The following theorem on the uniqueness of BTS is a consequence of the above results:

(5.5) THEOREM. *For any pair* (C, E) *where* $E\colon C \to EC$ *is a projection functor from an* E*-category* C *into a semi-classical category* EC *there exists a unique continuous* BTS (C, EC, SC, E, F). *It is unique in the following sense: If* $(C, EC, S'C, E, F')$ *is another continuous* BTS *(and* $S' = F' \circ E$*), then there exist functors (which are uniquely determined)* $H\colon SC \to S'C$ *and* $H'\colon S'C \to SC$ *such that*

$$H' \circ H\colon SC \to SC \quad \text{and} \quad H \circ H'\colon S'C \to S'C$$

are identity functors and

$$F' = H \circ F \quad \text{and} \quad F = H' \circ F'.$$

(5.6) Remark. We can say that a continuous functor of shape is a Dedekind section between the functors of shape and the continuous functors.

(5.7) Remark. It is clear that Theorem (5.1) holds for the contravariant functors G, G' also.

(5.8) EXAMPLE. Given an arbitrary BTS (C, EC, SC, E, F), let $Y \in E\text{-Ob}\,C$. Then, by Definition (2.1), $M^Y_{EC} \circ E\colon C \to \mathrm{Ens}$ is a continuous contravariant functor. Then, by Theorem (5.1), there exists a contravariant functor H such that $M^Y_{EC} = H \circ F$. It is easy to see that it must be $H = M^Y_{SC}$.

(5.9) EXAMPLE. Let $H\colon C \to HC$ be the homotopy functor from the topological category of compact pairs C to the homotopy category of compact pairs HC. Then H is a projection functor and C is an H-category. The Čech homology and cohomology functors and the cohomotopy functors π^n are H-invariant and continuous on C. Thus they are shape-invariant in the sense of Theorem (5.1) (see [2] and compare [3] and Example (5.8)). H-objects are precisely the pairs homotopically dominated by polyhedral pairs.

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Some results on fixed points — III

by

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Recently many authors have proved fixed point theorems (see for example [1], [4], [5], [8]) for operators mapping a Banach space X into itself. In each of these theorems it has been assumed that the mapping is non-expansive i.e., if φ maps the Banach space X into itself, then

(a) $$\|\varphi(x) - \varphi(y)\| \leqslant \|x - y\|, \quad \text{for } x, y \in X.$$

The main purpose of the present paper is to prove some fixed point theorems for operators mapping a Banach space into itself which, instead of the non-expansive property, possess the following: if φ is a mapping of a Banach space X into itself, then

(b) $$\|\varphi(x) - \varphi(y)\| \leqslant \tfrac{1}{2}\{\|x - \varphi(x)\| + \|y - \varphi(y)\|\} \quad \text{for } x, y \in X.$$

It may be noted that condition (a) implies the continuity of the operator in the whole space while condition (b) has no such implications. Moreover, it is known [6] that (a) and (b) are independent. For relevant works on fixed point theorems for operators mapping a metric space M into itself which satisfy condition (b) on M, one may refer to [6] and [7].

Before going into the theorems, we state the following well-known definitions and results.

DEFINITION ([2], p. 27). A norm in a normed linear space X is *uniformly convex* if

$$\|x_n\| = \|y_n\| = 1 \ (n = 1, 2, \ldots), \quad \lim_{n\to\infty} \|x_n + y_n\| = 2$$

imply

$$\lim_{n\to\infty} \|x_n - y_n\| = 0 \quad \text{for } x_n, y_n \in X.$$

THEOREM A ([2], p. 28). *Let* X *be a uniformly convex normed linear space and let* ε, M *be positive constants. Then there exists a constant* δ *with* $0 < \delta < 1$ *such that*

$$\|x\| \leqslant M, \quad \|y\| \leqslant M, \quad \|x - y\| \geqslant \varepsilon$$

imply

$$\|x + y\| \leqslant 2\delta \max(\|x\|, \|y\|).$$

THEOREM B [12]. *Every uniformly convex Banach space is norm-reflexive.*

THEOREM C [11]. *A necessary and sufficient condition that a Banach space X be reflexive is that:*

Every bounded descending sequence (transfinite) of non-empty closed convex subsets of X has a non-empty intersection.

