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A corrected correction

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The result of this note is that there exist an algebra A of real-valued functions on a set, closed under uniform convergence and all continuous finitary operations, and a homomorphism of A onto a non-archimedean ordered field which has a countable cofinal subset.

I denied this in [2] (omitting proof). In [1] we correctly deduced this from a claimed example of a completely regular space $\mathfrak{X}$, an increasing sequence of zero sets Z_i of $\beta\mathfrak{X}$ disjoint from $\mathfrak{X}$, and a point z each of whose neighborhoods meets every difference $Z_i - Z_{i-1}$. It remains to repair that example, and to thank Eleanor Aron for pointing out the error. (The algebra A consists of the continuous functions on $\mathfrak{X}$ having continuous real-valued extensions over the complement of some Z_i; the maximal ideal at z provides the ordered field, and the verification [1] is straightforward.)

Let $\mathfrak{X}$ be the product of $\aleph_0$ copies of the positive integers. Let p_i be the ith coordinate function, $q_i = p_1 + \ldots + p_i$, r_i the reciprocal of q_i, s_i the continuous extension of r_i over $\beta\mathfrak{X}$. Let Z_i be the zero set of s_i. Evidently the Z_i are increasing and disjoint from $\mathfrak{X}$. Consider all closed subsets of $\mathfrak{X}$ defined by conditions of the form $p_n > \varphi(p_1, \ldots, p_{n-1})$. No finite family of such conditions is inconsistent; so the closure in $\beta\mathfrak{X}$ of all these sets have a common point z. For any neighborhood U of z, for any index n, there are natural numbers $a_1, \ldots, a_{n-1}$ such that on the points of $U \cap \mathfrak{X}$ where $p_1 = a_1, \ldots, p_{n-1} = a_{n-1}$, p_n is unbounded; for a bound $\varphi(a_1, \ldots, a_{n-1})$ will yield a contradiction. So $(U \cap \mathfrak{X})^-$ meets $Z_n - Z_{n-1}$, and every neighborhood of z contains one of these.

References

[1] M. Henriksen, J. R. Isbell, D. G. Johnson, *Residue class fields of lattice-ordered algebras*, Fund. Math. 50 (1961), pp. 107–117.

[2] J. R. Isbell, *Algebras of uniformly continuous functions*, Ann. of Math. 68 (1958), pp. 96–125.

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