On some examples of monostratic λ-dendroids

by

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Abstract. A λ-dendroid X is said to be *monostratic* if it has no non-trivial upper semi-continuous decomposition into continua with an arcwise connected decomposition space. In the paper an example is described of a monostratic λ-dendroid N such that the set of all its terminal points is nowhere dense (a negative answer to the question asked in [6], p. 367) and such that there is an open mapping f of it onto an arc (a negative answer to the question asked in [5], p. 340). Moreover, we show an example of a λ-dendroid L such that it contains no non-degenerate monostratic λ-dendroid and has a stratum with a non-void interior (a negative answer to the question asked in [7], cf. [8], Problem 16); and an example of a λ-dendroid R such that each its stratum has a void interior and such that there is an open mapping of it onto a monostratic λ-dendroid (a negative answer to Problem 18 in [8]).

I. A continuum means a compact connected metric space. A hereditarily decomposable and hereditarily unicoherent continuum is said to be a *λ-dendroid* (see [3]). An arcwise connected λ-dendroid is said to be a *dendroid*.

It is proved in [3], Corollary 2, p. 29, that for every λ-dendroid X there exists a unique decomposition $\mathfrak{D}$ of X (called the *canonical decomposition*):

$$X = \bigcup \{S_d\colon\ d \in \Delta(X)\}$$

such that

(i) $\mathfrak{D}$ is upper semi-continuous,

(ii) the elements S_d of $\mathfrak{D}$ are continua,

(iii) the decomposition space $\Delta(X)$ of X is a dendroid,

(iv) $\mathfrak{D}$ is the finest possible decomposition among all decompositions satisfying (i), (ii) and (iii).

The elements S_d of $\mathfrak{D}$ are called *strata* of X. A λ-dendroid X is called *monostratic* if it consists of only one stratum, i.e., if $\Delta(X)$ is a point (see [2], p. 933 and [4], p. 75, where the term "monostratiform" was used in the same sense).

A point p of a continuum X is said to be a *terminal point* of X if every irreducible continuum in X which contains p is irreducible from p to some point (see [13], p. 190). It is known (see [6], p. 367) that each monostratic λ-dendroid has uncountably many terminal points. The following question is asked in [6], p. 367 (cf. [8], Problem 9): It is true that, for any monostratic λ-dendroid X, the set of all terminal points of X is dense in X? The answer is negative; moreover, it can happen that the set of all terminal points of a monostratic λ-dendroid is nowhere dense. This can be seen from Example 1.

Recall that a continuous mapping f from a topological space X onto a topological space Y is said to be

(i) *monotone* if for any subcontinuum Q in Y the set $f^{-1}(Q)$ is a continuum in X (see [10], p. 123);

(ii) *open* if f maps every open set in X onto an open set in Y;

(iii) *confluent* if for every continuum $Q \subset Y$ and each component C of the inverse image $f^{-1}(Q)$ we have $f(C) = Q$ (see [1], p. 213).

It is known (see [1], p. 214) that any monotone mapping is confluent, and that any open mapping of a compact space is confluent. It is proved (see [5], Property 7, p. 340; see also [7], Proposition 19) that monostraticity of λ-dendroids is an invariant under monotone mappings. Prof. J. B. Fugate has asked the following question (see [5], p. 340; cf. [8], Problems 10 and 11): is monostraticity of λ-dendroids an invariant under confluent or open mappings? The answer in both cases is negative. We define in Example 2 an open mapping f from a monostratic λ-dendroid N such that $f(N)$ is an arc. Hence, we have also a negative answer if f is confluent.

A λ-dendroid X is said to belong to the class $\mathfrak{L}$ if each stratum of X has a void interior (see [7]). It is known (see [7], Proposition 24) that if a λ-dendroid X is in the class $\mathfrak{L}$, then every monostratic λ-dendroid contained in X has a void interior. Dr. J. J. Charatonik has asked the following question (see [7]; cf. [8], Problem 16): does it follow that, if every monostratic λ-dendroid contained in a λ-dendroid X has a void interior, then X is in $\mathfrak{L}$? The answer is negative. It can be seen from Example 3.

In Example 4 we define an open mapping f from a λ-dendroid R belonging to the class $\mathfrak{L}$ such that $f(R)$ is a monostratic λ-dendroid. Therefore we see that an open image of a λ-dendroid of the class $\mathfrak{L}$ need not be in the class $\mathfrak{L}$. This is a negative answer to Problem 18 in [8].

