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On some applications of infinite-dimensional manifolds to the theory of shape (¹)

by

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1. Introduction. In this paper we apply some recent results concerning the point-set topology of infinite-dimensional manifolds to the concept of "shape", as introduced by Borsuk in [5].

Let the Hilbert cube I^∞ be represented by $\prod_{i=1}^{\infty} I_i$, where each I_i is the closed interval $[-1,1]$, and let s denote $\prod_{i=1}^{\infty} I_i^0$, where each I_i^0 is the open interval $(-1,1)$. We let S denote the category whose objects are compacta in s and whose morphisms are fundamental equivalence classes of fundamental sequences (in I^∞) between these compacta. (This constitutes a subcategory of the *fundamental category* introduced in [5].) We let P denote the category whose objects are subsets of I^∞, with complements in I^∞ which are compacta in s, and whose morphisms are weak proper homotopy classes of proper maps (see Section 2 for a more precise definition).

The first result we establish enables us to translate problems concerning the shape of compacta to problems concerning contractible open subsets of I^∞.

THEOREM 1. *There is a category isomorphism T from P onto S such that $T(X) = I^\infty \backslash X$, for each object X in P.*

We also show that the shape of a compactum in s depends on (and determines) the homeomorphism type of its complement in I^∞.

THEOREM 2. *If X and Y are compacta in s, then X and Y have the same shape* (i.e. $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$) *iff $I^\infty \backslash X$ and $I^\infty \backslash Y$ are homeomorphic* ($\cong$).

This result enables us to give a short proof of the following Corollary concerning fundamental absolute retracts (abbreviated FAR), as intro-

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duced in [6]. The Corollary is originally due to Hyman [11], who used different methods to prove it.

Corollary ([11]). *If X is a* FAR, *then X is the intersection of a decreasing sequence of Hilbert cubes.*

2. General preliminaries. Concerning the fundamental category $\mathcal{S}$ we will use the results and notation from [5] and [6].

Concerning the proper category P we define a map (i.e. a continuous function) $f\colon X\to Y$ to be *proper* iff for each compactum $B\subset Y$ there exists a compactum $A\subset X$ such that $f(X\setminus A)\cap B=\emptyset$. (This is just a reformulation of the usual notion of a proper map.) Then maps $f, g\colon X\to Y$ are said to be *weakly properly homotopic* iff for each compactum $B\subset Y$ there exists a compactum $A\subset X$ and a homotopy $F=\{F_t\}\colon X\times I\to Y$ (where $I=[0,1]$) such that $F_0=f$, $F_1=g$, and $F((X\setminus A)\times I)\cap B=\emptyset$. (If, in fact, there exists a proper map $F\colon X\times I\to Y$ which satisfies $F_0=f$ and $F_1=g$, then we say that f and g are *properly homotopic.*) We write $f\sim g$ to indicate that f and g are weakly properly homotopic.

If $f\colon X\to Y$ and $g\colon Y\to X$ are proper maps such that $f\circ g\sim \mathrm{id}_Y$ (the identity on Y), then we say that X *weakly properly homotopically dominates* Y. If, additionally, $g\circ f\sim \mathrm{id}_X$, then we say that X and Y have the same *weak proper homotopy type.* If $f\colon X\to Y$ is a proper map, then we use $\{f\}$ to denote the class of proper maps of X into Y which are weakly properly homotopic to f.

It is easy to see that $\sim$ is an equivalence relation on the class of proper maps from a space X to a space Y. It is also easy to see that if $f, f'\colon X\to Y$ and $g, g'\colon X\to Y$ are proper maps such that $f\sim f'$ and $g\sim g'$, then $g\circ f\sim g'\circ f'$. This verifies that the composition of the equivalence classes $\{f\}$ and $\{g\}$ can be well defined by $\{g\circ f\}$. Thus we can define a category P whose objects are subsets of I^∞, with complements in I^∞ which are compacta in s, and whose morphisms are weak proper homotopy equivalence classes of proper maps.

3. Infinite-dimensional preliminaries. We will need the following definition, as introduced by Anderson in [1]. A closed set K in a space X is said to be a *Z-set* in X iff for each non-null, homotopically trivial open set U in X, $U\setminus K$ is non-null and homotopically trivial. From [1] we find that compacta in s are Z-sets in s and I^∞ and compacta in $I^\infty\setminus s$ are Z-sets in I^∞. More generally it is easy to see that if K is a Z-set in a space X and U is open in X, then $U\cap K$ is a Z-set in U.

We will need the notion of a *Q-manifold*, which is a separable metric space which has an open cover by sets homeomorphic to open subsets of I^∞. In [2] it is shown that if X is a Q-manifold, then $X\times I^\infty\cong X$. Thus for each Q-manifold X we have $X\cong X\times[0,1]$. From [9] it follows that if X is a Q-manifold, $U\subset X$ is open, and $K\subset U$ is a Z-set in U which is closed in X, then K is also a Z-set in X. This fact will be used in the proof of Theorem 2. The following results on Q-manifolds are also established in [9].

Lemma 3.1. *If X is any Q-manifold, then there is a locally-compact polyhedron P such that $X\times[0,1)\cong P\times I^\infty$.*

Lemma 3.2. *If X is a Q-manifold, P is a locally-compact polyhedron, and $\varphi\colon P\to X$ is a closed embedding such that $\varphi(P)$ is a Z-set in X, then there exists a closed embedding $h\colon P\times I^\infty\to X$ such that $h(x,(0,0,\ldots))=\varphi(x)$, for all $x\in P$, and $\mathrm{Bd}(h(P\times I^\infty))=h(P\times W^+)$.*

(For the representation $I^\infty=\prod_{i=1}^{\infty} I_i$ as given in Section 1 we use the notation $W^+=\{(x_i)\in I^\infty|\ x_1=1\}$ and $W^-=\{(x_i)\in I^\infty|\ x_1=-1\}$. We also use Bd for the topological boundary operator.)

Let X and Y be spaces and let $\mathcal{U}$ be an open cover of Y. Then functions $f, g\colon X\to Y$ are said to be *$\mathcal{U}$-close* provided that for each $x\in X$ there exists a $U\in\mathcal{U}$ such that $f(x), g(x)\in U$. A function $F\colon X\times I\to Y$ is said to be *limited by* $\mathcal{U}$ provided that for each $x\in X$ there exists a $U\in\mathcal{U}$ such that $F(\{x\}\times I)\subset U$.

