potent, and so it is of the form $\sum_{i=1}^{m} x_i$, where $m$ is odd. All these operations are generated by the polynomial $s(x, y, z) = x + y + z$, and so $\mathcal{H}$ is the idempotent reduct of $\mathfrak{B}$.

By these three lemmas the theorem is proved.

We wish to add another fact about quasitrivial homogeneous operations, namely

**Theorem.** Every quasitrivial homogeneous operation on a finite set is generated (by composition) by the ternary discriminator

$$d(x, y, z) = \begin{cases} x & \text{if } x \neq y, \\ z & \text{if } z = y. \end{cases}$$

**Proof.** If the set $X$ is finite, then by [2] the algebra $\mathcal{H} = (X, d)$ is quasi-primal, which means that any operation preserving subalgebras and isomorphisms between subalgebras is a polynomial on $\mathcal{H}$. As any subset of $X$ forms a subalgebra of $\mathcal{H}$, that means that any quasitrivial homogeneous operation on $X$ is a polynomial on $\mathcal{H}$.

**References**


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Addition and correction to the paper "Diagonal algebras",

by

Jerry Plonka (Wrocław)

In this note we correct this mistake, namely the following is true:

Let $D_{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n} = (A_1 \times \cdots \times A_n; d(x_1, \ldots, x_n))$ be an $n$-dimensional diagonal algebra. Then the minimal cardinal number of sets of generators of $D_{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n}$ is equal to $\max(a_1, a_2, \ldots, a_n)$, where $a_0 = |A_j|$ ($j = 1, \ldots, n$).

**Proof.** If $G$ is a set of generators, it must contain at least one element of each coset in each direction (see [1]). Hence,

$$|G| \geq \max(a_1, a_2, \ldots, a_n).$$

We can assume without loss of generality that if $a_i < a_j$, then $A_i \subset A_j$.

Let us fix $a_0 = A_1 \cap A_2 \cap \cdots \cap A_n$. Then for any $a \in A_1 \cup A_2 \cup \cdots \cup A_n$, we define the $n$-tuple $(g_1, g_2, \ldots, g_n)$ as follows: $g_i = a$ if $a \in A_i$, and $g_i = a_0$ if $a \notin A_i$. Let $G_i$ be the set of all possible $n$-tuples $(g_1, g_2, \ldots, g_n)$. Then, by (i) from [1], $G_i$ is the set of generators of $D_{\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n}$ and

$$|G_i| = \max(a_1, a_2, \ldots, a_n).$$

Q.e.d.

Additionally, we show an interesting example of a diagonal algebra. We say that an algebra $\mathcal{H}_1 = (A; F_1)$ is a reduct of algebra $\mathcal{H}_2 = (A; F_2)$ if $F_1 \subset A(\mathcal{H}_2)$. We have

**Theorem.** For each $n \geq 2$ there exists an $n$-dimensional proper diagonal algebra which is a reduct of some abelian group.

**Proof.** Let $p_1, p_2, \ldots, p_n$ be a sequence of different prime integers. Let $G = (G_1, \cdots, G_n)$ be a diagonal group with the exponent $m = p_1 p_2 \cdots p_n$; i.e. $G$ satisfies $x^m = 1$ and does not satisfy any equality $x^k = 1$, where $k < m$. 

Let \( r_i = p_{i1}p_{i2} \cdots p_{ik} \) (\( i = 1, 2, \ldots, n \)). Moreover, let \( s_i \) be the smallest integer such that \( s_ir_i \equiv 1 \pmod{p_i} \). Then it is easy to check that the reduct \( (G; x_1^{s_1}, x_2^{s_2}, \ldots, x_n^{s_n}) \) is an \( n \)-dimensional proper diagonal algebra. Q.e.d.

References


A simpler set of axioms for polyadic algebras (1)

by

Charles Pinter (Lewisburg, Penns.)

Abstract. A new set of axioms for polyadic algebras is given. The new axioms are simple algebraic equations, having a clear algebraic content. From them are obtained some fresh insights into the structure of polyadic algebras.

1. Introduction. The purpose of this paper is to present a new, simpler set of axioms for polyadic algebras.

Polyadic algebras occupy a distinctive position in the scheme of algebraic logic, for they enjoy important properties which fail to hold for cylindric algebras, or even for polyadic algebras with equality. Notably, every polyadic algebra of infinite degree is representable in a very strong sense (see Daigneault and Monk [2]), and the class of all polyadic algebras has the amalgamation property (see J. Johnson [3]). Furthermore, polyadic algebras are, in a sense, richer structures than cylindric algebras, for they admit arbitrary cylindrifications as well as operations \( S(\tau) \) for arbitrary transformations \( \tau \).

It is unfortunate that, in one respect, polyadic algebras are less attractive to the mathematician than cylindric algebras: while the axioms for cylindric algebras are simple algebraic equations of a familiar kind, the axioms for polyadic algebras are more difficult to understand; two of them, in particular, fail to have a clear algebraic content. In our main result, we will show that these axioms may be replaced by simpler, more conventional algebraic equations. The new equations will then be used to obtain some fresh insights into the structure of polyadic algebras.

We assume the reader is acquainted with the basic papers, [3] and [4], of Halmos. In addition to the work of Halmos, we shall use an important result by P.-F. Jurie [6], which will be stated at the end of the next section.

2. Preliminaries. We shall use common set-theoretical notation and terminology. Small Greek letters will be used to denote transformations.

(1) The work reported in this paper was done while the author held an NSF Science Faculty Fellowship.