions with domain X and range in $\mathscr{E}$.

2.1. DEFINITION. If $\mathscr{E}$ is a class of spaces, then X is *$\mathscr{E}$-refinable* if, whenever $\mathscr{U}$ is an open cover of X, there is a member f of $C(X, \mathscr{E})$ so that the fibers of f refine $\mathscr{U}$. X is *$\mathscr{E}$-contractible* if there is a member f of $C(X, \mathscr{E})$ which is one-to-one.

In [1], $\mathscr{M}$-contractible spaces are called submetrizable spaces while in [3], [16], [17], and [23], an $\mathscr{M}$-contractible space is said to be contractible to a metric space. Obviously, an $\mathscr{E}$-contractible space is $\mathscr{E}$-refinable. In [30], it is shown that if $C(X, \mathscr{E}))$ determines the topology [3] on X and if $\mathscr{E}$ is hereditary and finitely productive, then X can be embedded as a closed subset in a product of members of $\mathscr{E}$. In particular then, any completely regular $\mathscr{M}$-refinable space is Dieudonné complete and any completely regular $\mathscr{SM}$-refinable space is realcompact.

2.2. THEOREM. *An $\mathscr{M}$-refinable M-space is paracompact.*

Proof. In [20], an M-space X is characterized as a space which admits a quasi-perfect member of $C(X, \mathscr{M})$; i.e., there is a closed map f taking X into a metric space with countably compact fibers. We will show that a closed countably compact subset of an $\mathscr{M}$-refinable space is compact. It will then follow that f is perfect and that X is paracompact.

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(2) A generalized manifold is a space that is locally Euclidean.

(3) The statement that $C(X, \mathcal{E})$ determines the topology on X means that $\{f^{-1}(U) \mid U$ open in the range of f, $f \in C(X, \mathcal{E})\}$ forms a subbasis for the topology on X.

Let H be a closed countably compact subset of X and suppose that H is not compact. Let $\mathscr{K}$ be a centered collection of closed subsets of H with no common part. Since X is $\mathscr{M}$-refinable, there is a member g of $C(X, \mathscr{M})$ whose fibers refine $\{X-K \mid K \in \mathscr{K}\}$. Since H is countably compact, $g(H)$ is compact and $\{g(K) \mid K \in \mathscr{K}\}$ is a centered family of closed subsets of $g(H)$. Thus, there is a point $x \in \bigcap \{g(K) \mid K \in \mathscr{K}\}$. This is a contradiction from which it follows that H is compact.

2.3. THEOREM. *X is $\mathscr{M}$-contractible ($\mathscr{SM}$-contractible; resp.) if and only if X is an $\mathscr{M}$-refinable ($\mathscr{SM}$-refinable; resp.) space with a G_δ-diagonal.*

Proof. Clearly, if X is $\mathscr{M}$-contractible or $\mathscr{SM}$-contractible, then X is $\mathscr{M}$-refinable or $\mathscr{SM}$-refinable, respectively; and also, X has a G_δ-diagonal. Suppose that X is $\mathscr{M}$-refinable ($\mathscr{SM}$-refinable; respectively) and X has a G_δ-diagonal. According to Ceder [9], there is a sequence $\mathscr{G}_1, \mathscr{G}_2, \ldots$ of open covers of X so that if $p \in X$, then $\bigcap_{i=1}^{\infty} \mathrm{St}(p, \mathscr{G}_i) = \{p\}$. For each i, let g_i be a function from X into the metric space M_i so that the fibers of g_i refine $\mathscr{G}_i$. Let $\varphi\colon X \to \prod_{i=1}^{\infty} M_i$ be defined by $\pi_i(\varphi(x)) = g_i(x)$ where π_i is the projection of $\prod_{j=1}^{\infty} M_j$ onto M_i. According to the Embedding Lemma (page 116, [15]), φ is continuous. Clearly, the g_i's distinguish points; and so, φ is one-to-one.