If X is a metric space and $K\subset X$ is closed, then from [3] there exists an open cover $\mathcal{U}$ of $X\setminus K$ such that if $h\colon X\setminus K\to X\setminus K$ is any homeomorphism which is $\mathcal{U}$-close to $\mathrm{id}_{X|K}$, then h can be extended to a homeomorphism $\tilde{h}\colon X\to X$ which satisfies $\tilde{h}|K=\mathrm{id}_K$. Such a cover of $X\setminus K$ will be called *normal* (with respect to K).

We will need the following mapping replacement result which appears in [4].

Lemma 3.3. *Let X be a Q-manifold, $\mathcal{U}$ be an open cover of X, A be a closed subset of a locally-compact separable metric space Y, and let $f\colon Y\to X$ be a proper map such that $f|A$ is a homeomorphism of A onto a Z-set in X. Then there exists an embedding $g\colon Y\to X$ such that $g(Y)$ is a Z-set, $g|A=f|A$, and g is $\mathcal{U}$-close to f.*

We will also need the following version of Lemma 3.3 for Q-manifolds X which are $[0,1)$-*stable* (i.e. $X\cong X\times[0,1)$). The proof is given in [4].

Lemma 3.4. *Let X be a Q-manifold which satisfies $X\cong X\times[0,1)$, A be a closed subset of a locally-compact separable metric space Y, and let $f\colon Y\to X$ be a map such that $f|A$ is a homeomorphism of A onto a Z-set in X. Then there exists an embedding $g\colon Y\to X$ such that $g(Y)$ is a Z-set in X, $g|A=f|A$, and $g\simeq f$ (i.e. g is homotopic to f).*

(Note that if X is any Q-manifold, then

$$(X\times[0,1))\times[0,1)\cong(X\times[0,1])\times[0,1)\cong X\times[0,1).$$

We will also need the following homeomorphism extension theorem which is established in [4]. For its statement we will need the following notation: if $\mathfrak{U}$ is a cover of a set X, then $\mathrm{St}^0(\mathfrak{U}) = \mathfrak{U}$ and $\mathrm{St}^{n+1}(\mathfrak{U})$ consists of all sets of the form $\bigcup\{U \in \mathfrak{U} \mid U \cap K \neq \emptyset\}$, where K runs through $\mathrm{St}^n(\mathfrak{U})$.

LEMMA 3.5. *Let X be a Q-manifold, $\mathfrak{U}$ be an open cover of X, A be a locally-compact separable metric space, and let $f, g\colon A \to X$ be closed embeddings such that $f(A)$ and $g(A)$ are Z-sets in X and such that there exists a proper homotopy $F\colon A\times I \to X$ which is limited by $\mathfrak{U}$ and which satisfies $F_0 = f$, $F_1 = g$. Then there exists a homeomorphism $h\colon X \to X$ which satisfies $h\circ f = g$ and which is $\mathrm{St}^4(\mathfrak{U})$-close to id_X.*

We now combine these results to prove the following lemma which will be needed in Section 5.

LEMMA 3.6. *Let X and Y be Q-manifolds such that $X \cong X\times[0,1)$ and let $f\colon X\to Y$ be any continuous function. Then there exists an open embedding $g\colon X\to Y$ which satisfies $g \simeq f$.*

Proof. Let $h\colon Y \to Y\times[0,1]$ be any homeomorphism. It is clear that $h\circ f$ is homotopic to a continuous function $f'\colon X\to Y\times[0,1)$. Let $Y' = h^{-1}(Y\times[0,1))$ (which is an open subset of Y) and define $f'' = h^{-1}\circ f'$, which is a continuous function of X into Y' which is homotopic to f. Note also that $Y' \cong Y'\times[0,1)$.

We know that $X \cong P\times I^\infty$, for some locally-compact polyhedron P. Thus without loss of generality assume that $X = P\times I^\infty$. Using Lemma 3.4 there exists an embedding $\varphi\colon P\times\{(0,0,\ldots)\}\to Y'$ such that $\varphi(P\times\{(0,0,\ldots)\})$ is a Z-set and $\varphi\simeq f''|P\times\{(0,0,\ldots)\}$. Using Lemma 3.2 there exists an open embedding $g\colon P\times(I^\infty\backslash W^+)\to Y'$ such that $g(x,(0,0,\ldots)) = \varphi(x,(0,0,\ldots))$, for all $x\in P$. Let $r\colon P\times(I^\infty\backslash W^+)\to P\times\{(0,0,\ldots)\}$ be the retraction which satisfies $r(x,t) = (x,(0,0,\ldots))$, for all $(x,t)\in P\times(I^\infty\backslash W^+)$. Then we observe that $r\simeq \mathrm{id}_{P\times(I^\infty\backslash W^+)}$. We thus have

$$g = g\circ\mathrm{id}\simeq g\circ r = \varphi\circ r\simeq (f''|P\times\{(0,0,\ldots)\})\circ r = f''\circ r\simeq f''\circ\mathrm{id} = f''.$$

We will also need the following result.

LEMMA 3.7. *Let X be a Q-manifold and let $K\subset X$ be a Z-set. Then there exists an open set $U\subset X$ such that $K\subset U$ and $U\cong U\times[0,1)$.*

Proof. From [8] it follows that there exists a homeomorphism $h\colon X\to X\times[0,1]$ such that $h(K)\subset X\times\{\frac12\}$. Then $U = h^{-1}(X\times[0,1))$ fulfills our requirements.

A subset K of a space X is said to be *bicollared* provided that there exists an open embedding $h\colon K\times(-1,1)\to X$ such that $h(x,0) = x$, for all $x\in K$. We will need the following result, which appears in [12].

LEMMA 3.8. *Let $f\colon I^\infty\to I^\infty$ be an embedding such that $f(I^\infty)$ is bicollared. Then $I^\infty\backslash f(I^\infty) = A\cup B$, where A and B are disjoint sets such that $\mathrm{Cl}(A)\cap\mathrm{Cl}(B) = f(I^\infty)$ and $\mathrm{Cl}(A)\cong\mathrm{Cl}(B)\cong I^\infty$, where Cl denotes closure.*

(Note that $f(I^\infty)$ is a Z-set in each of $\mathrm{Cl}(A)$ and $\mathrm{Cl}(B)$.)