We are now in a position to prove our theorems.

THEOREM 1. *Let X be a reflexive Banach space and let K be a non-empty closed convex bounded subset of X. If φ be a mapping of K into itself such that*

(i) $\|\varphi(x)-\varphi(y)\| \leqslant \frac{1}{2}\{\|x-\varphi(x)\|+\|y-\varphi(y)\|\}$, $x, y \in K$

and

(ii) $\sup_{y \in H} \|y-\varphi(y)\| \leqslant \dfrac{\delta(H)}{2}$,

where H is any non-empty convex subset of K which is mapped into itself by φ and $\delta(H)$ is the diameter of H, then φ has a unique fixed point in K.

For any non-empty closed convex subset F of K we define the following:

$$r_x(F) = \sup_{y \in F} \frac{\|x-y\|}{2} + \sup_{z \in F} \frac{\|z-\varphi(z)\|}{2}, \qquad x \in F,$$

$$r(F) = \inf_{x \in F} r_x(F)$$

and

$$F_c = \{x \in F\colon r_x(F) = r(F)\}.$$

We first prove the following lemma.

LEMMA. *F_c is non-empty, closed and convex.*

Proof of the lemma. For positive integer n, let

$$F(x, n) = \left\{y \in F\colon \frac{\|x-y\|}{2} \leqslant r(F) + \frac{1}{n} - \sup_{z \in F} \frac{\|z-\varphi(z)\|}{2}\right\}$$

and let $C_n = \bigcap_{x \in F} F(x, n)$.

It then follows that $\{C_n\}$ is a decreasing sequence of non-empty, closed, convex and bounded sets. Since X is reflexive, it follows by Theorem C that $F_c = \bigcap_n C_n$ is non-empty, closed and convex. This proves the lemma.

Proof of the theorem. Let $\mathfrak{F}$ denote the family of all non-empty, closed and convex subsets of K, each of which is mapped into itself by φ. By the result of Smulian [11] and Zorn's lemma it follows that $\mathfrak{F}$ has a minimal element, which we denote by F.

Let $x \in F_c$, the non-emptiness of F_c being a consequence of the lemma. Then

$$\|\varphi(x)-\varphi(y)\| \leqslant \frac{\|x-\varphi(x)\|}{2} + \frac{\|y-\varphi(y)\|}{2}, \qquad y \in F$$

$$\leqslant \sup_{y \in F} \frac{\|x-y\|}{2} + \sup_{y \in F} \frac{\|y-\varphi(y)\|}{2}$$

$$= r_x(F) = r(F).$$

So, $\varphi(F)$ is contained in a closed spherical ball $\overline{U}$ centred at $\varphi(x)$ and radius $r(F)$. Therefore $\varphi(F \cap \overline{U}) \subset F \cap \overline{U}$ and hence, by the minimality of F, we get $F \subset \overline{U}$. Hence for $y \in F$, $\|\varphi(x)-y\| \leqslant r(F)$.