2.4. COROLLARY. *X is metrizable if and only if X is an $\mathscr{M}$-refinable M-space with a G_δ-diagonal.*

Proof. In [21], Okuyama shows that a paracompact M-space with a G_δ-diagonal is metrizable.

Recall that, according to Burke [8], a space X is subparacompact if whenever $\mathscr{U}$ is an open cover of X there is a sequence $\mathscr{F}_1, \mathscr{F}_2, \ldots$ of discrete collections of closed sets such that $\bigcup_{i=1}^{\infty} \mathscr{F}_i$ covers X and refines $\mathscr{U}$. Subparacompact spaces were called F_σ-screenable spaces by McAuley in [18].

2.5. THEOREM. *If X is a perfectly normal subparacompact space such that $|X| \leqslant c$, then X is $\mathscr{SM}$-refinable.*

Proof. Let $\mathscr{U}$ be an open cover of X and let $\mathscr{F}_1, \mathscr{F}_2, \ldots$ be a sequence of discrete collections of closed subsets of X such that $\bigcup_{i=1}^{\infty} \mathscr{F}_i$ covers X and refines $\mathscr{U}$. Let $\mathscr{B}$ be a countable basis for $\mathscr{R}$ ($\mathscr{R}$ = real line). For each positive integer i, let f_i be a one-to-one function from $\mathscr{F}_i$ into $\mathscr{R}$. For each i and each $b \in \mathscr{B}$, let $H_{(i,b)} = \bigcup \{F \in \mathscr{F}_i \mid f_i(F) \in b\}$. For each i and each $b \in \mathscr{B}$, let $g_{(i,b)}$ be a continuous function from X into $[0, 1]$ so that $H_{i,b} = \{x \mid g_{(i,b)}(x) = 0\}$. For each (i, b), let $I_{(i,b)}$ denote a copy of $[0,1]$ and let $\pi_{(i,b)}$ denote the projection of $H = \prod \{I_{(i,b)} \mid i = 1, 2, \ldots; b \in \mathscr{B}\}$ onto $I_{(i,b)}$. Let φ denote the map taking X into H defined by $\pi_{(i,b)}(\varphi(x)) = g_{(i,b)}(x)$. Since each $g_{(i,b)}$ is continuous, φ is continuous (see the Embedding Lemma, page 116 of Kelley [15]).

Note that, for each $x \in X$, $\varphi^{-1}(\varphi(x)) = \bigcap\{g^{-1}_{(i,b)}(g_{(i,b)}(x))|\ i = 1, 2, \ldots; b \in \mathscr{B}\}$. To see that the fibers of φ refine $\mathscr{U}$, let $x \in X$ and let i denote an integer so that some member, F, of $\mathscr{F}_i$ contains x. Let $\mathscr{B}' = \{b \in \mathscr{B}|\ f_i(F) \in b\}$. Then we have that

$$F = \bigcap_{b \in \mathscr{B}'} \{x|\ g_{(i,b)}(x) = 0\}.$$

Thus, $\varphi^{-1}(\varphi(x)) \subset F$ and F is a subset of some member of $\mathscr{U}$.

From 2.5 and 2.3 we have:

2.6. THEOREM. *If X is a perfectly normal subparacompact space with a G_δ-diagonal such that $|X| \leqslant \mathfrak{c}$, then X is $\mathscr{S}\mathscr{M}$-contractible.*

2.7. Remark. The question has been raised by R. Hodel as to whether each Moore space has a point countable, point-separating open cover. The indexing technique employed in the proof of 2.5 can be used to show that if $|X| \leqslant \mathfrak{c}$ and X is a subparacompact space with a G_δ-diagonal, then X has a countable point-separating open cover. Hence, since each Moore space is subparacompact and has a G_δ-diagonal, the above question has a positive answer for Moore spaces with cardinality $\leqslant \mathfrak{c}$. For a discussion of this question with respect to higher cardinality, see [22].

