analytic sets, and Σ the class of Lebesgue-measurable sets or sets with the Baire property. From well-known facts it is easy to verify that the conditions of the above theorem are satisfied. Consequently, Theorem 1 of [1] follows from the above theorem. It also follows that there is no separable σ-algebra on I containing $\boldsymbol{A}$ and contained in $\boldsymbol{O}$, the class of sets with the Baire property. We believe that Theorem 2 says something more in the following sense: Fix any analytic non-Borel set A in I and let $\boldsymbol{A}_0$ be the σ-algebra on I generated by $\boldsymbol{B}$ and all the Borel isomorphs of A. Then $\boldsymbol{A}_0$ is also $\boldsymbol{B}$-mixing and hence the preceeding two special cases of Theorem 2 are still valid with $\boldsymbol{A}$ replaced by $\boldsymbol{A}_0$. However, we do not known whether $\boldsymbol{A}_0$ is properly contained in $\boldsymbol{A}$. We do not know whether any two analytic non-Borel subsets of I are Borel isomorphic.

The following theorem is a direct consequence of Theorem 2.

THEOREM 3. *Assume the hypothesis of Theorem 2. Let U be any subset of $X\times X$ such that the vertical sections of U generate $\boldsymbol{Z}$. Then $U \notin \boldsymbol{C}\times\boldsymbol{\Sigma}$. Here $\boldsymbol{C}$ is the class of all subsets of X.*

Clearly, Theorem 2 of [1] is a simple special case of the above theorem.

Remark 2. Assume the setup of Remark 1. If $\boldsymbol{C}$ is a $\boldsymbol{B}$-Souslin σ-algebra, then there is no separable σ-algebra containing $\boldsymbol{A}$. In fact, there is no such algebra containing $\boldsymbol{A}_0$ in that case. Thus, in particular, if one assumes the axiom of determinateness, then there is no separable σ-algebra containing $\boldsymbol{A}_0$ on I. However, we do not know whether, conversely, the non-existence of a separable σ-algebra containing $\boldsymbol{A}$ implies that $\boldsymbol{C}$ is a $\boldsymbol{B}$-Souslin σ-algebra.

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Note added in proof: Regarding non-isomorphic analytic sets see A. Maitra and C. Ryll-Nardzewski in Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phy. 18 (1970) pp. 177–178.

Reference

[1] B. V. Rao, *Remarks on analytic sets*, Fund. Math. 66 (1970), pp. (237–239).

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On uniform universal spaces

by

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The aim of the paper is to prove (Theorem 2) the existence of a universal space for the class of all uniform spaces whose uniformities have a dimension not greater than n and have a base of cardinality not greater than γ, consisting of coverings of cardinality not greater than τ, where n is a finite number, γ and τ are infinite cardinal numbers. A theorem of Nagata [6] concerning a universal metrizable space of a given topological dimension may be regarded as a special case of our theorem for $\gamma = \aleph_0$.

The condition limiting the cardinalities of the coverings from the base of the uniformities is necessary, because the class of uniform spaces of a given dimension and a fixed cardinality of bases for uniformities, such that each two spaces of the class are not uniformly homeomorphic, does not form a set in general. For example, the class consisting of all discrete spaces (they have uniformities consisting of single-point-set coverings) do not form a set.

The proof of the existence of this universal space is based on Theorem 1, which presents a strenghtened form of a factorization theorem from [3].

I wish to express my gratitude to Docent J. Mioduszewski for helpful conversations during the writing of this paper.

§ 1. Preliminaries. A *pseudouniformity* U on set X is a family of coverings of X such that:

(1) U is directed with respect to star refinement,

(2) if $P \in U$ and $P \succ P'$, then $P' \in U$ ($P \succ P'$ — this means that P is a refinement of P').

A subfamily B of U such that each $P' \in U$ has a refinement $P \in B$ is said to be a *base* of U.

If a pseudouniformity U is such that:

(3) for each distinct point x' and x'' from X there exists a $P \in U$ such that $x'' \notin \operatorname{st}(x', P)$,

then U is said to be a *uniformity*.