4. Proof of Theorem 1. We will need the following result in the proof of Theorem 1.

LEMMA 4.1. *If $X\subset I^\infty$ is a Z-set, then there exists a homotopy $F\colon I^\infty\times I\to I^\infty$ which satisfies the following properties:*

(1) $F_0 = \mathrm{id}$,

(2) *for each open neighborhood U of X there exists a $t_1\in(0,1)$ such that $F_t(U)\subset U$, for $0\leqslant t\leqslant t_1$,*

(3) *$F_t(I^\infty)\cap X = \emptyset$, for all $t\in(0,1]$.*

Proof. Using Lemma 3.5 we can assume that $X\subset W^+$. Then the construction of F is straightforward.

We will use the notation $\mathcal{F}(X)$ to denote the class of homotopies $F\colon I^\infty\times I\to I^\infty$ as described in Lemma 4.1.

We now construct an isomorphism T from P onto S. As indicated in the statement of Theorem 1 we let $T(X) = I^\infty\backslash X$, for each X in P. We now show how T assigns morphisms.

Let $\{f\}\colon X\to Y$ be a morphism in P, choose any $F\in\mathcal{F}(I^\infty\backslash X)$, and for each integer $k>0$ let $f_k = f\circ F_{1/k}$. We show that $\underline{f} = \{f_k, I^\infty\backslash X, I^\infty\backslash Y\}$ is a fundamental sequence. To see this let $V\subset I^\infty$ be an open neighborhood of $I^\infty\backslash Y$ and use the fact that f is proper to choose an open neighborhood $U\subset I^\infty$ of $I^\infty\backslash X$ which satisfies $f(U\cap X)\subset V$. Now choose $t_1\in(0,1)$ such that $F_t(U)\subset U$, for $0\leqslant t\leqslant t_1$. If k, l are positive integers such that $1/k, 1/l\leqslant t_1$, then $f_k|U = f\circ F_{1/k}|U\simeq f\circ F_{1/l}|U$ (in V) $= f_l|U$, as we wanted. Thus $\underline{f}$ is a fundamental sequence.

To see that $\underline{f}$ is uniquely defined in terms of F choose $F'\in\mathcal{F}(I^\infty\backslash X)$ and let $\underline{f}' = \{f\circ F'_{1/k}, I^\infty\backslash X, I^\infty\backslash Y\}$ be similarly defined. We show that $\underline{f}\simeq\underline{f}'$. Let $V\subset I^\infty$ be an open neighborhood of $I^\infty\backslash Y$ and choose $U\subset I^\infty$ an open neighborhood of $I^\infty\backslash X$ satisfying $f(U\cap X)\subset V$. Choose $t_1\in(0,1)$ such that $F_t(U)\subset U$ and $F'_t(U)\subset U$, for $0\leqslant t\leqslant t_1$. If k is a positive integer satisfying $1/k\leqslant t_1$ we clearly have $F_{1/k}|U\simeq F'_{1/k}|U$ (in U), with the image of the homotopy possibly intersecting $I^\infty\backslash X$. If this is the case we cannot use f to transfer this homotopy to one joining $f\circ F_{1/k}|U$ to $f\circ F'_{1/k}|U$.

To remedy this let $G\colon U\times I\to U$ be a homotopy which satisfies $G_0 = F_{1/k}|U$, $G_1 = F'_{1/k}|U$, and let $H\colon U\times I\to U$ be defined by $H_t = F_{t(1-t)}\circ G_t$. We note that $H_0 = F_{1/k}|U$, $H_1 = F'_{1/k}|U$, and for $0<t<1$ we have $H_t(U) = F_{t(1-t)}(G_t(U))\subset F_{t(1-t)}(U)\subset U\cap X$. Thus $f\circ H_t$ defines a homotopy which joins $f\circ F_{1/k}|U$ to $f\circ F'_{1/k}|U$. This means that $\underline{f}\simeq\underline{f}'$.

This gives a means of assigning to each proper map $f\colon X\to Y$ (where $I^\infty\setminus Y$ and $I^\infty\setminus X$ are compacta in s) a fundamental sequence $\underline{f}$ from $I^\infty\setminus X$ to $I^\infty\setminus Y$. In order to see that this assignment depends only on the weak proper homotopy class of f assume that $g\colon X\to Y$ is proper and $f\sim g$. We wish to show that if $F\in F(I^\infty\setminus X)$, $\underline{f}=\{f\circ F_{1/k}, I^\infty\setminus X, I^\infty\setminus Y\}$, $\underline{g}=\{g\circ F_{1/k}, I^\infty\setminus X, I^\infty\setminus Y\}$, then $\underline{f}\simeq\underline{g}$. To see this let $V\subset I^\infty$ be an open neighborhood of $I^\infty\setminus Y$ and choose a compact set $A\subset X$ and a homotopy $G\colon X\times I\to Y$ such that $G_0=f$, $G_1=g$, and $G\big((U\cap X)\times I\big)\subset V$, where $U=I^\infty\setminus A$. Let $t_1\in(0,1)$ be chosen so that $F_t(U)\subset U$, for $0\leqslant t\leqslant t_1$. Then for each positive integer k satisfying $1/k\leqslant t_1$ we find that $G_t\circ F_{1/k}|U$ gives a homotopy (in V) which joins $f\circ F_{1/k}|U$ to $g\circ F_{1/k}|U$ (in V), as we needed.

Thus to each morphism $\{f\}\colon X\to Y$ in P we have shown how to assign a unique morphism $[\underline{f}]\colon I^\infty\setminus X\to I^\infty\setminus Y$ in S, and we write $T(\{f\})=[\underline{f}]$. We now demonstrate that T is a functor and it is an isomorphism from P onto S. To show that $T(\mathrm{id})=\mathrm{id}$ choose an object X in P and $F\in F(I^\infty\setminus X)$, and let $\underline{f}=\{F_{1/k}, I^\infty\setminus X, I^\infty\setminus X\}$. We must show that $\underline{f}\simeq\underline{i}$, the identity fundamental sequence on $I^\infty\setminus X$. Choose an open set U containing $I^\infty\setminus X$ and $t_1\in(0,1)$ such that $F_t(U)\subset U$, for $0\leqslant t\leqslant t_1$. Clearly $F_{1/k}|U\simeq \mathrm{id}_{I^\infty}|U$ (in U), for all positive integers k satisfying $1/k\leqslant t_1$.

