seulement pour $h>0$ n'affaiblit l'hypothèse que d'un point de vue étroitement formal.

Notons encore que le théorème en § 2, que nous avons donné sous la forme de condition suffisante, est tout de suite ramené à la forme de condition nécessaire au moyen de la décomposition:

$$\frac{F(x+kh)-F(x-h)}{(k+1)h}=\frac{F(x+kh)-F(x)}{(k+1)h}+\frac{F(x)-F(x-h)}{(k+1)h}.$$

Ça suffit pour en déduire que l'existence de la dérivée k-pseudo-symétrique pour une valeur fixée de $k>0$ entraîne son existence pour toute valeur positive de k.

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Concerning product of paracompact spaces

by

R. Telgársky (Bratislava)

This paper is a continuation of [9]. It has 3 sections. Section 1 deals with absolute paracompactness (see [9], Section 3) for which a product theorem is proved (Th. 1.1). Section 2, based on an analysis of a construction of E. Michael ([6], Examples 1.4 and 1.5), treats of the property C'' (see [4], p. 527, Th. 5) and of some singular spaces. Section 3 contains some positive facts about the Hurewicz property (see [5], p. 209), e.g. that it is σ-additive (Th. 3.3), perfect (Cor. 3.11) and productive if at least one of the two factors is C-scattered (Th. 3.4 and 3.5).

The topological terminology is that of [1]. N denotes the set of all positive integers.

1. A Cartesian product of an absolutely paracompact space (see [9], Section 3) by a discrete space is absolutely paracompact. This assertion can be generalized as follows:

THEOREM 1.1. *If X is a scattered paracompact space and Y is an absolutely paracompact space, then the product space $X\times Y$ is absolutely paracompact.*

Proof. Let Z be a paracompact space such that $X\times Y$ is a closed subspace of Z. Let $\mathcal{B}$ be an outer base for $X\times Y$ in Z. We shall prove by transfinite induction over α such that $X^{(\alpha)}=0$ that $\mathcal{B}$ contains a locally finite covering of $X\times Y$ in Z.

If $X^{(0)}=0$, then $X=0$, and so the theorem is trivially true.

If $X^{(\alpha+1)}=0$, then $X^{(\alpha)}$ is a closed discrete set in X. Clearly, $X^{(\alpha)}\times Y$ is closed in Z and absolutely paracompact as a free union of absolutely paracompact spaces. So $\mathcal{B}$ contains a locally finite covering $\mathcal{A}_1$ of $X^{(\alpha)}\times Y$ in Z. Since $(X\times Y)-\bigcup\mathcal{A}_1$ is closed in Z, it is paracompact. Now it is sufficient to prove that $(X\times Y)-\bigcup\mathcal{A}_1$ has a relatively open cover by sets whose closures are absolutely paracompact. If $\langle x,y\rangle\in(X\times Y)-\bigcup\mathcal{A}_1$, then $x\notin X^{(\alpha)}$. Since X is regular, there is an open nbhd U_x of x in X such that $\overline{U_x}\cap X^{(\alpha)}=0$. So $\overline{U_x}^{(\beta)}=0$ for some $\beta<\alpha$. Hence $\overline{U_x}\times Y$ is absolutely paracompact by the inductive assumption. Now, $(U_x\times Y)-\bigcup\mathcal{A}_1$ is an open nbhd of $\langle x,y\rangle$ in $(X\times Y)-\bigcup\mathcal{A}_1$ whose closure is absolutely paracompact, because each closed subset of an absolutely paracompact space is absolutely paracompact. So $\mathcal{B}$ contains a locally finite covering $\mathcal{A}_2$ of $(X\times Y)-\bigcup\mathcal{A}_1$ in Z. Clearly, $\mathcal{A}_1\cup\mathcal{A}_2$ is locally finite in Z and covers $X\times Y$.