Recall that a mapping f from a topological space X to a topological space Y is said to be a *local homeomorphism* if for every point $x \in X$ there exists a neighborhood U of x such that $f(U)$ is a neighborhood of $f(x)$ and such that f restricted to U is a homeomorphism between U and $f(U)$ (see [14], p. 199). It is proved (see [12], Corollary 10) that a local homeomorphism of a λ-dendroid is a homeomorphism. Therefore we get the following immediate corollaries.

(1) The monostraticity of λ-dendroids is an invariant under local homeomorphism (cf. [8], Problem 12).

(2) If a λ-dendroid X belongs to $\mathfrak{L}$ and f is a local homeomorphism defined on X, then $f(X)$ is also in the class $\mathfrak{L}$ (cf. [8], Problem 17).

The author is very much indebted to Dr. J. J. Charatonik, who contributed to these investigations.

II. In each example described in this section let (x, y, z) denote a point of the Euclidean 3-space E^3 having x, y and z as its rectangular coordinates.

EXAMPLE 1. We denote the straight line interval joining points u and v of E^3 by uv, and its length by $|u-v|$.

To describe the example we first define some three subsets A, B and C of the plane $z = 0$. To do this, let P be an arbitrary rectangle (lying in the plane $z = 0$) with the ratio of its sides equal to $1:3$. Denote the vertices of P by a, b, c, d in such a way that ab, bc, cd, da are sides of P and $|a-b| = |c-d| = 3|b-c| = 3|d-a|$. For $i = 1, 2$ take points a_i, $b_i \in ab$ and c_i, $d_i \in cd$ such that

$$|a-a_1| = |a_1-b_1| = |b_1-b| = 3|a_1-a_2| = 3|a_2-b_2| = 3|b_2-b_1|$$

and

$$|d-d_1| = |d_1-c_1| = |c_1-c| = 3|d_1-d_2| = 3|d_2-c_2| = 3|c_2-c_1|\,.$$

Further, take points $e \in ad$ and $e_1 \in a_1d_1$ such that $2|a-e| = |e-d|$ and $2|a_1-e_1| = |e_1-d_1|$ (see Fig. 1).

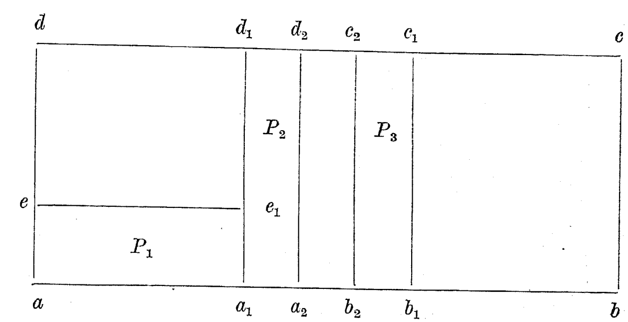


Fig. 1

Observe that points a, a_1, e_1, e are the vertices of some rectangle with the ratio of its sides equal to $1:3$. Denote this rectangle by P_1. Similarly, points a_1, a_2, d_2, d_1 and b_1, b_2, c_2, c_1 are the vertices of rectangles P_2 and P_3 which are congruent to P_1.

Remark that the choice of rectangles P_1, P_2 and P_3 for the rectangle P depends on the choice of one longer side of P and on the choice of one end-point of this side. Denote by (P, ab, a) the rectangle P with the distinguished side ab and with the distinguished end-point a of ab.

Let $\mathfrak{R}$ be the family of all triples (P, ab, a) described above. We define a five-valued function $\Phi\colon \mathfrak{R} \to \mathfrak{R}$ which assigns to a triple (P, ab, a) five triples (elements of $\mathfrak{R}$) as follows:

$$\Phi\big((P, ab, a)\big)$$
$$= \{(P_1, aa_1, a), (P_2, a_1d_1, a_1), (P_2, a_1d_1, d_1), (P_3, b_1c_1, b_1), (P_3, b_1c_1, c_1)\}.$$

As usual in the theory of multi-valued functions we define

$$(*) \qquad \Phi(\mathcal{A}) = \bigcup \{\Phi(A)\colon A \in \mathcal{A}\}$$

for every subset $\mathcal{A}$ of $\mathfrak{R}$.