To show that T preserves compositions choose morphisms $\{f\}\colon X\to Y$ and $\{g\}\colon Y\to Z$ in P and choose $F\in F(I^\infty\setminus X)$, $G\in F(I^\infty\setminus Y)$. We must show that $\{g\circ f\circ F_{1/k}, I^\infty\setminus X, I^\infty\setminus Z\}\simeq\{g\circ G_{1/k}\circ f\circ F_{1/k}, I^\infty\setminus X, I^\infty\setminus Z\}$.

Choose open neighborhoods $U\subset I^\infty$ of $I^\infty\setminus X$, $V\subset I^\infty$ of $I^\infty\setminus Y$, and $W\subset I^\infty$ of $I^\infty|Z$ such that $f(U\cap X)\subset V$ and $g(V\cap Y)\subset W$. Also choose $t_1\in(0,1)$ such that $F_t(U)\subset U$ and $G_t(V)\subset V$, for $0\leqslant t\leqslant t_1$. Then for each positive k satisfying $1/k\leqslant t_1$ we have $g\circ G_{1/k}\circ f\circ F_{1/k}|U\simeq g\circ f\circ F_{1/k}|U$ (in W).

To show that T is an isomorphism we show first that if $\{f\}\colon X\to Y$ and $\{g\}\colon X\to Y$ are morphisms in P such that $T(\{f\})=T(\{g\})$, then $\{f\}=\{g\}$. Choose $F\in F(I^\infty\setminus X)$ and note that $\{f\circ F_{1/k}, I^\infty\setminus X, I^\infty\setminus Y\}\simeq\{g\circ F_{1/k}, I^\infty\setminus X, I^\infty\setminus Y\}$. Choose $B\subset Y$ a compact set and put $V=I^\infty\setminus B$. Then there exists an open neighborhood $U\subset I^\infty$ of $I^\infty\setminus X$ and an integer $n_1>0$ such that $k\geqslant n_1$ implies that $f\circ F_{1/k}|U\simeq g\circ F_{1/k}|U$ (in V) and $t\leqslant 1/n_1$ implies that $F_t(U)\subset U$. Clearly $f|U\cap X\simeq f\circ F_{1/k}|U\cap X$ (in V), for each $k\geqslant n_1$. Similarly $g|U\cap X\simeq g\circ F_{1/k}|U\cap X$, hence $f|U\cap X\simeq g|U\cap X$ (in V).

Choose an open neighborhood $U'\subset I^\infty$ of $I^\infty\setminus X$ such that $\mathrm{Cl}(U')\subset U$ and use the above remarks to obtain a homotopy $G\colon\big(\mathrm{Cl}(U')\cap X\big)\times I\to V$ which satisfies $G_0=f|\mathrm{Cl}(U')\cap X$ and $G_1=g|\mathrm{Cl}(U')\cap X$. Let $A=(\mathrm{Cl}(U')\cap X)\times I\cup\big((X\setminus\mathrm{Cl}(U'))\times\{0,1\}$, which is a closed subset of $X\times I$, and let $\alpha\colon A\to I^\infty$ be defined by $\alpha|\big(\mathrm{Cl}(U')\cap X\big)\times I=G$, $\alpha(x,0)=f(x)$, and $\alpha(x,1)=g(x)$, for all $x\in X\setminus\mathrm{Cl}(U')$. Extend α to a continuous function $\beta\colon X\times I\to I^\infty$. Then for $t\in I$ let $\gamma_t=F_{t(1-t)}\circ\beta_t$. We see that $\gamma\colon X\times I\to Y$ is a continuous function which satisfies $\gamma_0=f$, $\gamma_1=g$, and $\gamma\big(\mathrm{Cl}(U')\times I\big)\subset V$. This implies that $f\sim g$.

Now choose a morphism $[\underline{f}]\colon X\to Y$ in S. We must show that there exists a morphism $\{f\}\colon I^\infty\setminus X\to I^\infty\setminus Y$ in P such that $T(\{f\})=[\underline{f}]$. Using techniques like those used above we can choose a representative $\underline{f}=\{f_k, X, Y\}$ from the class $[\underline{f}]$ such that $f_k(I^\infty)\cap Y=\emptyset$, for all $k>0$. Choose a sequence $\{U_i\}_{i=1}^\infty$ of open sets in I^∞ such that $X=\bigcap_{i=1}^\infty U_i$ and $U_i\supset\mathrm{Cl}(U_{i+1})$, for all $i>0$. Also choose a sequence $\{V_i\}_{i=1}^\infty$ of open subsets of I^∞ such that $Y=\bigcap_{i=1}^\infty V_i$. We can pick a sequence $\{n_i\}_{i=1}^\infty$ of positive integers such that $n_1<n_2<\ldots$ and for each $i\geqslant 0$ and $k,l\geqslant n_i$, we have $f_k|\mathrm{Cl}(U_{n_i})\simeq f_l|\mathrm{Cl}(U_{n_i})$ (in V_i).

Let $\varphi_i\colon I^\infty\to[0,1]$ be a continuous function such that $\varphi_i(x)=0$, for $x\in I^\infty\setminus U_{n_i}$, and $\varphi_i(x)=1$, for $x\in\mathrm{Cl}(U_{n_{i+1}})$. Let $F^i\colon\mathrm{Cl}(U_{n_i})\times I\to V_i$ be a homotopy such that $F_0^i=f_{n_i}|\mathrm{Cl}(U_{n_i})$ and $F_1^i=f_{n_{i+1}}|\mathrm{Cl}(U_{n_i})$. Using tricks similar to those already employed we can additionally require that $F^i\big(\mathrm{Cl}(U_{n_i})\times I\big)\cap Y=\emptyset$, for all $i>0$. Then define $f\colon I^\infty\setminus X\to I^\infty\setminus Y$ by $f(x)=f_{n_1}(x)$, for $x\in I^\infty\setminus U_{n_1}$, and $f(x)=F^i_{\varphi_i(x)}(x)$, for $x\in\mathrm{Cl}(U_{n_i})\setminus U_{n_{i+1}}$. It then follows that f is a proper map. It remains to be shown that $T(\{f\})=[\underline{f}]$.