A pair (X, U) is said to be a pseudouniform (uniform) space.

A map $f\colon X \to Y$ is said to be *uniform* with respect to a pseudouniformity (uniformity) U on X and a pseudouniformity (uniformity) V on Y, $f\colon (X, U) \to (Y, V)$ iff for each $P \in V$ we have $f^{-1}(P) \in U$, where $f^{-1}(P)$ means the covering $\{f^{-1}(V)\colon V \in P\}$.

If for a pseudouniformity U there exists a base of cardinality $\leqslant \gamma$ consisting of coverings of cardinality $\leqslant \tau$, then U is said to be *of double weight* $\leqslant (\gamma, \tau)$; $\operatorname{dweight} U \leqslant (\gamma, \tau)$.

If a pseudouniformity U contains a base consisting of coverings of order $\leqslant n+1$, then it is said to be *of dimension* $\leqslant n$, $\dim U \leqslant n$.

§ 2. A factorization theorem. We shall show here a theorem and some propositions, which are analogous to those of [3], where instead of the weight appears the double weight. We shall give proofs of these propositions, but we shall omit some inessential details which do not directly concern the double weight.

Proposition 1. *Let (X, U) be a pseudouniform space. Then there exists a uniform map $q\colon (X, U) \to (X_U, U_q)$ onto a uniform space (X_U, U_q) such that*:

$$\dim U_q \leqslant \dim U, \qquad \operatorname{dweight} U_q \leqslant \operatorname{dweight} U$$

and

$$\textit{for each } P \in U \textit{ there exists a } {}_qP \in U_q \textit{ such that } q^{-1}({}_qP) \succ P.$$

Proof. The family X_U of subsets of X

$$[x] = \bigcap \{\operatorname{st}(x, P)\colon P \in U\},$$

forms a partition of X. Let $q\colon X \to X_U$, $q(x) = [x]$ be the quotient map onto the partition. For each $P \in U$ the family

$${}_qP = \{X_U - q(X - V)\colon V \in P\}$$

forms a covering of X_U of cardinality $\leqslant \operatorname{card} P$ and of order $\leqslant \operatorname{ord} P$. We regard the family $\{{}_qP\colon P \in B\}$, where B is a base for U, as a base for U_q.

Proposition 2. *For each two coverings P', P such that $P' \succ P$ there exists a covering P'' such that*

$$P' \succ P'' \succ P,$$

$$\operatorname{card} P'' \leqslant \operatorname{card} P \quad \textit{and} \quad \operatorname{ord} P'' \leqslant \operatorname{ord} P'.$$

Proof. Let $\varphi\colon P' \to P$ be a map such that for each $V \in P'$ we have $V \subset \varphi(V) \in P$. The existence of φ follows from the axiom of choice. For each $U \in P$, let

$$U_\varphi = \bigcup \{V \in P'\colon \varphi(V) = U\}.$$

We define P'' as $P'' = \{U_\varphi\colon U \in P\}$.

Proposition 3 (cf. Isbell [2] for the case where P is finite). *Let U be a pseudouniformity. For each point-finite covering $P \in U$ there exists a covering $P' \in U$ such that $P' \underset{*}{\succ} P$, (where $P' \underset{*}{\succ} P$ means that P' is a star refinement of P), and*

$$\operatorname{card} P' \leqslant \operatorname{card} P \quad \textit{if} \quad \operatorname{card} P \textit{ is infinite},$$

$$\operatorname{card} P < \aleph_0 \quad \textit{if} \quad \operatorname{card} P < \aleph_0.$$

Proof. Let Q be a covering from U such that $Q \underset{*}{\succ} P$. We define an equivalence relation r on Q assuming that for each $V, V' \in Q$

$$(VrV') \Longleftrightarrow (\text{for each } U \in P,\ V \subset U \text{ iff } V' \subset U).$$

Let $[Q]$ mean the covering consisting of sets of the form

$$[V] = \bigcup \{V'\colon VrV'\}.$$

It is obvious that $Q \succ [Q] \succ P$. The cardinality of $[Q]$ is not greater than $\operatorname{card} P$, since $\operatorname{card} Q$, being equal to the cardinality of the family of all equivalence classes of r, is not greater than the cardinality of the family of finite subsets of P. In fact, let P_V means the maximal family consisting of elements of P such that V is contained in each element of P_V. From the definition of r it follows that each P_V uniquely determines $[V]$. The family P_V, $V \in Q$, is finite, since P is a point-finite covering.