So,

(A) $$\sup_{y \in F} \|\varphi(x)-y\| \leqslant r(F).$$

Now

$$r_{\varphi(x)}(F) = \sup_{y \in F} \frac{\|\varphi(x)-y\|}{2} + \sup_{z \in F} \frac{\|z-\varphi(z)\|}{2},$$

so,

(B) $$r_{\varphi(x)}(F) \leqslant \frac{r(F)}{2} + \sup_{z \in F} \frac{\|z-\varphi(z)\|}{2} \qquad \text{(by (A))}.$$

Also

$$\sup_{z \in F} \frac{\|z-\varphi(z)\|}{2} = \sup_{z \in F} \frac{\|z-\varphi(z)\|}{4} + \sup_{z \in F} \frac{\|z-\varphi(z)\|}{4}$$

$$\leqslant \frac{\delta(F)}{8} + \sup_{z \in F} \frac{\|z-\varphi(z)\|}{4}, \qquad \text{by condition (ii)}$$

$$= \sup_{z,t \in F} \frac{\|z-t\|}{8} + \sup_{z \in F} \frac{\|z-\varphi(z)\|}{4}.$$

So,

$$\sup_{z \in F} \frac{\|z-\varphi(z)\|}{2} \leqslant \sup_{z \in F} \frac{\|z-x\|}{8} + \sup_{t \in F} \frac{\|t-x\|}{8} + \sup_{z \in F} \frac{\|z-\varphi(z)\|}{4}$$

$$= \sup_{z \in F} \frac{\|z-x\|}{4} + \sup_{z \in F} \frac{\|z-\varphi(z)\|}{4}$$

$$= \frac{r_x(F)}{2} = \frac{r(F)}{2}.$$

So, from (B), $r_{\varphi(x)}(F) \leqslant r(F)$, which implies that

$$r_{\varphi(x)}(F) = r(F) \text{ i.e., } \varphi(x) \in F_c.$$

(C) Hence φ maps F_c into itself.

We now show that, if F contains more than one element, F_c is a proper subset of F. Otherwise, let $F_c = F$. Then for $x, y \in F$

$$r_x(F) = r_y(F) = r(F).$$

So, $\sup_{t\in F}\|x-t\| = \sup_{t\in F}\|y-t\|$ for $x, y \in F$. This implies that $\sup_{t\in F}\|x-t\| = M$, a constant, for all $x \in F$. Hence $\delta(F) = \sup_{x,t\in F}\|x-t\| = M$, where $\delta(F)$ denotes the diameter of F. This, however, implies for $x \in F$ that

$$\text{(D)} \qquad \sup_{t\in F}\|\varphi(x)-t\| = \delta(F).$$

Again,

$$\|\varphi(x)-\varphi(y)\| \leqslant \frac{\|x-\varphi(x)\|}{2} + \frac{\|y-\varphi(y)\|}{2}, \quad y \in F$$
$$\leqslant \frac{\delta(F)}{2}, \quad \text{by condition (ii).}$$

Proceeding in the same manner as in obtaining (A), we now get $\sup_{y\in F}\|\varphi(x)-y\| \leqslant \frac{\delta(F)}{2}$, which contradicts (D) because F contains more than one element.

Hence we infer that if F contains more than one element, then F_c is a proper subset of F. But this, in view of (C), contradicts the minimality of F. Hence F contains only one element. Since φ maps F into itself, φ has a fixed point in K.

The unicity may be proved as follows.

Suppose that $\varphi(x) = x$, $\varphi(y) = y$, where $x, y \in K$. Then

$$\|\varphi(x)-\varphi(y)\| \leqslant \frac{\|x-\varphi(x)\|}{2} + \frac{\|y-\varphi(y)\|}{2} = 0.$$

Hence $x = \varphi(x) = \varphi(y) = y$. This completes the proof.

Note. Kirk [8] has proved a fixed point theorem with the help of Theorem C and using the concept of normal structure (which is defined in [3]) where, however the unicity is not guaranteed.

THEOREM 2. *Let K be a non-empty, bounded, closed and convex subset of a uniformly convex Banach space X. Let φ be a mapping of K into itself such that*

(i) $\|\varphi(x)-\varphi(y)\| \leqslant \frac{\|x-\varphi(x)\|}{2} + \frac{\|y-\varphi(y)\|}{2}$, $x, y \in K$

and

(ii) $\sup_{z\in F}\|z-\varphi(z)\| \leqslant \frac{\delta(F)}{2}$, *where F is any non-empty convex subset of K which is mapped into itself by φ.*

Then the sequence $\{x_n\}$, where $x_{n+1} = \frac{x_n+\varphi(x_n)}{2}$, converges to the fixed point of φ in K, where x_0 is any arbitrary point of K.