To see this choose $F\in F(X)$ and note that $T(\{f\})=[\{f\circ F_{1/k}, X, Y\}]$. Thus we must show that $\underline{f}\simeq\{f\circ F_{1/k}, X, Y\}$. If V is an open neighborhood of Y, then we can choose i and $n\geqslant n_i$ such that $k,l\geqslant n_i$ implies that $f_k|U_{n_i}\simeq f_l|U_{n_i}$ (in V) and such that $0\leqslant t\leqslant 1/n$ implies that $F_t(U_{n_i})\subset U_{n_i}$. If we can show that $k\geqslant n$ implies that $f_k|U_{n_i}\simeq f\circ F_{1/k}|U_{n_i}$ (in V), then we will be done. For such a fixed $k\geqslant n$ we have $F_{1/k}(U_{n_i})\subset\mathrm{Cl}(U_{n_i})\setminus U_{n_j}$, for some $j>i$. We can now use a finite induction to conclude that $f|F_{1/k}(U_{n_i})\simeq f_{n_i}|F_{1/k}(U_{n_i})$ (in V). In order to see the induction define functions $g_l\colon\mathrm{Cl}(U_{n_i})\setminus U_{n_j}\to V$, for $i\leqslant l\leqslant j$, as follows:

$$g_l=\begin{cases} f, & \text{on } \mathrm{Cl}(U_{n_l})\setminus U_{n_l},\\ f_{n_l}, & \text{on } \mathrm{Cl}(U_{n_l})\setminus U_{n_j}.\end{cases}$$

Note that $g_i=f_{n_i}|\mathrm{Cl}(U_{n_i})\setminus U_{n_j}$ and $g_j=f|\mathrm{Cl}(U_{n_i})\setminus U_{n_j}$. Thus all we need do is prove that $g_l\simeq g_{l+1}$ (in V), for $i\leqslant l\leqslant j-1$. To this end note that

$$g_{l+1}(x)=\begin{cases} f(x), & x\in\mathrm{Cl}(U_{n_i})\setminus U_{n_l},\\ F^l_{\varphi_l(x)}(x), & x\in\mathrm{Cl}(U_{n_l})\setminus U_{n_j}.\end{cases}$$

Define $h\colon (\mathrm{Cl}(U_{n_l})\setminus U_{n_j})\times I\to V$ by

$$h_t(x)=\begin{cases} f(x), & x\in \mathrm{Cl}(U_{n_l})\setminus U_{n_l},\\ F^l_{t\varphi_l(x)}(x), & x\in \mathrm{Cl}(U_{n_l})\setminus U_{n_j}.\end{cases}$$

Then it easily follows that $h_0=g_l$ and $h_1=g_{l+1}$. Hence $f\circ F_{1/k}|U_{n_l}\simeq f_k\circ F_{1/k}|U_{n_l}$ (in V) $\simeq f_k|U_{n_l}$ (in V), and we are done.

5. Relative fundamental sequences. We will need to define a relative notion of a fundamental sequence. Let A and B be subsets of a space X. Then a *relative fundamental sequence* $\underline{f}$ from A to B in X consists of an open set G containing A and a sequence $\{f_k\}_{k=1}^{\infty}$ of continuous functions, $f_k\colon G\to X$, such that the following properties are satisfied.

(1) f_k is homotopic to the inclusion map of G into X, for all $k\geqslant 1$ (we will incorrectly write this as $f_k\simeq \mathrm{id}_G$),

(2) for each open neighborhood V of B there exists an open neighborhood $U\subset G$ of A and an integer $n_1>0$ such that if $k,l\geqslant n_1$ are integers, then $f_k|U\simeq f_l|U$ (in V).

If $X=I^\infty$ and $\underline{f}=\{f_k,A,B\}$ is a fundamental sequence, then it is clear that $\{f_k|G,A,B,G\}$ is a relative fundamental sequence, for each open neighborhood G of A. If A,B,C are subsets of X and $\{f_k,A,B,G\}$, $\{g_k,B,C,H\}$ are relative fundamental sequences, then there exists an integer $n_1>0$ and an open set G' satisfying $A\subset G'\subset G$ such that $\{g_k\circ f_k|G',A,C,G'\}_{k=n_1}^{\infty}$ is a relative fundamental sequence. We will agree to identify relative fundamental sequences $\{f_k,A,B,G\}$ and $\{g_k,A,B,H\}$ provided that there exists an open neighborhood $G'\subset G\cap H$ of A such that $f_k|G'=g_k|G'$, for all but finitely many values of k. Thus composition is well defined.

If $\underline{f}=\{f_k,A,B,G\}$ and $\underline{g}=\{g_k,A,B,H\}$ are relative fundamental sequences then we write $\underline{f}\simeq\underline{g}$ iff for each open neighborhood V of B there exists an open neighborhood $U\subset G\cap H$ of A and an integer $n_1>0$ such that $f_k|U\simeq g_k|U$ (in V), for all integers $k\geqslant n_1$. In analogy with [5] we say that A *relatively fundamentally dominates* B (in X) iff there exist relative fundamental sequences $\underline{f}=\{f_k,A,B,G\}$ and $\underline{g}=\{g_k,B,A,H\}$ such that $\underline{f}\circ\underline{g}\simeq\underline{i}_B$, i.e. for each open neighborhood V of B there exists an open neighborhood $U\subset V\cap H$ of B and an integer $n_1>0$ such that $k\geqslant n_1$ implies that U is in the domain of $f_k\circ g_k$ and $f_k\circ g_k|U\simeq \mathrm{id}_U$ (in V). In like manner we can also define what is meant by *relative fundamental equivalence*.

We now establish a result which plays a key role in the inductive step in the proof of Theorem 2. We do it in two steps.