Now we show that for each $x \in X$, $\operatorname{st}(x, [Q]) \subset U$, where U is an element of P. In fact, $\operatorname{st}(x, [Q]) = \bigcup \{[V]\colon x \in [V] \in [Q]\}$. Let us take for each $[V]$, $x \in [V] \in [Q]$, a V' for which $V'rV$, $x \in V'$. Since $Q \underset{*}{\succ} P$, hence the sum of such V' is contained in some $U \in P$. From the definition of r it follows that $\operatorname{st}(x, [Q]) \subset U$. Now let us take a covering $Q' \in U$ such that $Q' \underset{*}{\succ} [Q]$. It is easy to verify that the covering $P' = [Q']$ is such that $P' \underset{*}{\succ} P$ and $\operatorname{card} P' \leqslant \operatorname{card} P$.

In the case where P is a finite covering, the covering P' obtained as above is finite, not necessarily of the same cardinality as P. Thus we have in this case $P' \underset{*}{\succ} P$ and $\operatorname{card} P' < \aleph_0$.

Proposition 4. *If U is a pseudouniformity with a finite dimension, then for each $P \in U$ there exists a covering $P' \in U$ such that*

$$P' \underset{*}{\succ} P, \qquad \operatorname{ord} P' \leqslant 1 + \dim U,$$

$$\operatorname{card} P' \leqslant \operatorname{card} P \quad \textit{if} \quad \operatorname{card} P \geqslant \aleph_0,$$

$$\operatorname{card} P' < \aleph_0 \quad \textit{if} \quad \operatorname{card} P < \aleph_0.$$

Proof. Let us take $Q \underset{*}{\succ} P$ such that $\operatorname{card} Q \leqslant \operatorname{card} P$ (see Proposition 3). According to Proposition 2 there exists a $P' \succ Q$ such that ord P'

$\leqslant 1+\dim U$, $\operatorname{card} P' \leqslant \operatorname{card} Q$ if P is infinite and $\operatorname{card} P' < \aleph_0$ if P is a finite covering.

PROPOSITION 5. *Let the dimension of a pseudouniformity U be finite and let B be a subfamily of U.*

If $\operatorname{card} B \leqslant \gamma$ and B consists of coverings of cardinality $\leqslant \tau$, then there exists a pseudouniformity $\tilde{U}$ such that:

$$B \subset \tilde{U} \subset U,$$

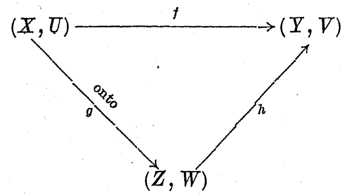
$$\dim \tilde{U} \leqslant \dim U, \quad \operatorname{dweight} \tilde{U} \leqslant (\gamma, \tau).$$

In addition, if the coverings from B are finite, then the base for $\tilde{U}$ consists of finite coverings and the assumption of finite dimension of U may be omitted.

Proof. For each two coverings $P'', P' \in B$ let us take a covering $P \in U$ such that $P \underset{*}{>} P'$ and $P \underset{*}{>} P''$, $\operatorname{ord} P \leqslant 1+\dim U$ and $\operatorname{card} P \leqslant \tau$. The covering P may be obtained by applying Proposition 3 to $P' \wedge P'' = \{V' \cap V'' \colon V' \in P', V'' \in P''\}$. Let us denote by B_1 the family of all such coverings. Let us assume that families $B_1, \ldots, B_n$ are already defined. Applying to $\bigcup \{B_i \colon i=1, \ldots, n\}$ the above operation, we shall get B_{n+1}. The family $\bigcup \{B_i \colon i=1,2,\ldots\}$ is the base for the desired pseudouniformity $\tilde{U}$.