Note. One may refer to a theorem of Krasnoselski ([2], p. 30 and [9]), where the same conslusion as above is obtained under different assumptions.

Proof. The existence of the fixed point of φ in K is given by Theorem 1. We consider the sequence $\{x_n - \varphi(x_n)\}$. Two cases arise.

Case I. There exists an $\varepsilon > 0$ such that $\|x_n - \varphi(x_n)\| \geqslant \varepsilon$ for all $n > N$. Let y be the fixed point of φ in K. Now

$$\big\|(x_n-y)-(\varphi(x_n)-y)\big\| = \|x_n-\varphi(x_n)\| \geqslant \varepsilon, \quad n > N.$$

Since X is uniformly convex and $x_n \in K$, we have

$$\|x_{n+1}-y\| = \left\|\frac{x_n+\varphi(x_n)}{2} - \frac{y+\varphi(y)}{2}\right\|$$
$$\leqslant \delta\max\big(\|x_n-y\|, \|\varphi(x_n)-\varphi(y)\|\big), \quad n > N,\ 0<\delta<1.$$

Now

$$\|\varphi(x_n)-\varphi(y)\| \leqslant \tfrac{1}{2}[\|x_n-\varphi(x_n)\| + \|y-\varphi(y)\|]$$
$$\leqslant \tfrac{1}{2}[\|x_n-y\| + \|y-\varphi(y)\| + \|\varphi(y)-\varphi(x_n)\|].$$

So, $\|\varphi(x_n)-\varphi(y)\| \leqslant \|x_n-y\|$.

Hence $\|x_{n+1}-y\| \leqslant \delta\|x_n-y\|$, $n > N$, $0<\delta<1$.

$\therefore$ $\{\|x_n-y\|\}$, $n > N$, is a monotone decreasing sequence tending to zero. Hence $\lim x_n = y$ and this proves the theorem.

Case II. There exists a sequence of integers $\{n_k\}$ such that

$$\lim_{k\to\infty}\|x_{n_k}-\varphi(x_{n_k})\| = 0.$$

Now

$$\|\varphi(x_{n_k})-\varphi(x_{n_l})\| \leqslant \frac{\|x_{n_k}-\varphi(x_{n_k})\|}{2} + \frac{\|x_{n_l}-\varphi(x_{n_l})\|}{2}.$$

$\therefore$ $\{\varphi(x_{n_k})\}$ is a Cauchy sequence and hence it converges, say, to u. So $\lim x_{n_k} = \lim\varphi(x_{n_k}) = u$.

Also

$$\|u-\varphi(u)\| \leqslant \|u-x_{n_k}\| + \|x_{n_k}-\varphi(x_{n_k})\| + \|\varphi(x_{n_k})-\varphi(u)\|.$$

So,

$$\frac{\|u-\varphi(u)\|}{2} \leqslant \|u-x_{n_k}\| + \|x_{n_k}-\varphi(x_{n_k})\| + \frac{\|x_{n_k}-\varphi(x_{n_k})\|}{2}$$

for each positive integer k.

This implies that $u = \varphi(u)$, i.e., u is the fixed point of φ in K.

Also

$$\|x_{n+1}-u\| = \left\|\frac{x_n+\varphi(x_n)}{2}-\frac{u+\varphi(u)}{2}\right\|$$
$$\leqslant \tfrac{1}{2}\|x_n-u\|+\tfrac{1}{2}\|\varphi(x_n)-\varphi(u)\|.$$

But

$$\|\varphi(x_n)-\varphi(u)\| \leqslant \frac{\|x_n-\varphi(x_n)\|}{2}+\frac{\|u-\varphi(u)\|}{2}$$
$$\leqslant \frac{\|x_n-u\|}{2}+\frac{\|\varphi(x_n)-\varphi(u)\|}{2}.$$

Therefore $\|\varphi(x_n)-\varphi(u)\| \leqslant \|x_n-u\|$.