Lemma 5.1. *Let X be a Q-manifold and let A, B be compact Z-sets in X such that A relatively fundamentally dominates B in X. If W is an open subset of X containing B, then there exists an embedding $\varphi\colon A\to W$ such that $\varphi(A)$ is a Z-set, $\varphi\simeq\mathrm{id}_A$, and $\varphi(A)$ relatively fundamentally dominates B in W.*

Proof. Choose relative fundamental sequences $\underline{f}=\{f_k,A,B,G\}$ and $\underline{g}=\{g_k,B,A,H\}$ such that $\underline{f}\circ\underline{g}\simeq\underline{i}_B$. Choose an integer $n_1>0$ and an open set U such that $A\subset\overline{U}\subset G$, $f_k(U)\subset H\cap W$, and $f_k|U\simeq f_l|U$ (in $H\cap W$), for all $k,l\geqslant n_1$. Using Lemma 3.7 we may assume that $U\cong U\times[0,1)$. Now apply Lemma 3.6 to get an open embedding $\Phi\colon U\to W$ such that $\Phi\simeq f_{n_1}|U$ (in W). We can find an open neighborhood $V\subset H\cap W$ of B and an integer $n_2\geqslant n_1$ such that $g_k(V)\subset U$, for all $k\geqslant n_2$, $g_k|V\simeq g_l|V$ (in U), for all $k,l\geqslant n_2$, and $f_k\circ g_k|V\simeq\mathrm{id}_V$ (in $H\cap W$), for all $k\geqslant n_2$. Now let $\varphi=\Phi|A$, $G'=\Phi(U)$, $H'=V$, $f'_k=f_k\circ\Phi^{-1}$, and $g'_k=\Phi\circ g_k|V$, for all $k\geqslant n_2$.

To see that $\underline{f}'=\{f'_k,\varphi(A),B,G'\}$ is a relative fundamental sequence in W first note that for each $k\geqslant n_2$ we have $f'_k=f_k\circ\Phi^{-1}\simeq f_{n_1}\circ\Phi^{-1}$ (in W) $\simeq\Phi\circ\Phi^{-1}$ (in W) $=\mathrm{id}_{G'}$. Now let $V'\subset W$ be an open neighborhood of B and choose an open neighborhood $U'\subset U$ of A and an integer $n_3\geqslant n_2$ such that $f_k|U'\simeq f_l|U'$ (in V'), for all $k,l\geqslant n_3$. Then $\Phi(U')$ is an open set in W containing $\varphi(A)$ such that $f'_k|\Phi(U')\simeq f'_l|\Phi(U')$ (in V'), for all $k,l\geqslant n_3$.

To see that $\underline{g}'=\{g'_k,B,\varphi(A),H'\}$ is a relative fundamental sequence in W we have $g'_k=\Phi\circ(g_k|V)\simeq f_k\circ(g_k|V)$ (in W) $\simeq\mathrm{id}_V$ (in W), for all $k\geqslant n_2$. Now let U' be an open set in W containing $\varphi(A)$ and choose an integer $n_3\geqslant n_2$ and an open set $V'\subset V$ containing B such that $g_k(V')\subset\Phi^{-1}(U'\cap\Phi(U))$, for all $k\geqslant n_3$, and $g_k|V'\simeq g_l|V'$ (in $\Phi^{-1}(U'\cap\Phi(U))$), for all $k,l\geqslant n_3$. Then it follows that $g'_k|V'\simeq g'_l|V'$ (in U'), for all $k,l\geqslant n_3$.

To see that $\underline{f}'\circ\underline{g}'\simeq\underline{i}_B$ choose an open neighborhood $V'\subset W$ of B. Now choose an open neighborhood $V''\subset V'\cap V$ of B and an integer $n_3\geqslant n_2$ such that $f_k\circ g_k|V''\simeq\mathrm{id}_{V''}$ (in V'), for all $k\geqslant n_3$. Then it easily follows that $f'_k\circ g'_k|V''\simeq\mathrm{id}_{V''}$ (in V'), for all $k\geqslant n_3$. Thus $\varphi(A)$ relatively fundamentally dominates B in W. It is clear that $\varphi=\Phi|A\simeq f_{n_1}|A\simeq\mathrm{id}_A$ and it follows from the remarks proceeding Lemma 3.1 that $\varphi(A)$ is a Z-set in X. Using a similar argument we can establish the following result.

Lemma 5.2. *Let X be a Q-manifold and let A, B be compact Z-sets in X such that A and B are relatively fundamentally equivalent in X. If W is an open subset of X containing B, then there exists an embedding $\varphi\colon A\to W$ such that $\varphi(A)$ is a Z-set, $\varphi\simeq\mathrm{id}_A$, and $\varphi(A)$ is relatively fundamentally equivalent to B (in W).*

6. Proof of Theorem 2. We note that if $I^\infty\backslash X \cong I^\infty\backslash Y$, then $I^\infty\backslash X$ has the same weak proper homotopy type as $I^\infty\backslash Y$, and we can thus use Theorem 1 to conclude that $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$.

On the other hand assume that $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$, where X and Y are compacta in s. We will inductively construct sequences $\{U_i\}_{i=1}^\infty$ and $\{V_i\}_{i=1}^\infty$ of open subsets of I^∞ and a sequence $\{h_i\}_{i=1}^\infty$ of homeomorphisms of I^∞ onto itself such that the following properties are satisfied.

(1) $$X = \bigcap_{i=1}^\infty U_i \quad \text{and} \quad U_{i+1} \subset U_i, \quad \text{for all } i > 0,$$

(2) $$Y = \bigcap_{i=1}^\infty V_i \quad \text{and} \quad V_{i+1} \subset V_i, \quad \text{for all } i > 0,$$

(3) $$h_{2i-1} \circ \ldots \circ h_1(X) \subset V_i, \quad \text{for all } i > 0,$$

(4) $$h_j | I^\infty\backslash V_i = \mathrm{id}, \quad \text{for all } j > 2i-1,$$

(5) $$h_{2i} \circ \ldots \circ h_1(U_i) \supset Y, \quad \text{for all } i > 0,$$

(6) $$h_j | I^\infty\backslash h_{2i} \circ \ldots \circ h_1(U_i) = \mathrm{id}, \quad \text{for all } j > 2i.$$

Before proceeding with the construction of these sequences we will show how to use them to construct our desired homeomorphism of $I^\infty\backslash X$ onto $I^\infty\backslash Y$.

