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Remarks on a paper by Bernstein

by

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A. Bernstein [1] introduced the concept of D-compactness. We note here how the concept can be expressed in a convenient equivalent form so that the analogue of the Tychonoff theorem becomes immediate. In addition, other kinds of compactness suggest themselves.

Let D be a non-principal ultrafilter on the set I of positive integers, and let βI be the Stone–Čech compactification of I regarded as a discrete space. Then $I \cup \{D\} \subset \beta I$. Then definition 3.1 on page 187 in [1] says that x is a D-limit of $\langle x_i \rangle$ precisely when the mapping $I \cup \{D\} \to X$ such that $f(i) = x_i$ and $f(D) = x$ is continuous. Thus definition 3.2 on page 188 in [1] is equivalent of the following:

X is *D-compact* if and only if every map $I \to X$ can be extended to a continuous map $I \cup \{D\} \to X$.

In general let $A \subset B$ be two topological spaces. Call a space X, *(A, B)-compact* if every continuous map from A into X can be extended to B. By considering projections, it is immediate that any product of (A, B)-compact spaces is (A, B)-compact. (A, B)-compactness is most interesting when A is completely regular and B is a subspace of the Stone–Čech compactification of A. In this case all compact spaces are (A, B)-compact.

Definition 3.3 on page 188 in [1] says X is *ultracompact* if and only if X is D-compact for every D. By an exercise 6H in [2], p.95, if a map from I into X can be extended to a continuous map $I \cup \{D\} \to B$ for every D, then it can be extended to βI. The converse is obvious. Thus ultracompactness can be expressed in the following form: X is *ultracompact* if and only if every map $I \to X$ can be extended to a continuous map $\beta I \to X$. This finally suggests the problem of studying $(A, \beta A)$-compactness where A is some well-known completely regular space other than I.

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Remarks on open-closed mappings

by

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I this paper we shall investigate the invariance and the inverse invariance of some topological properties under open-closed mappings. In the first section we shall prove a generalization of theorems proved by N. Dykes [4] and A. V. Arhangel'skiĭ [2] which shows a connection between the classes of open-closed and open-perfect mappings. In the second section we shall consider inverse images of complete spaces (1) under open-closed, open, and closed mappings. The last section is devoted to an investigation of invariance and inverse invariance of axioms of separation. In particular, we generalize (and simplify) an example of M. Henriksen and J. R. Isbell from [12] to show that an inverse image of a normal, complete space under an open-perfect mapping need not be completely regular.

We shall use the terminology and notation from [5]. By a *mapping* we always mean a continuous function. Perfect mappings will be assumed to be defined on Hausdorff spaces.

We shall often use the following simple criteria for the openness and closedness of mappings: A mapping $f\colon X\to Y$ is open iff the image of each element of a base of X is open in Y; f is closed iff $f(X)=Y$ and for each point y in Y and an open set U in X which contains $f^{-1}(y)$ there exists a neighbourhood V of y in Y such that $f^{-1}(V)\subset U$.

1. Invariance of real-compactness and strong paracompactness. In this section the following notion will be widely applied.

Definition 1.1. A space Y will be called a *c-space* iff for each non-isolated point $y\in Y$ and each sequence $\{U_n\}_{n=1}^{\infty}$ of neighbourhoods of y there exists a set $\{y_n\}_{n=1}^{\infty}$ which is not closed and is such that $y_n\in U_n$ for every n.

(1) By a *complete space* we always mean a space which is complete in the sense of Čech. A space with the topology induced by complete uniformity will be called a *uniformly complete space* (such spaces are also called complete in the sense of Dieudonné).