2.8. EXAMPLE. The authors do not know if the condition that closed sets be G_δ-sets can be eliminated from the hypothesis of 2.5 or 2.6. However, the following example shows that in 2.6 the cardinality condition can not be relaxed even if $\mathscr{S}\mathscr{M}$-contractible is replaced by $\mathscr{M}$-contractible: Let H be the example H as described by Bing in [6]; however, let the set P described by Bing have cardinality greater than $\mathfrak{c}$. The space H is a perfectly normal σ-space (and hence, X is a subparacompact space with a G_δ-diagonal). Suppose that H is $\mathscr{M}$-contractible. Let f be a one-to-one continuous function from H into the metric space M. Since P has more than c points, $f(P)$ is not separable. Let P' be an uncountable subset of P so that $f(P')$ has no limit point. Since M is metric, there is a collection $\{U(p)|\ p \in P'\}$ of mutually exclusive open subsets of M such that $f(p) \in U(p)$ for each $p \in P'$. Then $\{f^{-1}(U(p))|\ p \in P'\}$ is an uncountable collection of mutually exclusive open sets, each member of which intersects P. But Bing shows that every collection of mutually exclusive open sets, each member of which intersects P, is countable. This contradicts the assumption that H is $\mathscr{M}$-contractible.

3. Locally connected and locally compact spaces.

3.1. LEMMA. *If X is a connected first countable space such that each point, p, is contained in a connected open set $U(p)$ with $|U(p)| \leqslant \mathfrak{c}$, then $|X| \leqslant \mathfrak{c}$.*

Proof. Let Ω denote the countable ordinals and let $q \in X$. Define $V\colon \Omega \to 2^X$ as follows:

(a) Let $V(1) = U(q)$.

(b) Having $V(\alpha)$ for $\alpha < \beta$, let $V(\beta) = \bigcup\{U(x)|\ x \in \mathrm{cl}(\bigcup_{\alpha<\beta} V_\alpha)$.

Then V satisfies the following properties:

(i) Each $V(\alpha)$ is open and connected.

(ii) If $\alpha < \beta$, then $\overline{V(\alpha)} \subset V(\beta)$.

(iii) For each α, $|V(\alpha)| \leqslant \mathfrak{c}$.

That (i) and (ii) hold is clear. To see that (iii) holds, suppose otherwise. Let β be the first ordinal so that $|V(\beta)| > \mathfrak{c}$. Then $|\bigcup_{\alpha<\beta} V(\alpha)| \leqslant \mathfrak{c}$. Since X is first countable, $\mathrm{cl}(\bigcup_{\alpha<\beta} V_\alpha) \leqslant \mathfrak{c}$.

Thus, $|\{U(x)|\ x \in \mathrm{cl}(\bigcup_{\alpha<\beta} V_\alpha)\}| \leqslant \mathfrak{c}$.

It follows that $|V(\beta)| \leqslant \mathfrak{c}$ which is a contradiction from which (iii) follows. Hence, $|X| \leqslant \mathfrak{c}$ since $X = \bigcup_{\alpha\in\Omega} V(\alpha)$.

3.2. THEOREM. *Suppose that X is perfectly normal and subparacompact. If X is locally connected and locally compact, then X is $\mathscr{S}\mathscr{M}$-refinable.*

Proof. According to 2.5 and 3.1, each component of X is $\mathscr{S}\mathscr{M}$-refinable. Since X is perfectly normal and locally compact, by [4], X has local cardinality $\leqslant \mathfrak{c}$. Thus, X is the union of a discrete collection of open $\mathscr{S}\mathscr{M}$-refinable subsets. It follows that X is $\mathscr{S}\mathscr{M}$-refinable.

3.3. THEOREM. *Suppose that X is a perfectly normal and subparacompact space with a G_δ-diagonal. If X is locally connected and locally compact, then X is metrizable.*

Proof. According to 3.2 and 2.4, X is $\mathscr{M}$-contractible; in particular then, X has a regular G_δ-diagonal (i.e., there is a sequence of open sets $U_1, U_2, \ldots$ in $X \times X$ so that $\Delta = \bigcap_{i=1}^{\infty} U_i = \bigcap_{i=1}^{\infty} \overline{U}_i$, where Δ is the diagonal of X). It now follows from Theorem 5 of [28] that X is metrizable.