For each $x \in I^\infty\backslash X$ we have $x \notin U_i$, for some $i > 0$. Thus $h_{2i} \circ \ldots \circ h_1(x) \notin h_{2i} \circ \ldots \circ h_1(U_i)$ and we therefore have $h_j \circ \ldots \circ h_1(x) = h_{2i} \circ \ldots \circ h_1(x)$, for all $j > 2i$. This means that $h(x) = \lim_{j\to\infty} h_j \circ \ldots \circ h_1(x)$ is defined, for all $x \in I^\infty\backslash X$. It follows from (5) above that $h(x) \in I^\infty\backslash Y$. Thus we have defined a function $h\colon I^\infty\backslash X \to I^\infty\backslash Y$. To prove that h is onto choose $y \in I^\infty\backslash Y$ and choose an integer $i > 0$ such that $y \notin V_i$. By (4) above we have $h_j(y) = y$, for all $j > 2i-1$. Put $x = (h_{2i-1} \circ \ldots \circ h_1)^{-1}(y)$ and note that (3) implies that $x \in I^\infty\backslash X$. Thus $h(x) = \lim_{j\to\infty} h_j \circ \ldots \circ h_1(x) = h_{2i-1} \circ \ldots \circ h_1(x) = y$, which implies that h is onto. To prove that h is one-to-one choose $x_1, x_2 \in I^\infty\backslash X$ such that $x_1 \neq x_2$ and an integer $i > 0$ such that $x_1, x_2 \in I^\infty\backslash U_i$. Thus $h_{2i} \circ \ldots \circ h_1(x_1)$, $h_{2i} \circ \ldots \circ h_1(x_2) \in I^\infty\backslash h_{2i} \circ \ldots \circ h_1(U_i)$. By (6) we have $h_j(h_{2i} \circ \ldots \circ h_1(x_1)) = h_{2i} \circ \ldots \circ h_1(x_1)$ and $h_j(h_{2i} \circ \ldots \circ h_1(x_2)) = h_{2i} \circ \ldots \circ h_1(x_2)$, for all $j > 2i$. Thus $h(x_1) = h_{2i} \circ \ldots \circ h_1(x_1)$ and $h(x_2) = h_{2i} \circ \ldots \circ h_1(x_2)$. Since $h_{2i} \circ \ldots \circ h_1$ is one-to-one we have $h(x_1) \neq h(x_2)$, as we needed. To prove that h is continuous choose $y \in I^\infty\backslash Y$ and an open neighborhood $V \subset I^\infty$ of y. Then there exists an open neighborhood $V' \subset V$ of y such that $V' \cap V_i = \emptyset$, for some integer $i > 0$. By (4) above this implies that $h^{-1}(V') = (h_{2i-1} \circ \ldots \circ h_1)^{-1}(V')$. Since $h_{2i-1} \circ \ldots \circ h_1$ is continuous this implies that $h^{-1}(V')$ is open, as we needed. Similarly one uses (6) to prove that h is an open map. Thus h is a homeomorphism of $I^\infty\backslash X$ onto $I^\infty\backslash Y$.

Now for the construction of the necessary sequences. We start by choosing $\{U_i'\}_{i=1}^\infty$ and $\{V_i'\}_{i=1}^\infty$ to be decreasing sequences of open subsets of I^∞ such that $X = \bigcap_{i=1}^\infty U_i'$ and $Y = \bigcap_{i=1}^\infty V_i'$. For any integer $n > 0$ let $\{a_i\}_{i=1}^n$, $\{b_i\}_{i=1}^n$ be increasing sequences of positive integers and let us choose $\{h_i\}_{i=1}^{2n}$ to be a sequence of homeomorphisms of I^∞ onto itself. Then we say that the triple $(\{a_i\}_{i=1}^n, \{b_i\}_{i=1}^n, \{h_i\}_{i=1}^{2n})$ satisfies property P_n provided that the following five conditions are met:

(i) $h_{2i-1} \circ \ldots \circ h_1(U'_{a_i}) \subset V'_{b_i}$, for $1 \leqslant i \leqslant n$,

(ii) $h_j | I^\infty\backslash V'_{b_i} = \mathrm{id}$, for $1 \leqslant i \leqslant n$ and $2i-1 < j \leqslant 2n$,

(iii) $h_{2i} \circ \ldots \circ h_1(U'_{a_i}) \supset V'_{b_{i+1}}$, for $1 \leqslant i < n$,

(iv) $h_j | I^\infty\backslash h_{2i} \circ \ldots \circ h_1(U'_{a_i}) = \mathrm{id}$, for $1 \leqslant i \leqslant n$ and $2i < j \leqslant 2n$,

(v) $h_{2n} \circ \ldots \circ h_1(X)$ is relatively fundamentally equivalent to Y (in $h_{2n} \circ \ldots \circ h_1(U'_{a_n})$).

We first show that there exist positive integers a_1, b_1 and homeomorphisms h_1, h_2 of I^∞ onto itself such that $(\{a_1\}, \{b_1\}, \{h_i\}_{i=1}^2)$ satisfies property P_1. Choose $b_1 = 1$ and use Lemma 5.2 to get an embedding $f_1\colon X \to V'_{b_1}$ such that $f_1(X)$ is a Z-set in V'_{b_1} and $f_1(X)$ is relatively fundamentally equivalent to Y (in V'_{b_1}). Then extend f_1 to a homeomorphism $h_1\colon I^\infty \to I^\infty$. Choose a_1 to be a positive integer large enough so that $U'_{a_1} \subset h_1^{-1}(V'_{b_1})$. Once more using Lemma 5.2 let $f_2\colon Y \to h_1(U'_{a_1})$ be an embedding such that $f_2 \simeq \mathrm{id}_Y$ (in V'_{b_1}), $f_2(Y)$ is a Z-set in V'_{b_1}, and $f_2(Y)$ is relatively fundamentally equivalent to $h_1(X)$ (in $h_1(U'_{a_1})$). Since $f_2 \simeq \mathrm{id}_Y$ (in V'_{b_1}) we can extend f_2 to a homeomorphism $f_2'\colon V'_{b_1} \to V'_{b_1}$ which in turn can be extended to a homeomorphism $f_2''\colon I^\infty \to I^\infty$ which satisfies $f_2'' | I^\infty\backslash V'_{b_1} = \mathrm{id}$. The construction of f_2' requires an application of Lemma 3.5, where f_2' is limited by an open cover of V'_{b_1} which is normal with respect to $I^\infty\backslash V'_{b_1}$. Then we put $h_2 = (f_2'')^{-1}$ and note that $(\{a_1\}, \{b_1\}, \{h_i\}_{i=1}^2)$ satisfies property P_1 (with (iii), (iv) being vacuously satisfied).