Put

$$\Phi^0\big((P, ab, a)\big) = (P, ab, a)$$

and

$$\Phi^{n+1}\big((P, ab, a)\big) = \Phi\Big(\Phi^n\big((P, ab, a)\big)\Big).$$

Thus, in particular, Φ^n is a multi-valued function from $\mathfrak{R}$ into $\mathfrak{R}$ which assigns to a triple (P, ab, a) 5^n triples which are elements of $\mathfrak{R}$.

Now we define on $\mathfrak{R}$ three multi-valued functions whose values are some closed subsets of the plane $z = 0$. Namely, let

$$\Psi\big((P, ab, a)\big) = a_1b_1 \subset P,$$

$$\Omega\big((P, ab, a)\big) = ab \subset P, \quad \text{and}$$

$$\Sigma\big((P, ab, a)\big) = \{a, b\} \subset P.$$

If $\mathcal{A}$ is a set of triples which is contained in $\mathfrak{R}$, then $\Psi(\mathcal{A})$, $\Omega(\mathcal{A})$ and $\Sigma(\mathcal{A})$ are defined in the same way as it was done for $\Phi(\mathcal{A})$ by $(*)$.

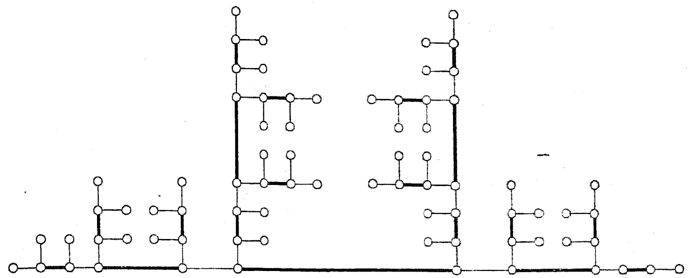
Now take in the plane $z = 0$ the rectangle Q with vertices $p = (0, 0, 0)$, $q = (1, 0, 0)$, $r = (1, \frac{1}{3}, 0)$, $s = (0, \frac{1}{3}, 0)$ and the subset $\mathcal{B}$ contained in the family of triples $\mathfrak{R}$ defined as follows:

$$\mathcal{B} = \bigcup_{n=0}^{\infty} \big[\Phi^n\big((Q, pq, p)\big) \cup \Phi^n\big((Q, pq, q)\big)\big].$$

Put

$$A = \Psi(\mathcal{B}), \quad B = \Omega(\mathcal{B}) \quad \text{and} \quad C = \Sigma(\mathcal{B}).$$

The set A is represented in Figure 2, in which the thickest lines represent the set B and the little circle the points of the set C.



Fig, 2

Observe that C is the set of end-points of the distinguished sides of all the rectangles which are considered in the construction. We have

(1.1) *The set C is a totally disconnected closed set.*

Now let us go into the 3-space and define

$$D = \{(x, y, 1) \in E^3\colon (x, y, 0) \in \overline{B \setminus A}\},$$

$$E = \{(x, y, z) \in E^3\colon (x, y, 0) \in C \text{ and } 0 \leqslant z \leqslant 1\}, \quad \text{and}$$

$$M = A \cup D \cup E.$$

Thus M is the union of the sets A and D and of all the straight line intervals joining points $(x, y, 0)$ of C with $(x, y, 1)$ respectively. These intervals are components of the set E.

Observe that the intersection of the set M and of the plane $y = 0$ is homeomorphic to the plane continuum H defined by the formula

$$H = \{(x, y, 0) \in E^3\colon 0 < x \leqslant 1 \text{ and } y = \sin(\pi/x)\} \cup$$
$$\cup \{(x, y, 0) \in E^3\colon 1 \leqslant x < 2 \text{ and } y = -\sin(\pi/(2-x))\} \cup$$
$$\cup \{(x, y, 0) \in E^3\colon x = 0 \text{ or } x = 2, \text{ and } -1 \leqslant y \leqslant 1\}.$$

Further,

(1.2) *Each component of the set E is one of two non-degenerate layers of some irreducible continuum contained in M and homeomorphic to H.*

It is easy to observe that

(1.3) *The set M is a hereditarily decomposable continuum.*

Since for each two points $x, y \in M$ there exists a unique subcontinuum of M irreducible between x and y, we have by Theorem 1.1 in [13], p. 179 that

(1.4) *The continuum M is hereditarily unicoherent.*

Thus, by (1.3) and (1.4),

(1.5) *M is a λ-dendroid.*

It follows from (1.2) and Theorem 5 in [3], p. 26, that

(1.6) *The strata S_d of the canonical decomposition of M have the following form:*

(i) *if $S_d \cap E \neq \emptyset$, then S_d is a component of the set E,*

(ii) *if $S_d \cap E = \emptyset$, then S_d is a one-point set.*

Let $u, v \in M$. We define the equivalence relation ϱ on M as follows: $u \varrho v$ if and only if either $u = v$ or the points u and v both belong to the same component of the set $A \cup D$.