If $X^{(\lambda)}=0$ for the limit λ, then $\{X-X^{(\alpha)}: \alpha<\lambda\}$ is an open cover of X. Since X is paracompact and scattered (see [7], p. 569, Cor. 3, dispersed = scattered), we can take a discrete refinement $\mathcal{A}$ of the covering $\{X-X^{(\alpha)}: \alpha<\lambda\}$ of X. For each $A\in\mathcal{A}$ there is an $\alpha<\lambda$ such that $A\subseteq X-X^{(\alpha)}$ and hence $A^{(\alpha)}=0$. So $A\times Y$ is absolutely paracompact by the inductive assumption for each $A\in\mathcal{A}$. $X\times Y$ is covered by the discrete family $\{A\times Y: A\in\mathcal{A}\}$ of absolutely paracompact sets, and so $X\times Y$ is absolutely paracompact as well.

The proof is complete.

Remark 1.2. I do not know whether "C-scattered" can be taken instead of "scattered" in Theorem 1.1 (cf. [9], Problem 3.3).

For comparing the absolute paracompactness and the total hypocompactness ([8], p. 625) we give the following

EXAMPLE 1.3. Let $X=\{0\}\cup\{1/m+1/n: m\in N \,\&\, n\in N\}$, equipped with the relative topology, and Y be the Cantor set. Then X is totally hypocompact, Y is compact, $X\times Y$ is absolutely paracompact, but $X\times Y$ is not totally hypocompact.

Indeed, X is totally hypocompact, because the complement of any nbhd of 0 is a discrete closed-open set and therefore each open basis contains a disjoint covering of X. $X\times Y$ is absolutely paracompact, because it is paracompact and C-scattered (see [9], Cor. 1.4 and Th. 3.1). Finally, $X\times Y$ is not totally hypocompact, because it is not locally compact (see [8], Th. 3).

2. E. Michael [6] considered regular spaces X having the following property: there is a countable set $X_1\subseteq X$ such that for each open set $U\supseteq X_1$ in X the set $X-U$ is countable. In the terminology of [4] these spaces are called "concentrated about a countable subset" (see p. 526, Def. 2). It is known that each space X concentrated about a countable subset has the property C'': if $\{\mathcal{A}_n: n\in N\}$ is a sequence of open covers of X, then there is a selector $\{A_n: n\in N\}$ from $\{\mathcal{A}_n: n\in N\}$ such that $\{A_n: n\in N\}$ covers X (see [4], p. 527, Th. 5). We shall prove that the Lindelöf space X^n considered in Lemma 5.1 of [6] has C'', although it is not concentrated about a countable subset. What follows now is a refinement of the construction in the proof of Lemma 5.1 of [6].

DEFINITION 2.1. A regular space X is said to be *chain-concentrated about a countable set* $A\subseteq X$, if there is an $n\in N$ and $\{X_k: 0\leqslant k\leqslant n\}$ such that $X=X_0\supseteq X_1\supseteq X_2\supseteq\ldots\supseteq X_n=A$ and X_k is concentrated about X_{k+1} for each $0\leqslant k\leqslant n-1$.

THEOREM 2.2. *If X is chain-concentrated about a countable subset A, then X has the property C''.*

Proof. If X is chain-concentrated about a countable subset A, then there is an $n\in N$ and $\{X_k: 0\leqslant k\leqslant n\}$ such that $X=X_0\supseteq X_1\supseteq\ldots\supseteq X_n=A$ and X_k is concentrated about X_{k+1} for each $0\leqslant k\leqslant n-1$. Let $\{\mathcal{A}_i: i\in N\}$ be a sequence of open covers of X. Let $N=\bigcup\{M_k: 0\leqslant k\leqslant n\}$ be a decomposition of N such that M_k is infinite for each $0\leqslant k\leqslant n$. Since $X_n=A$ is countable, we can choose $\{A_i: i\in M_n\}$ such that $A_i\in\mathcal{A}_i$ for each $i\in M_n$ and $\bigcup\{A_i: i\in M_n\}\supseteq X_n$. Hence $X_{n-1}-\bigcup\{A_i: i\in M_n\}$ is countable, because X_{n-1} is concentrated about X_n. Step by step we shall come to a covering $\{A_i: i\in M_k \,\&\, 0\leqslant k\leqslant n\}$ of X which has the desired properties. The proof is complete.

