potent, and so it is of the form $\sum_{i=1}^{m} x_i$, where m is odd. All these operations are generated by the polynomial $s(x,y,z) = x+y+z$, and so $\mathfrak{A}$ is the idempotent reduct of $\mathfrak{B}$.

By these three lemmas the theorem is proved.

We wish to add another fact about quasitrivial homogeneous operations, namely

THEOREM. *Every quasitrivial homogeneous operation on a finite set is generated (by composition) by the ternary discriminator*

$$d(x,y,z) = \begin{cases} x & \text{if} \quad x \neq y, \\ z & \text{if} \quad x = y. \end{cases}$$

Proof. If the set X is finite, then by [2] the algebra $\mathfrak{A} = (X, d)$ is quasi-primal, which means that any operation preserving subalgebras and isomorphisms between subalgebras is a polynomial on $\mathfrak{A}$. As any subset of X forms a subalgebra of $\mathfrak{A}$, that means that any quasitrivial homogeneous operation on X is a polynomial on $\mathfrak{A}$.

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[4] H. Werner, *Produkte von Kongruenzklassengeometrien universeller Algebren*, Math. Zeitschr. 121 (171), pp. 111–140.

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Addition and correction to the paper "Diagonal algebras", Fund. Math. 58 (1966), pp. 309-321

by

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In the paper quoted in the title the second part of Theorem 2 was formulated wrongly. That was observed by Dawid Kelly (Ludwigshafen). In this note we correct this mistake, namely the following is true:

Let $\mathfrak{D}_{A_1,A_2,\ldots,A_n} = (A_1 \times \ldots \times A_n;\ d^*(x_1,\ldots,x_n))$ *be an n-dimensional diagonal algebra. Then the minimal cardinal number of sets of generators of* $\mathfrak{D}_{A_1,A_2,\ldots,A_n}$ *is equal to* $\max(a_1, a_2, \ldots, a_n)$, *where* $a_p = |A_p|$ $(p = 1, \ldots, n)$.

Proof. If G is a set of generators, it must contain at least one element of each coset in each direction (see [1]). Hence,

$$|G| \geqslant \max(a_1, a_2, \ldots, a_n).$$

We can assume without loss of generality that if $a_i \leqslant a_j$, then $A_i \subset A_j$. Let us fix $a_0 \in A_1 \cap A_2 \cap \ldots \cap A_n$. For any $a \in A_1 \cup A_2 \cup \ldots \cup A_n$ we define the n-tuple $[q_1, q_2, \ldots, q_n]$ as follows: $q_i = a$ if $a \in A_i$ and $q_i = a_0$ if $a \notin A_i$. Let G_0 be the set of all possible n-tuples $[q_1, q_2, \ldots, q_n]$. Then, by (i) from [1], G_0 is the set of generators of $\mathfrak{D}_{A_1,A_2,\ldots,A_n}$ and

$$|G_0| = \max(a_1, a_2, \ldots, a_n). \qquad \text{Q.e.d.}$$

Additionally we show an interesting example of a diagonal algebra. We say that an algebra $\mathfrak{A}_1 = (A; F_1)$ is a *reduct* of algebra $\mathfrak{A}_2 = (A; F_2)$ if $F_1 \subset A(\mathfrak{A}_2)$. We have

THEOREM. *For each* $n \geqslant 2$ *there exists an n-dimensional proper diagonal algebra which is a reduct of some abelian group.*

Proof. Let $p_1, p_2, \ldots, p_n$ be a sequence of different prime integers. Let $\mathfrak{G} = (G;\ \cdot,\ ^{-1})$ be an abelian group with the exponent $m = p_1 p_2 \ldots p_n$, i.e. $\mathfrak{G}$ satisfies $x^m = 1$ and does not satisfy any equality $x^k = 1$, where $k < m$.

Let $r_i = p_1 p_2 \dots p_{i-1} p_{i+1} \dots p_n$ $(i = 1, 2, \dots, n)$. Moreover, let s_i be the smallest integer such that $s_i r_i \equiv 1 (\bmod p_i)$. Then it is easy to check that the reduct $(G;\ x_1^{s_1 r_1} \cdot x_2^{s_2 r_2} \cdot \dots \cdot x_n^{s_n r_n})$ is an n-dimensional proper diagonal algebra. Q.e.d.

References

[1] J. Płonka, *Diagonal algebras*, Fund. Math. 58 (1966), pp. 309–321.

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A simpler set of axioms for polyadic algebras (1)

by

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Abstract. A new set of axioms for polyadic algebras is given. The new axioms are simple algebraic equations, having a clear algebraic content. From them are obtained some fresh insights into the structure of polyadic algebras.

1. Introduction. The purpose of this paper is to present a new, simpler set of axioms for polyadic algebras.

Polyadic algebras occupy a distinctive position in the scheme of algebraic logic, for they enjoy important properties which fail to hold for cylindric algebras, or even for polyadic algebras with equality. Notably, every polyadic algebra of infinite degree is representable in a very strong sense (see Daigneault and Monk [2]), and the class of all polyadic algebras has the amalgamation property (see J. Johnson [5]). Furthermore, polyadic algebras are, in a sense, richer structures than cylindric algebras, for they admit arbitrary cylindrifications as well as operations $S(\tau)$ for arbitrary transformations τ.

It is unfortunate that, in one respect, polyadic algebras are less attractive to the mathematician than cylindric algebras: while the axioms for cylindric algebras are simple algebraic equations of a familiar kind, the axioms for polyadic algebras are more difficult to understand; two of them, in particular, fail to have a clear algebraic content. In our main result, we will show that these axioms may be replaced by simpler, more conventional algebraic equations. The new equations will then be used to obtain some fresh insights into the structure of polyadic agebras.

We assume the reader is acquainted with the basic papers, [3] and [4], of Halmos. In addition to the work of Halmos, we shall use an important result by P.-F. Jurie [6], which will be stated at the end of the next section.

2. Preliminaries. We shall use common set-theoretical notation and terminology. Small Greek letters will be used to denote *transformations*,

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