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Connectivity retracts of unicoherent Peano continua in R^n

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If X is a topological space with the connectivity function fixed point property, then each connectivity retract of X has the continuous function fixed point property. (See [7] for the definitions and background.) Since I^2 has the connectivity function fixed point property (see [6]), it was hoped that any non-separating planar continuum might be a connectivity retract of I^2 and hence have the fixed point property. Cornette, however, demonstrated in [3] that the class of unicoherent Peano continua was closed under connectivity retraction. In a subsequent joint paper, Cornette and Girolo (see [4]) raise the question: "Is there a k-coherent Peano continuum that has a connectivity retract that is not a continuous retract?".

That this question has a negative answer for sets in R^2 was proven in [2]. The purpose of this paper is to describe an example in R^n, $n \geqslant 3$, which provides an affirmative answer to the major question of [4].

In the subsequent discussion, we will use these results:

(1) If a Peano continuum X fails to be unicoherent, there is a simple closed curve which is a retract of X, and thus $H_1(X, Z) \neq 0$ (Čech homology, integral coefficients). (See [5].)

(2) If each of X and Y are unicoherent Peano continua and $X \cap Y$ is connected, then $X \cup Y$ is unicoherent. (See [8], Chapter 9.)

(3) If X is a unicoherent Peano continuum with no cut points, and $f\colon X \to X$ is a peripherally continuous function, then f is a connectivity function. (A function $f\colon X \to Y$ is peripherally continuous if for each point $p \in X$ and each pair of open sets U, V about p and $f(p)$ respectively, there is an open set W, $p \in W \subset U$ and $f(\mathrm{Bd}\,W) \subset V$. See [9].)

Let $\theta = (0, \ldots, 0)$ in R^{n-1}, and the set

$$B_k = \{x \in R^{n-1}\colon d[x, (3/2^{k+1}, 0, \ldots, 0)] \leqslant 1/2^{k+1}\}, \quad \text{and} \quad B = \{\theta\} + \sum_{k=1}^{\infty} B_k.$$

Then with the set H_k equal to the sum

$$\sum_{i=1}^{2^{k-1}} \{y \in R^n\colon d[y, (3/2^{k+1}, 0, \ldots, 0, (2i-1)/2^k)] < 1/2^{k+1}\},$$

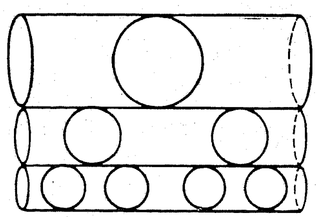
and viewing R^n as $R^{n-1}\times R$, let $X = B\times I - \sum_{k=1}^{\infty} H_k$. X is a Peano continuum and is unicoherent, for $X = M+N$ where $M = \{(x_1, \ldots, x_n) \in X\colon x_2 \leqslant 0\}$ and $N = \{(x_1, \ldots, x_n) \in X\colon x_2 \geqslant 0\}$. These sets each contain an $(n-1)$-cell as a deformation retract, and hence are themselves unicoherent. Because M and N are unicoherent Peano continua and $M \cap N$ is connected, $X = M+N$ is unicoherent.

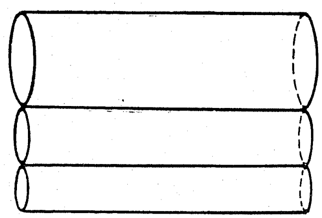
Now the set X is $1h^{n-2}$; that is, for each point $x \in X$ and each $\varepsilon > 0$, there is a $\gamma > 0$ such that any homeomorphism of S^{n-2} into a γ-neighborhood of x can be extended to a mapping of E^{n-1} into the ε-neighborhood of x. Moreover, this property is invariant under continuous retracts, a proposition which follows from a slight modification of a theorem on page 26 of [1].

On $\{B_k\times I\} - H_k$, let r_k be a continuous retract whose image is the set $\{\mathrm{Fr}\, B_k\times I + B_k\times(1)\}$. Then set $r\colon X \to \{B\times(1) + (\theta)\times I + \sum_{k=1}^{\infty}[\mathrm{Fr}\, B_k\times I]\}$, where r is the connectivity retract which is r_k on each $\{B_k\times I\} - H_k$ and is the identity on $\{\theta\}\times I$.

That r is peripherally continuous follows from the fact that r is continuous except on $\{\theta\}\times I$, and on this limit line, each point has small neighborhoods whose boundaries are $(n-2)$-dimensional and stay fixed under r. Therefore r is a connectivity retraction. The set $r(X)$ is not $1h^{n-2}$ on $\{\theta\}\times I$, so $r(X)$ cannot be a continuous retract of X.

If $n = 3$, X and $r(X)$ are indicated below. (The space X is solid, except for the omission of the open balls whose cross-sections are drawn in, and the retract $r(X)$ is a sequence of successively tangent tubes, capped at one end, which converge to a line.)





The classification of connectivity retracts of the n-cell, $n > 2$, is an interesting question. We ask the question: Is there a acyclic, locally contractible continuum that has a connectivity retract that is not a continuous retract?

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