residual and F_1 is 1st category. If, for every $\beta < \alpha$, x_β has been chosen such that $x_\beta \in A - \bigcup_{\gamma\leq\beta} F_\gamma$ and such that $x_\beta \neq x_\gamma$ for every $\gamma < \beta$, choose $x_\alpha \in A - \bigcup_{\beta<\alpha} F_\beta \cup \{x_\beta\}_{\beta<\alpha}$. Since $\bigcup_{\beta<\alpha} F_\beta$ is a countable union of first category sets, $\{x_\beta\}_{\beta<\alpha}$ a countable set, such a point x_α can be chosen and the induction is complete. Let $X = \{x_\alpha\}_{\alpha<\Omega}$ and let g be a map with domain X and image the real numbers. Let $f(x) = 0$ if $x \notin X$, $f(x) = g(x)$ if $x \in X$. Then f satisfies condition (N′) since if F is a closed set of measure 0, $F = F_{\alpha_0}$ for some $\alpha_0 < \Omega$, and $f(F)$ is an at most countable set. However f does not satisfy condition (N) since $f(A)$ is the real line.

This same example can be constructed if the continuum hypothesis is replaced by both of the following:

i) the union of fewer than τ sets of measure 0 is of measure 0,

ii) the union of fewer than τ sets of 1st category is of 1st category,

where τ is the power of the continuum.

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Accepté par la Rédaction le 14. 3. 1974

A comment on Balbes' representation theorem for distributive quasi-lattices

by

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This short note points out that Balbes' [1] representation theorem for distributive quasi-lattices may be proven from rather general considerations. (A distributive quasi-lattice is an algebra with two semi-lattice operations connected by the distributive laws.) Recall his Theorem 4 (rewritten slightly): *an algebra $\mathfrak{D} = \langle D; +, \cdot\rangle$ with two binary operations is a distributive quasi-lattice iff there are two families Y, X of sets closed to intersection and union, respectively, and two one-to-one correspondences ψ: $D \leftrightarrow Y$ and φ: $D \leftrightarrow X$ such that*

$$a+b = \psi^{-1}(\psi a \cup \psi b)\,,$$

$$a\cdot b = \varphi^{-1}(\varphi a \cap \varphi b)\,,$$

$$a\cdot(b+c) = a\cdot b + a\cdot c\,,$$

$$a+b\cdot c = (a+b)\cdot(a+c)\,,$$

for all $a, b, c \in D$. This is true because any semi-lattice is isomorphic to a family of subsets closed to intersection (or union) [2].

Since this representation theorem for semi-lattices is equivalent to saying that each semi-lattice is a subdirect power of the two-element semi-lattice, the technique of Theorem 4 is generalizable to any algebra

$$\mathfrak{A} = \langle A; f_1, \ldots, f_k, g_1, \ldots, g_m, h_1, \ldots, h_n, \ldots\rangle$$

of which each reduct

$$\mathfrak{A}_f = \langle A; f_1, \ldots, f_k\rangle\,,$$

$$\mathfrak{A}_g = \langle A; g_1, \ldots, g_m\rangle\,,$$

$$\ldots\ldots\ldots\ldots\ldots$$

is representable as a subdirect power.

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Accepté par la Rédaction le 14. 3. 1974

Function spaces with intervals as domain spaces

by

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Abstract. An example is given of a pseudo-complete, separable metric space Y such that the space of continuous functions from the closed unit interval into Y is of first category, where the topology on the function space may be taken to be any of the following: supremum metric, compact-open, pointwise convergence. Then conditions are given which guarantee that a function space with an interval as domain space and with compact-open topology be pseudo-complete, and hence of second category.

A well-known theorem in topology and analysis says that the supremum metric on a function space is complete whenever the metric on the range space is complete (the converse is also true). In this paper we take a particular space — the closed unit interval I — and consider the general question as to what "complete-type" properties can one obtain on a function space with domain space I when the property of completeness on the range space is relaxed. An example is given showing that even if the range space is a pseudo-complete, separable metric space, with no further conditions the function space with domain space I may be of first category — far from complete. However, we then give certain conditions on the range space (which do not imply completeness) insuring that the function space with I as domain space be pseudo-complete, and hence of second category.

1. Basic definitions. A subset of the topological space X is *of first category in* X provided that it can be written as the countable union of nowhere dense subsets of X (i.e., subsets of X whose closures have no interior points). If a subset of X is not of first category in X, then it is *of second category in* X. A space is *of first category* (*second category*, respectively) if it is of first category (second category, respectively) in itself. A space having the property that every open subspace is of second category is called a *Baire space*.

The Baire Category Theorem says that every complete metric space is a Baire space. In some cases one needs to have a complete space only to use such a theorem as the Baire Category Theorem, so that a natural question is whether one may weaken the completeness property on the range space and still retain some generalization of completeness, such as