LEMMA 2.3. *If $n\in N$, $X=\bigcup\{Y_i: i\in N\}$ and Y_i is chain-concentrated about a countable set $A_i\subseteq Y_i$ with the length of the chain $\leqslant n$ for each $i\in N$, then X is chain-concentrated about $A=\bigcup\{A_i: i\in N\}$ with the length of the chain $\leqslant n$.*

Proof. Let $\{X_{i,k}: 0\leqslant k\leqslant n\}$ be a chain in Y_i such that $Y_i=X_{i,0}\supseteq X_{i,1}\supseteq\ldots\supseteq X_{i,n}=A_i$ and $X_{i,k}$ is concentrated about $X_{i,k+1}$ for $0\leqslant k\leqslant n-1$ and $i\in N$. It is easy to check that $X_k=\bigcup\{X_{i,k}: i\in N\}$, where $0\leqslant k\leqslant n$, form the desired chain in X. The proof is complete.

THEOREM 2.4. *If A is a countable subset of X and $n\in N$ such that X^m is concentrated about $X^m-(X-A)^m$, for each $1\leqslant m\leqslant n$, then X^n is chain-concentrated about A^n.*

Proof. We prove by induction that X^m is chain-concentrated about A^m for all $1\leqslant m\leqslant n$. This is clear for $m=1$, because $X-(X-A)=A$, and so X is concentrated about A and the length of the chain is $\leqslant 1$. Assume that for some $m<n$ X^m is concentrated about A^m and the length of the chain is $\leqslant m$. Let us remark (or, see [6], Lemma 1.4) that $X^{m+1}-(X-A)^{m+1}=\{x\in X^{m+1}: x_i\in A \text{ for some } i,\ 1\leqslant i\leqslant m+1\}=\bigcup\{Y_{i,a}: 1\leqslant i\leqslant m+1 \,\&\, a\in A\}$, where $Y_{i,a}=\{x\in X^{m+1}: x_i=a\}$. Clearly, $Y_{i,a}$ is chain-concentrated about $Z_{i,a}=\{x\in A^{m+1}: x_i=a\}$ and the length of the chain is $\leqslant m$, because $Y_{i,a}$ is homeomorphic to X^m. Hence, by Lemma 2.3, $X^{m+1}-(X-A)^{m+1}$ is chain-concentrated about $A^{m+1}=\bigcup\{Z_{i,a}: 1\leqslant i\leqslant m+1 \,\&\, a\in A\}$ and the length of the chain is $\leqslant m$. Since, by the assumption, X^{m+1} is concentrated about $X^{m+1}-(X-A)^{m+1}$, X^{m+1} is chain-concentrated about A^{m+1} and the length of the chain is $\leqslant m+1$. The proof is complete.

And now follows the refinement of Lemma 5.1 of [6], as a corollary to Theorems 2.2 and 2.4:

COROLLARY 2.5. *If A is a countable subset of X and $n\in N$ such that for each m, $1\leqslant m\leqslant n$, X^m is concentrated about $X^m-(X-A)^m$, then X^n has the property C''.*

Hence, the Examples 1.4 and 1.5 constructed by E. Michael [6] are, from the point of view of covering properties, much stronger, as is asserted in [6].

Remark 2.6. The concept of a space concentrated about a countable subset can easily be generalized, in another way, as follows: we say that a regular space X fulfils (⁎) if there is a σ-compact subset A of X such that $X-U$ is σ-compact whenever U is an open nbhd of A in X. It can be proved that if X fulfils (⁎), then X is strongly Hurewicz (see [5], p. 210).

Remark 2.7. A relation of the property C'' to the Lindelöf property and to other properties gives the following sequence of implications (for regular spaces): a countable space $\Rightarrow$ a space concentrated about a countable subset $\Rightarrow$ a space chain-concentrated about a countable subset $\Rightarrow C'' \Rightarrow$ strongly Hurewicz $\Rightarrow$ Hurewicz $\Rightarrow$ Lindelöf $\Rightarrow$ hypocompact $\Rightarrow$ $\Rightarrow$ hypo-lindelöf $\Rightarrow$ paracompact.