In [6], Bing shows that every Moore space is subparacompact, thus, we have the following result mentioned in the introduction:

3.4. THEOREM. *Every normal, locally connected, locally compact Moore space is metrizable.*

3.5. LEMMA. *X is paracompact if and only if for every open cover $\mathscr{U}$ of X, there is a sequence $\mathscr{G}_1, \mathscr{G}_2, \ldots$ of open covers of X so that, if $x \in X$, then there are an n and an open set V containing x so that* $\mathrm{St}(V, \mathscr{G}_n)$ *is a subset of some member of $\mathscr{U}$* (Arhangel'skiĭ [2]).

3.6. THEOREM. *If X is a locally connected and locally peripherally compact $\mathscr{M}$-refinable space, then X is paracompact.*

Proof. Let $\mathscr{U}$ be an open cover of X. Let $\mathscr{V}$ be an open refinement of $\mathscr{U}$ so that the boundary of each member of $\mathscr{V}$ is compact. Let $\mathscr{W}$ be an open refinement of $\mathscr{V}$ so that the members of $\mathscr{W}$ are connected. Let $f \in C(X,\mathscr{M})$ so that the fibers of f refine $\mathscr{W}$. Then there is a normal sequence $\mathscr{G}_1, \mathscr{G}_2, \ldots$ of open covers of X so that for each x,

$$f^{-1}(f(x)) = \bigcap_{i=1}^{\infty} \mathrm{St}(x, \mathscr{G}_i) = \bigcap_{i=1}^{\infty} \mathrm{St}^2(x, \mathscr{G}_i).$$

For each i, let $\mathscr{G}_i'$ denote the collection of components of members of $\mathscr{G}_i$.

CLAIM 1. *$\mathscr{G}_1', \mathscr{G}_2', \ldots$ is a normal sequence.*

Proof. Let n be an integer. Let $x \in \mathscr{U} \in \mathscr{G}_n$ so that $\mathrm{St}^2(x, \mathscr{G}_{n+1}) \subset U$. It follows that $\mathrm{St}^2(x, \mathscr{G}_{n+1}')$ is a subset of the component of x in U and the component of x in U is in $\mathscr{G}_{n+1}'$.

CLAIM 2. *If $x \in X$, then there are an integer n and a $V \in \mathscr{V}$ so that* $\mathrm{St}^2(x, G_n') \subset V$

Proof. Let V be a member of $\mathscr{V}$ that contains $\bigcap_{n=1}^{\infty} \mathrm{St}(x, \mathscr{G}_n')$. Since $\mathscr{G}_1', \mathscr{G}_2', \ldots$ is a normal sequence,

$$\bigcap_{n=1}^{\infty} \mathrm{St}(x, \mathscr{G}_n') = \bigcap_{n=1}^{\infty} \mathrm{cl}\big(\mathrm{St}^2(x, \mathscr{G}_n')\big).$$

Suppose that, for each n,

$$\mathrm{cl}\big(\mathrm{St}^2(x, \mathscr{G}_n')\big) \cap (X - V) \neq \emptyset.$$

Since $\mathrm{cl}\big(\mathrm{St}^2(x, \mathscr{G}_n')\big)$ is connected, for each n,

$$\mathrm{cl}\big(\mathrm{St}^2(x, \mathscr{G}_n')\big) \cap \mathrm{Bd}(V) \neq \emptyset.$$

Since $\mathrm{Bd}(V)$ is compact, there is a point

$$q \in \bigcap_{n=1}^{\infty} \mathrm{cl}\big(\mathrm{St}^2(x, \mathscr{G}_n')\big) \cap \mathrm{Bd}(V).$$

Thus, q is a point in $[\bigcap_{n=1}^{\infty} (\mathrm{St}(x, \mathscr{G}_n'))] - V$ which contradicts the choice of V. That X is paracompact now follows from Lemma 3.5.