Now suppose that for some $n > 0$ we have a triple $(\{a_i\}_{i=1}^n, \{b_i\}_{i=1}^n, \{h_i\}_{i=1}^{2n})$ which satisfies property P_n. We will construct integers a_{n+1}, b_{n+1} and homeomorphisms h_{2n+1}, h_{2n+2} of I^∞ onto itself such that $(\{a_i\}_{i=1}^{n+1}, \{b_i\}_{i=1}^{n+1}, \{h_i\}_{i=1}^{2n+2})$ satisfies property P_{n+1}. To construct b_{n+1} note that (v) implies that $Y \subset h_{2n} \circ \ldots \circ h_1(U'_{a_n})$, thus we can choose $b_{n+1} > b_n$ large enough so that $V'_{b_{n+1}} \subset h_{2n} \circ \ldots \circ h_1(U'_{a_n})$. Since Y is a Z-set in I^∞ and $h_{2n} \circ \ldots \circ h_1(U'_{a_n})$ is a neighborhood of Y, it follows from the first paragraph of Section 3 that Y is a Z-set in $h_{2n} \circ \ldots \circ (U'_{a_n})$. Also $h_{2n} \circ \ldots \circ h_1(X)$ is a Z-set in $h_{2n} \circ \ldots \circ h_1(U'_{a_n})$. Using Lemma 5.2 there exists an embedding $f_{2n+1}\colon h_{2n} \circ \ldots \circ h_1(X) \to V'_{b_{n+1}}$ such that $f_{2n+1} \simeq \mathrm{id}$ (in $h_{2n} \circ \ldots \circ h_1(U'_{a_n})$), $f_{2n+1}(h_{2n} \circ \ldots \circ h_1(X))$ is a Z-set in $h_{2n} \circ \ldots \circ h_1(U'_{a_n})$, and $f_{2n+1}(h_{2n} \circ \ldots \circ h_1(X))$ is relatively fundamentally equivalent to Y (in

$V'_{b_{n+1}}$). As in the construction of h_2 we can extend f_{2n+1} to a homeomorphism $h_{2n+1}\colon I^\infty \to I^\infty$ such that $h_{2n+1}|I^\infty \setminus h_{2n}\circ \dots \circ h_1(U'_{a_n}) = \mathrm{id}$. Now choose $a_{n+1} > a_n$ to be an integer such that $h_{2n+1}\circ \dots \circ h_1(U'_{a_{n+1}}) \subset V'_{b_{n+1}}$. Applying the same procedure once more we can construct a homeomorphism $h_{2n+2}\colon I^\infty \to I^\infty$ such that $h_{2n+2}\circ \dots \circ h_1(X)$ is relatively fundamentally equivalent to Y (in $h_{2n+1}\circ \dots \circ h_1(U'_{a_{n+1}})$) and $h_{2n+2}|I^\infty \setminus V'_{b_{n+1}} = \mathrm{id}$. It is now easily seen that $(\{a_i\}_{i=1}^{n+1}, \{b_i\}_{i=1}^{n+1}, \{h_i\}_{i=1}^{2n+2})$ satisfies property P_{n+1}.

We have just shown above that we can inductively construct infinite increasing sequences $\{a_i\}_{i=1}^\infty$, $\{b_i\}_{i=1}^\infty$ of positive integers and a sequence $\{h_i\}_{i=1}^\infty$ of homeomorphisms of I^∞ onto itself such that $(\{a_i\}_{i=1}^n, \{b_i\}_{i=1}^n, \{h_i\}_{i=1}^{2n})$ satisfies property P_n, for all $n > 0$. If we put $\{U_i\}_{i=1}^\infty = \{U'_{a_i}\}_{i=1}^\infty$ and $\{V_i\}_{i=1}^\infty = \{V'_{b_i}\}_{i=1}^\infty$, then these sequences together with $\{h_i\}_{i=1}^\infty$ satisfy the properties (1)–(6) as we wanted.

7. Proof of corollary. Let X be a FAR and without loss of generality assume that $X \subset s$. Using Theorem (7.1) of [7] we have $\mathrm{Sh}(X) = \mathrm{Sh}(\{\text{point}\})$. Using Theorem 2 there is a homeomorphism $h\colon I^\infty \setminus W^+ \to I^\infty \setminus X$. Then $I^\infty \setminus X = h\left[\bigcup_{i=1}^\infty \left(\left[-1, 1-\frac{1}{i}\right] \times \prod_{i=2}^\infty I_i\right)\right]$. We note that each $h\left(\left\{1-\frac{1}{i}\right\} \times \prod_{i=2}^\infty I_i\right)$ is a bicollared copy of I^∞ in $I^\infty \setminus X$. Thus $I^\infty \setminus h\left(\left\{1-\frac{1}{i}\right\} \times \prod_{i=2}^\infty I_i\right) = A_i \cup B_i$, where A_i and B_i are disjoint sets such that $\mathrm{Cl}(A_i) \cap \mathrm{Cl}(B_i) = h\left(\left\{1-\frac{1}{i}\right\} \times \prod_{i=2}^\infty I_i\right)$ and $\mathrm{Cl}(A_i) \cong \mathrm{Cl}(B_i) \cong I^\infty$. Choose notation so that $\mathrm{Cl}(A_i) = h\left(\left[-1, 1-\frac{1}{i}\right] \times \prod_{i=2}^\infty I_i\right)$ and thus we have $X = \bigcap_{i=1}^\infty \mathrm{Cl}(B_i)$, a decreasing sequence of Hilbert cubes.

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