It is easy to verify that

(1.7) *The decomposition of M into equivalence classes of the relation ϱ is upper semi-continuous, and the equivalence classes of the relation ϱ are continua.*

It follows from (1.7) by Theorem 4 in [3], p. 25, that

(1.8) *The decomposition space $N = M/\varrho$ is a λ-dendroid.*

Denote the canonical mapping from M onto M/ϱ by ψ. We infer by (1.6) and by the definition of ϱ that

(1.9) *The image under ψ of a stratum of M is contained in some stratum of N.*

Observe that since each component of E is a stratum of M by (1.6), and since the mapping ψ identifies the two end-points of each component of $A \cup D$ (these components are straight line intervals), we infer by (1.9) that the image under ψ of an arbitrary stratum of M is contained in the same stratum of N. Therefore

(1.10) *The λ-dendroid N is monostratic.*

Further, it is easy to observe that

(1.11) *The set $\psi(A \cup D)$ is nowhere dense in N.*

In fact, the set $A \cup D$ is closed in M; thus the set $\psi(A \cup D)$ is closed in N, and we have $N \setminus \psi(A \cup D) = \psi\big(M \setminus (A \cup D)\big)$. Since $M \setminus (A \cup D)$ is dense in M, we conclude that $\psi\big(M \setminus (A \cup D)\big)$ is dense in N, i.e., $N \setminus \psi(A \cup D)$ is dense in N; thus the set $\psi(A \cup D)$ is nowhere dense in N.

(1.12) *If a point of N is a terminal point of N, then it belongs to the set $\psi(A \cup D)$.*

Indeed, if $w \in N \setminus \psi(A \cup D)$, then, by the definition of ψ, $\psi^{-1}(w)$ is a one-point set and $\psi^{-1}(w) \in M \setminus (A \cup D)$. Therefore, there exists an arc J contained in $M \setminus (A \cup D)$ such that $\psi^{-1}(w) \in J$ and $\psi^{-1}(w)$ is neither of the two end-points of the arc J. Then the point w belongs to the arc $\psi(J)$ and it is not an end-point of $\psi(J)$; thus w is not a point of the irreducibility of the arc $\psi(J) \subset N$, i.e., w is not a terminal point of N.

We conclude, by (1.11) and (1.12), that the set of all terminal points of N is contained in a set which is nowhere dense in N. Since each subset of the set which is nowhere dense is also nowhere dense, we have

(1.13) *The set of all terminal points of the monostratic λ-dendroid N is nowhere dense in N.*

Example 2. Adopt the notation of Example 1. Define a projection mapping f^* of the λ-dendroid M onto the straight line interval $I = \{(0, 0, z)\colon\ 0 \leqslant z \leqslant 1\}$:

$$f^*\colon M \to I$$

by

$$f^*\big((x, y, z)\big) = (0, 0, z) \quad \text{for each point } (x, y, z) \text{ of } M.$$

Observe that

(2.1) *The mapping f^* is continuous.*

Since each non-degenerate equivalence class with respect to the relation ϱ is contained in the set $A \cup D$, and since $f^*(A) = \{(0, 0, 0)\}$ and $f^*(D) = \{(0, 0, 1)\}$, we have

(2.2) *The equivalence classes with respect to the relation ϱ are mapped under f^* onto points in I.*

It is easy to see that

(2.3) *The mapping f^* is such that for each open set U of N the set $f^*\big(\psi^{-1}(U)\big)$ is open in I.*

Define the mapping f of the λ-dendroid N onto I as follows:

$$(**) \qquad f(w) = f^*\big(\psi^{-1}(w)\big) \quad \text{for each point } w \in N.$$

Let G be an open subset of I. It follows from (2.1) that $(f^*)^{-1}(G)$ is open in M. Moreover, (2.2) implies that $(f^*)^{-1}(G)$ is such that if an equivalence class with respect to the relation ϱ has a non-empty intersection with $(f^*)^{-1}(G)$, then it is contained in $(f^*)^{-1}(G)$. Therefore $\psi\big((f^*)^{-1}(G)\big)$ is open in N, i.e., $f^{-1}(G)$ is open in N. This implies that