3.7. THEOREM. *Suppose that X is locally connected and locally compact. Then the following are equivalent*:

a. *X is metrizable,*

b. *X is $\mathscr{M}$-contractible,*

c. *X has a regular G_δ-diagonal,*

d. *X^2 is perfectly normal,*

e. *X is a perfectly normal and subparacompact space with a G_δ-diagonal.*

Proof. That (a) $\Rightarrow$ (b), (b) $\Rightarrow$ (d), (d) $\Rightarrow$ (c) and (a) $\Rightarrow$ (e) are obvious. That (c) $\Rightarrow$ (a) was done in Theorem 5 of [29]. That (e) $\Rightarrow$ (b) now follows from 3.2 and 2.4.

3.8. THEOREM. *Every perfectly normal subparacompact generalized manifold X is metrizable.*

Proof. From 3.2 and 3.6, it follows that X is paracompact. Hence, since X is locally metrizable, it follows from Smirnov's Metrization Theorem that X is metrizable.

Note. Ryszard Engelking has observed that the consistency of the Souslin continuum shows that the G_δ-diagonal property can not be removed from (e) of Theorem 3.7. Also, David Lutzer has kindly pointed out to the authors that, in [28], Worrell and Wicke show that if a space X locally has a base of countable order, X has a base of countable order. Since a θ-refinable space with a base of countable order is a Moore space, we have that a normal θ-refinable generalized manifold is metrizable.

Example B, p. 376, [19] is an example of a generalized manifold which is a Moore space (and hence subparacompact) but it is not metrizable.

Added in proof. (1) Assuming (CH), Rudin and Zenor have displayed an example of a perfectly normal nonmetrizable manifold [31].

(2) In [32], it is shown that every perfectly normal manifold is collectionwise normal with respect to Lindelöf sets. This provides an alternate proof to our Theorem 3.4.

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Metacompactness and the class MOBI

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Abstract. We construct examples of open compact mappings which are defined on metacompact complete Moore spaces. The examples show that the range of such a mapping can be either a Moore space which is not metacompact or a regular nondevelopable space. This solves some problems connected with the class MOBI.

Let MOBI_i denote the minimal class of T_i spaces containing all metric spaces and closed under open compact mappings (see [1, Definition 5.4], and [10]).

It is known that MOBI_2 contains hereditarily paracompact nonmetrizable spaces [13, Example 2] (a similar example is constructed in [2]) and nondevelopable nonmetacompact spaces [13, Example 3] (a similar example is constructed in [3]).

On the other hand, it is shown in [13, Theorem 2] implicitly (and independly in [10]) that the paracompact members of MOBI_3 are metrizable.

The purpose of this note is to construct a space Y in $\mathrm{MOBI}_{3\frac{1}{2}}$ which is neither metacompact nor developable.

More exactly, we shall construct an example of an open compact mapping of a completely regular metacompact developable Čech complete space X onto a completely regular space Y which is not a p-space and contains a closed subset which is not a G_δ-subset; moreover, Y has not a G_δ-diagonal (Examples 2.2 and 2.4).

From the results of the generalized base of countable order theory of H. H. Wicke and J. M. Worrell, Jr., it follows that Y is not θ-refinable (see [6] for simpler proofs and definitions); hence Y is neither metacompact nor subparacompact.

The example gives an answer to Problems 7.1,2,3,5,6 and, partially, to 12 (1) from [10] (see also Question 2 from [2]), and some questions from [3].

In the first section we present a general method of constructing open compact mappings. This method is used in the second section to construct various spaces in $\mathrm{MOBI}_{3\frac{1}{2}}$.

We shall use the terminology and notation from [7].

(1) It is easy to see that Problem 7.1 is equivalent to the negation of Problem 7.5.