$\therefore$ $\|x_{n+1}-u\| \leqslant \|x_n-u\|$, and since $\lim x_{n_k} = u$, we have $\lim x_n = u$. This proves the theorem.

THEOREM 3. *Let X be a uniformly convex Banach space and let φ be a mapping of X into itself such that*

(i) $\|\varphi(x)-\varphi(y)\| \leqslant \dfrac{\|x-\varphi(x)\|}{2}+\dfrac{\|y-\varphi(y)\|}{2}$, $x, y \in X$

and

(ii) $\sup_{y\in H}\|y-\varphi(y)\| \leqslant \dfrac{\delta(H)}{2}$, *where H is any non-empty convex subset of X which is mapped into itself by φ.*

Then if φ has a fixed point u in X, the sequence $\{x_n\}$ given by $x_{n+1} = \dfrac{x_n+\varphi(x_n)}{2}$, where x_0 is any arbitrary point of X, converges to u.

Proof. Consider the closed sphere K with u as centre and d $(=\|u-x_0\|)$ as radius. If $y \in K$, then we get

$$\|\varphi(y)-u\| = \|\varphi(y)-\varphi(u)\|$$
$$\leqslant \frac{\|y-\varphi(y)\|}{2}+\frac{\|u-\varphi(u)\|}{2}$$
$$\leqslant \frac{\|y-u\|}{2}+\frac{\|u-\varphi(y)\|}{2}.$$

So, $\|\varphi(y)-u\| \leqslant \|y-u\| \leqslant d$.

Hence $\varphi(y) \in K$, i.e., φ maps K into itself. Also K is bounded, closed, convex and non-empty. Hence, by Theorem 1, φ has a unique fixed point in K and, by Theorem 2, $\{x_n\}$ converges to u. This proves the theorem.

THEOREM 4. *Let X be a Banach space and x_0 an arbitrary point of X. Let φ be a mapping of X into itself such that*

$$\|\varphi(x)-\varphi(y)\| \leqslant \frac{\|x-\varphi(x)\|}{2}+\frac{\|y-\varphi(y)\|}{2}, \quad x, y \in X.$$

Then if the sequence $\{x_n\}$, where $x_{n+1} = \dfrac{x_n+\varphi(x_n)}{2}$, converges to ξ, then ξ is the unique fixed point of φ in X.

Proof. We define an operator φ_1 as follows

$$\varphi_1(x) = \frac{x}{2}+\frac{\varphi(x)}{2}.$$

Then φ_1 maps X into itself and the sequence $\{x_n\}$ becomes the sequence of iterates of x_0 by φ_1.

Now for $x, y \in X$ we have

$$\|\varphi_1(x)-\varphi_1(y)\| \leqslant \frac{\|x-y\|}{2}+\frac{\|x-\varphi(x)\|}{4}+\frac{\|y-\varphi(y)\|}{4}$$
$$= \frac{\|x-y\|}{2}+\frac{\|x-\varphi_1(x)\|}{2}+\frac{\|y-\varphi_1(y)\|}{2}.$$

Hence

$$\|x_{n+1}-\varphi_1(\xi)\| \leqslant \|\varphi_1(x_n)-\varphi_1(\xi)\|$$
$$\leqslant \frac{\|x_n-\xi\|}{2}+\frac{\|x_n-\varphi_1(x_n)\|}{2}+\frac{\|\xi-\varphi_1(\xi)\|}{2}$$
$$\leqslant \frac{\|x_n-\xi\|}{2}+\frac{\|x_n-x_{n+1}\|}{2}+\frac{\|\xi-x_{n+1}\|}{2}+\frac{\|x_{n+1}-\varphi_1(\xi)\|}{2}.$$

$\therefore$ $\|x_{n+1}-\varphi_1(\xi)\| \leqslant \|x_n-\xi\|+\|x_n-x_{n+1}\|+\|\xi-x_{n+1}\|$.