3. Hurewicz spaces (see [5], p. 209) are very special cases of Lindelöf spaces. Each σ-compact regular space is a Hurewicz space and any closed subset of a Hurewicz space is a Hurewicz space. Although the product of two Hurewicz spaces need not be normal (see [5], p. 216, Example), there are some positive facts about Hurewicz spaces and we shall prove here some of them in a general setting, without the assumption of metrizability.

THEOREM 3.1. *If there is a continuous map from a Hurewicz space onto a regular space X, then X is a Hurewicz space.*

The proof is easy from the definition of the Hurewicz space, and so it is left as an excercise.

THEOREM 3.2. *If a Hurewicz space X is complete in the sense of Čech, then X is σ-compact and C-scattered.*

Proof. Since X is Hurewicz, X is paracompact. Hence, by Theorem 3 of Z. Frolík [2], there is a perfect map f from X onto a complete metric space Y. Since f is continuous, Y is also a Hurewicz space by Theorem 3.1. Now, by Satz 20 of W. Hurewicz [3] ($E^*=$ Hurewicz property), Y is σ-compact. Since Y is a σ-compact complete metric space, by Theorem 1.7 in [9] Y is C-scattered. Since f is perfect, X is σ-compact (σ-compactness is a perfect property) and X is also C-scattered by Theorem 1.3 in [9]. The proof is complete.

THEOREM 3.3. *If a regular space X is a union of a countable family of Hurewicz spaces, then X is a Hurewicz space; i.e. the Hurewicz property is σ-additive.*

Proof. Let $X=\bigcup\{X_n\colon n\in N\}$, where each X_n is a Hurewicz space. Let $\{\mathcal{A}_k\colon k\in N\}$ be a sequence of open covers of X. For each $n\in N$, $\{\{A\cap X_n\colon A\in\mathcal{A}_k\}\colon k\geqslant n\}$ is a sequence of open covers of X_n, and so there is a sequence $\{\mathcal{B}_{n,k}\colon k\geqslant n\}$ such that $\mathcal{B}_{n,k}\subseteq\mathcal{A}_k$, $\mathcal{B}_{n,k}$ is finite and $\{A\cap X_n\colon A\in\mathcal{B}_{n,k}\ \&\ k\geqslant n\}$ covers X_n. Let us put $\mathcal{B}_k=\bigcup\{\mathcal{B}_{n,k}\colon n\leqslant k\}$. Then each $\mathcal{B}_k$ is finite, $\mathcal{B}_k\subseteq\mathcal{A}_k$ and $\bigcup\{\mathcal{B}_k\colon k\in N\}$ covers X, because it covers each X_k. So X is a Hurewicz space. The proof is complete.

THEOREM 3.4. *If X is a Lindelöf C-scattered space, then X is a Hurewicz space.*

This theorem follows from a more general product

THEOREM 3.5. *If X is a Lindelöf C-scattered space and Y is a Hurewicz space, then $X\times Y$ is a Hurewicz space.*

First we prove

LEMMA 3.6. *If X is a compact space and Y is a Hurewicz space, then $X\times Y$ is a Hurewicz space.*

Proof. Let $\{\mathcal{A}_n\colon n\in N\}$ be a sequence of open covers of $X\times Y$. For each $n\in N$ and $y\in Y$ we choose a finite subfamily $\mathcal{A}_{n,y}$ of $\mathcal{A}_n$ such that $X\times\{y\}\subseteq\bigcup\mathcal{A}_{n,y}$. Let $U_{n,y}$ be an open nbhd of y in Y such that $X\times U_{n,y}\subseteq\bigcup\mathcal{A}_{n,y}$. Then $\mathcal{U}_n=\{U_{n,y}\colon y\in Y\}$ is an open cover of Y, for each $n\in N$. Since Y is a Hurewicz space, each $\mathcal{U}_n$ contains a finite subfamily $\mathcal{V}_n$ such that $\bigcup\{\mathcal{V}_n\colon n\in N\}$ covers Y. Define $\mathcal{B}_n=\{A\in\mathcal{A}_n\colon A\in\mathcal{A}_{n,y}\ \&\ U_{n,y}\in\mathcal{V}_n\}$. Each $\mathcal{B}_n$ is finite, because each $\mathcal{A}_{n,y}$ and each $\mathcal{V}_n$ is finite. $\bigcup\{\mathcal{B}_n\colon n\in N\}$ covers $X\times Y$, because $\{X\times U_{n,y}\colon U_{n,y}\in\mathcal{V}_n\ \&\ n\in N\}$ covers $X\times Y$ and if $\langle x,y\rangle\in X\times Y$, then there is an $n\in N$ and $z\in Y$ such that $y\in U_{n,z}\in\mathcal{V}_n$, and so from $X\times U_{n,z}\subseteq\bigcup\mathcal{A}_{n,z}$ it follows that $\langle x,y\rangle\in A$, for some $A\in\mathcal{A}_{n,z}$, hence $\langle x,y\rangle\in\bigcup\mathcal{B}_n$. It follows that $X\times Y$ is a Hurewicz space. The proof is complete.