(2.4) *The mapping f is continuous.*

Further, let U be an open subset of N. It follows from (2.3) that $f^*\big(\psi^{-1}(U)\big)$ is open in I, i.e., $f(U)$ is open in I; thus

(2.5) *The mapping f is open.*

Since the λ-dendroid N is monostratic (cf. (1.10) here), we infer by (2.5) that

(2.6) *The open mapping f maps the monostratic λ-dendroid N onto the straight line interval I.*

Remark. Another proof of the openness of f runs as follows. It is easy to observe that the mapping f defined by $(**)$ is confluent and 0-dimensional (i.e., light, see [14], p. 130) onto a locally connected space.

Thus it follows from Corollary 5.2 in [11] that f is a light OM-mapping. Therefore f is open.

Example 3. Adopt the notation of Example 1. For each component K of the set A we take in the half-space $z \leqslant 0$ a square (a 2-cell) S_K such that the component K is a side of S_K and S_K is perpendicular to the plane $z = 0$. In this square S_K we take a continuum H_K such that H_K is the union of K and the side of S_K opposite to K, and a line lying in the square S_K which approximates both sides. Observe that H_K is homeomorphic to the continuum H described in Example 1.

Further, for each component K of the set D we take in the half-space $z \geqslant 1$ a square S_K such that the component K is a side of S_K and S_K is perpendicular to the plane $z = 1$. In this square S_K we take a continuum H_K such that as above, i.e., H_K is a continuum lying in S_K which is homeomorphic to H and which approximates K and the side of S_K opposite to K.

Put

$$L = M \cup \bigcup \{H_K\colon K \text{ is a component of } A \cup D\}.$$

It is easy to see by construction (in the same way as for the continuum M) that

(3.1) *The continuum L is a λ-dendroid.*

Moreover,

(3.2) *The λ-dendroid L contains no non-degenerate monostratic λ-dendroid.*

In fact, let Q' be an arbitrary non-degenerate subcontinuum of the λ-dendroid L. If the intersection $Q' \cap (L \setminus M)$ is non-empty, then there are points of $Q' \cap (L \setminus M)$ which are strata of Q', i.e., Q' is not monostratic. If the continuum Q' is contained in M, then either $Q' \cap (M \setminus E) \neq \emptyset$ or $Q' \subset E$ holds. If $Q' \cap (M \setminus E) \neq \emptyset$, then points of the set $Q' \cap (M \setminus E)$ are the strata of Q', i.e., Q' is not monostratic. If $Q' \subset E$, then Q' is an arc, i.e., Q' is not monostratic. Therefore the λ-dendroid L contains no non-degenerate monostratic λ-dendroid.

Obviously we have

(3.3) *The continuum M has a non-void interior in L.*

Now, we shall observe that

(3.4) *The continuum M is a stratum of L.*

Indeed, each straight line interval contained in E is a layer of some irreducible continuum which is contained in M and is homeomorphic to H (cf. (1.2)); similarly, each component K of $A \cup D$ is a layer of the irreducible continuum H_K described above. Moreover, they intersect in the sense that for each component of E there is a component of $A \cup D$ which intersects it and vice versa. Thus we infer by Theorem 5 in [3], p. 26, that the continuum M must be contained at some stratum of L. It is easy to verify that the inverse inclusion also holds.

It follows from (3.3) and (3.4) that

(3.5) *The λ-dendroid L is not contained in the class $\mathfrak{L}$.*

Thus, by (3.2), we have

(3.6) *The λ-dendroid L contains no non-degenerate monostratic λ-dendroid and it is not contained in the class $\mathfrak{L}$.*

Example 4. Take a plane monostratic λ-dendroid T (for example that one which is described in [4]) lying in the triangle which has points $(0,0,0)$, $(0,1,0)$ and $(0,\frac{1}{2},\frac{1}{2})$ as its vertices, and is such that the straight line interval joining points $(0,0,0)$ and $(0,1,0)$ is contained in T.

Let N_V be the irreducible continuum lying in the unit square with vertices $(0,0,0)$, $(0,1,0)$, $(1,1,0)$ and $(1,0,0)$ described by B. Knaster in [9], Section 2, p. 570 (see also [10], § 48, I, Example 5, p. 191).