Since $\lim x_n = \xi$, the above inequality implies $\xi = \varphi_1(\xi)$. So $\xi = \varphi_1(\xi) = \dfrac{\xi}{2}+\dfrac{\varphi(\xi)}{2}$, which gives $\xi = \varphi(\xi)$. This proves the theorem.

Browder and Petryshyn [10] have proved the following:

Let X be a uniformly convex Banach space and let φ be a mapping of X into itself such that

$$\|\varphi(x)-\varphi(y)\| \leqslant \|x-y\|, \quad x, y \in X.$$

Then a necessary and sufficient condition for $u = \varphi(u)$ to have a solution in X is that the sequence of iterates $\{x_n\}$, $x_{n+1} = \varphi(x_n)$, with x_0 arbitrary, be bounded in X.

Combining Theorems 3 and 4, we obtain

THEOREM 5. *Let φ be a mapping of a uniformly convex Banach space X into itself such that*

(i) $\|\varphi(x)-\varphi(y)\| \leqslant \dfrac{\|x-\varphi(x)\|}{2}+\dfrac{\|y-\varphi(y)\|}{2}$, $x, y \in X$

and

(ii) $\sup_{y\in F}\|y-\varphi(y)\| \leqslant \dfrac{\delta(F)}{2}$, *where F is any non-empty convex subset of X which is mapped into itself by φ.*

Then φ has a fixed point u in X if and only if the sequence $\{x_{n+1}\}$, $x_{n+1} = \frac{x_n+\varphi(x_n)}{2}$, x_0 being an arbitrary point in X, converges to u.

Finally we prove the following theorem.

THEOREM 6. *Let $\{f_n\}$ be a sequence of elements in a Banach space X. Let v_n be the unique solution of the equation $u-\varphi(u) = f_n$ where φ is a mapping of X into itself such that*

$$\|\varphi(x)-\varphi(y)\| \leqslant \frac{\|x-\varphi(x)\|}{2}+\frac{\|y-\varphi(y)\|}{2}, \quad x, y \in X.$$

If $\|f_n\|\to 0$ as $n\to\infty$, the sequence $\{v_n\}$ converges to the solution of the equation $u = \varphi(u)$.

Proof. We have

$$\begin{aligned}\|v_n-v_m\| &= \|v_n-\varphi(v_n)\|+\|\varphi(v_n)-\varphi(v_m)\|+\|v_m-\varphi(v_m)\| \\ &\leqslant \|f_n\|+\frac{\|v_n-\varphi(v_n)\|}{2}+\frac{\|v_m-\varphi(v_m)\|}{2}+\|f_m\| \\ &= \|f_n\|+\frac{\|f_n\|}{2}+\frac{\|f_m\|}{2}+\|f_m\|.\end{aligned}$$

It follows, therefore, that $\{v_n\}$ is a Cauchy sequence in X. Hence it converges, say, to $v \in X$. Also,

$$\begin{aligned}\|v-\varphi(v)\| &\leqslant \|v-v_n\|+\|v_n-\varphi(v_n)\|+\|\varphi(v_n)-\varphi(v)\| \\ &\leqslant \|v-v_n\|+\|f_n\|+\frac{\|v_n-\varphi(v_n)\|}{2}+\frac{\|v-\varphi(v)\|}{2}.\end{aligned}$$

$\therefore$ $\|v-\varphi(v)\| \leqslant 2\|v-v_n\|+3\|f_n\|$ for arbitrary positive integer n. Hence it follows that $v = \varphi(v)$ and this completes the proof.

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