LEMMA 3.7. *If a regular space X has a Hurewicz subspace X_0 such that, for each open set U in X, $X-U$ is Hurewicz whenever $U\supseteq X_0$, then X is a Hurewicz space.*

Proof. Let $\{\mathcal{A}_n\colon n\in N\}$ be a sequence of open covers of X. Then $\{\{A\cap X_0\colon A\in\mathcal{A}_n\}\colon n\in N\}$ is a sequence of open covers of X_0, and so there is a sequence $\{\mathcal{B}_n\colon n\in N\}$ such that each $\mathcal{B}_n$ is finite, $\mathcal{B}_n\subseteq\mathcal{A}_n$ and $\bigcup\{\mathcal{B}_n\colon n\in N\}$ covers X_0. Let us put $X_1=X-\bigcup\{B\in\mathcal{B}_n\colon n\in N\}$. Then, similarly as for X_0, we can choose $\{\mathcal{C}_n\colon n\in N\}$ such that each $\mathcal{C}_n$ is finite, $\mathcal{C}_n\subseteq\mathcal{A}_n$ and $\bigcup\{\mathcal{C}_n\colon n\in N\}$ covers X_1, because X_1 is a Hurewicz space. Let us put $\mathcal{D}_n=\mathcal{B}_n\cup\mathcal{C}_n$. Then $\{\mathcal{D}_n\colon n\in N\}$ is the desired sequence showing that X is a Hurewicz space. The proof is complete.

Proof of Theorem 3.5. We prove by transfinite induction over a such that $X^{(a)}=0$ that $X\times Y$ is a Hurewicz space.

If $X^{(0)}=0$, then $X=0$, and so the theorem is trivially true.

If $X^{(a+1)}=0$, then $X^{(a)}$ is a locally compact closed Lindelöf subset of X. Hence X has a countable closed covering $\{F_n\colon n\in N\}$ such that each $F_n^{(\alpha)}$ is compact (or void). According to Theorem 3.3 it is sufficient to prove that each $F_n \times Y$ is Hurewicz. So, without loss of generality, we can assume that $X^{(\alpha)}$ is compact. Let U be an open set in $X \times Y$ such that $U \supseteq X^{(\alpha)} \times Y$. For each $y \in Y$ there is an open set V_y in X containing $X^{(\alpha)}$ and an open set W_y in Y containing y such that $X^{(\alpha)} \times \{y\} \subseteq V_y \times W_y \subseteq U$. By Lemma 3.6 $X^{(\alpha)} \times Y$ is Hurewicz and therefore Lindelöf. So there is a countable subset A of Y such that $\{V_y \times W_y: y \in A\}$ covers $X^{(\alpha)} \times Y$. But $(X - V_y)^{(\alpha)} = 0$, and so $(X - V_y) \times \overline{W}_y$ is Hurewicz by the inductive assumption. Let $\langle x, y\rangle \in (X \times Y) - U$. Then $\langle x, y\rangle \in (X \times Y) - \bigcup \{V_z \times W_z: z \in A\}$. However, $\{W_z: z \in A\}$ covers Y, and so there is a $z \in A$ such that $y \in W_z$. It follows that $x \notin V_z$; so $x \in X - V_z$ and hence $\langle x, y\rangle \in (X - V_z) \times W_z \subseteq (X - V_z) \times \overline{W}_z$. So, since $(X \times Y) - U$ is a closed subset of $\bigcup \{(X - V_z) \times \overline{W}_z: z \in A\}$, $(X \times Y) - U$ is a Hurewicz space by Theorem 3.3. Hence $X \times Y$ is a Hurewicz space by Lemma 3.7.