We define a continuum R as follows:

$$R = \{(x,y,z) \in E^3\colon (x,y,0) \in N_V \text{ and } (0,y,z) \in T\},$$

i.e., R consists of the continuum N_V and of the homeomorphic images of T such that each of them has in common with N_V only exactly one maximal straight line interval contained in N_V, and each maximal straight line interval contained in N_V is also contained in exactly one monostratic λ-dendroid, which is a homeomorphic image of T.

In the same way as for M in Example 1 we conclude that

(4.1) *The continuum R is a λ-dendroid.*

It follows by construction that

(4.2) *Each stratum of R has a void interior in R, i.e., $R \in \mathfrak{L}$.*

We define a projection mapping f of R onto T as follows

$$f\big((x,y,z)\big) = (0,y,z) \quad \text{for each } (x,y,z) \in R.$$

It is easy to verify that

(4.3) *The mapping f is open.*

Therefore, we have by (4.2)

(4.4) *The open mapping f maps the λ-dendroid R belonging to the class $\mathfrak{L}$ onto the monostratic λ-dendroid T.*

References

[1] J. J. Charatonik, *Confluent mappings and unicoherence of continua*, Fund. Math. 56 (1964), pp. 213–220.

[2] — *Fixed point property for monotone mappings of hereditarily stratified λ-dendroids*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 16 (1968), pp. 931–936.

[3] J. J. Charatonik, *On decompositions of λ-dendroids*, Fund. Math. 67 (1970), pp. 15–30.
[4] — *An example of a monostratiform λ-dendroid*, Fund. Math. 67 (1970), pp. 75–87.
[5] — *Remarks on some class of continuous mappings of λ-dendroids*, Fund. Math. 67 (1970), pp. 337–344.
[6] — *Irreducible continua in monostratiform λ-dendroids*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 19 (1971), pp. 365–367.
[7] — *On decompositions of continua*, Fund. Math. 79 (1973), pp. 113–130.
[8] — *Some problems concerning monotone decompositions of continua*, Proceedings of the International Colloquium on Topology held in Keszthely, 1972, pp. 145–154.
[9] B. Knaster, *Un continu irréductible à décomposition continue en tranches*, Fund. Math. 25 (1935), pp. 568–577.
[10] K. Kuratowski, *Topology* II, New York–London–Warszawa 1968.
[11] A. Lelek and D. R. Read, *Compositions of confluent mappings and some other classes of functions*, Colloq. Math. 29 (1974), pp. 101–112.
[12] T. Maćkowiak, *A note on local homeomorphisms*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 21 (1973), pp. 854–858.
[13] H. C. Miller, *On unicoherent continua*, Trans. Amer. Math. Soc. 69 (1950), pp. 179–194.
[14] G. T. Whyburn, *Analytic Topology*, New York 1942.

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On a question of Borsuk concerning non-continuous retracts I*

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Abstract. We are concerned with the question as to whether an acyclic plane continuum is an almost continuous retract of a 2-cell. It is shown that there exists a non-locally connected almost continuous retract of the unit square. On the other hand, it is shown that no pseudo-arc is an almost continuous retract of a Peano continuum.

In [5] Stallings states a question due to Borsuk which suggests the possibility of proving fixed point theorems using non-continuous retracts. The question is whether an acyclic plane continuum is an almost continuous retract of a 2-cell. An affirmative answer to this question would settle the long-standing question of whether such continua have the fixed point property for continuous functions, since an almost continuous retract of an n-cell has the fixed point property for continuous functions.

In the present paper we obtain partial solutions to Borsuk's question. We show the existence of a non-locally connected almost continuous retract of the unit square. On the other hand, we show that no pseudo-arc is an almost continuous retract of a Peano continuum. The first result contrasts sharply to a result of Cornette [1] that a connectivity retract of a unicoherent Peano continuum is a unicoherent Peano continuum.

Preliminaries. Suppose $f: A \to B$. We make no distinction between f and its graph. If each open set containing f also contains a continuous function with domain A, then f is *almost continuous*. If $f|C$ is a connected set whenever C is a connected subset of A then f is a *connectivity function*. Every connectivity function on an n-cube, with $n > 1$, is almost continuous [3]. Now suppose $B \subset A$. We say that B is an *almost continuous* (*connectivity*) *retract* of A if there exists an almost continuous (connectivity) function $f: A \to B$ such that $f(b) = b$ for each b in B. The statement that the subset K of $A \times B$ is a *blocking set of* f means that K is closed, K contains no point of f and K intersects g whenever $g: A \to B$ is

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