If $X^{(\lambda)} = 0$ for the limit λ, then for each $x \in X$ there is an $\alpha < \lambda$ such that $x \in X^{(\alpha)} - X^{(\alpha+1)}$; hence there is an open nbhd U_x of x in X such that $\overline{U}_x \cap X^{(\alpha+1)} = 0$. So $\overline{U}_x^{(\alpha+1)} = 0$ and therefore $\overline{U}_x \times Y$ is Hurewicz by the inductive assumption. Since $\{U_x: x \in X\}$ covers X and X is Lindelöf, then there is a countable subset A of X such that $\{U_x: x \in A\}$ covers X. Hence $X \times Y = \bigcup \{\overline{U}_x \times Y: x \in A\}$ and therefore $X \times Y$ is Hurewicz by Theorem 3.3.

The proof is complete.

From Theorem 3.3 and Lemma 3.6 we have the following

COROLLARY 3.8. *If X is a σ-compact regular space and Y is a Hurewicz space, then $X \times Y$ is a Hurewicz space.*

Remark 3.9. Similarly, as in the proof of Theorem 3.5, one can prove that if X is a Lindelöf C-scattered space and Y is a Lindelöf space, then $X \times Y$ is a Lindelöf space.

THEOREM 3.10. *If there is a perfect map f from a regular space X onto a Hurewicz space Y, then X is a Hurewicz space.*

Proof. First let us remark that X is homeomorphic to $f = \{\langle x, f(x)\rangle: x \in X\}$. Now it is sufficient to prove that f is a closed subset of $\beta X \times Y$, according to Lemma 3.6. Let $\langle x, y\rangle \in (\beta X \times Y) - f$. Then (a) $x \in \beta X - X$, or (b) $x \in X$ and $f(x) \neq y$. If (a) holds, then there is an open set U in βX such that $U \supseteq f^{-1}(y)$ and $x \notin \overline{U}$ (the closure by means of βX). Let us put $V = Y - f(X - U)$. Then V is an open nbhd of y in Y and

$$f^{-1}(V) = f^{-1}(Y) - f^{-1}f(X - U) = X - f^{-1}f(X - U) \subseteq X - (X - U)$$
$$= X \cap U \subseteq U \subseteq \overline{U}.$$

Hence $x \notin \overline{f^{-1}(V)}$ and so there is an open nbhd W of x in βX such that $W \cap f^{-1}(V) = 0$. But $W \cap f^{-1}(V) = 0$ implies $(W \times V) \cap f = 0$. So $W \times V$ is an open nbhd of $\langle x, y\rangle$ in $\beta X \times Y$ disjoint with f. If (b) holds, then $f^{-1}f(x) \cap f^{-1}(y) = 0$. Since $f^{-1}f(x)$ and $f^{-1}(y)$ are compact, there are two open disjoint sets U and V in βX such that $f^{-1}f(x) \subseteq U$ and $f^{-1}(y) \subseteq V$. Let us put $W = Y - f(X - V)$. Then W is an open nbhd of y in Y and

$$f^{-1}(W) = X - f^{-1}f(X - V) \subseteq X - (X - V) = X \cap V \subseteq V.$$

Clearly, $\langle x, y\rangle \in U \times W$. But $U \cap f^{-1}(W) \subseteq U \cap V = 0$, and so $(U \times W) \cap f = 0$. From the results in (a) and (b) we conclude that f is a closed subset of $\beta X \times Y$; so X, being homeomorphic to f, is a Hurewicz space. The proof is complete.

From Theorem 3.1 and Theorem 3.10 we have

COROLLARY 3.11. *If f is a perfect map from a regular space X onto a regular space Y, then X is a Hurewicz space iff Y is a Hurewicz space; i.e., the Hurewicz property is perfect.*

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INSTITUTE OF MATHEMATICS
SLOVAK ACADEMY OF SCIENCES

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