Now we state a factorization theorem, a stronger one than Theorem 1 from [3]. The proof follows from Proposition 5 and Proposition 1.

THEOREM 1. *If $f\colon (X, U) \to (Y, V)$ is a uniform map with respect to a pseudouniformity U on X and a uniformity V on Y, then there exists a uniform factorization*



of f (i.e. $f = h \cdot g$) with respect to a uniformity W on Z such that

$$\dim W \leqslant \dim U \quad \text{and} \quad \operatorname{dweight} W \leqslant \operatorname{dweight} V.$$

PROPOSITION 6. *If U is a pseudouniformity such that $\dim U \leqslant n$ and $\operatorname{dweight} U \leqslant (\gamma, \tau)$, then there exists a base B for U, such that $\operatorname{card} B \leqslant \gamma$ and B consists of coverings of order $\leqslant n+1$ and cardinalities of these coverings are not greater than τ.*

Proof. Let B_0 be a base for U, such that $\operatorname{card} B_0 \leqslant \gamma$ and B_0 consists of coverings of cardinality $\leqslant \tau$. Applying to B_0 the countable operation as in proof of Proposition 5, we get B.

PROPOSITION 7 (cf. Isbell [2]). *If U is a uniformity such that $\dim U \leqslant n$, and $\operatorname{dweight} U \leqslant (\gamma, \tau)$, then there exists a uniformity $U^* \subset U$, inducing the same topology on X as U, having a base consisting of finite coverings of order $\leqslant n+1$, and the cardinality of the base is not greater than $\max(\gamma, \tau)$.*

Proof. Let B be a base for U consisting of coverings of order $\leqslant n+1$ and of cardinality $\leqslant \tau$, and let $\operatorname{card} B \leqslant \gamma$.

For each $U \in P$ where $P \in B$ let us form a covering consisting of two elements; the first one is $\operatorname{st}(U, P)$ and the second one is the sum $\bigcup \{U' \colon U \cap U' = \emptyset, U' \in P\}$. The family

$$B^* = \{U(P) \colon U \in P \in B\}$$

is contained in U, $\operatorname{card} B^* \leqslant \gamma\tau$, and B^* satisfies the condition from the definition of uniformity, i.e. for each two distinct points x', x'' there exists a $U(P) \in B^*$ such that $x'' \in \operatorname{st}[x', U(P)]$.

Applying Proposition 5 to B^*, we receive U^*.

Let us note that the family of interiors of elements from coverings belonging to a base of uniformity U on X forms a base for the topology induced by U. From Proposition 7 follows

PROPOSITION 8. *If U is a pseudouniformity on X such that $\dim U \leqslant n$ and $\operatorname{dweight} U \leqslant (\gamma, \tau)$, then there exists a compactification αX of X such that*

$$\dim \alpha X \leqslant n \quad \text{and} \quad \operatorname{weight} \alpha X \leqslant \max(\gamma, \tau).$$

An analogous proposition to Proposition 7 and Proposition 8 may be found in Isbell's book [2], but without the consideration of the double weight.

§ 3. Universal space. Now we shall construct a uniform space $R(\gamma, \tau, n)$ with a uniformity having a double weight $\leqslant (\gamma, \tau)$. The uniform space $R(\gamma, \tau, n)$ will be such that each uniform space (X, U) with $\dim U \leqslant n$ and $\operatorname{dweight} U \leqslant (\gamma, \tau)$ may be uniformly embedded into $R(\gamma, \tau, n)$.

Let $R(\tau, n)$ be a subset of a product of τ copies of the segment $[0,1]$, such that each point belonging to $R(\tau, n)$ has at most $n+1$ coordinates different from zero. Let us define a metric ϱ^* on $R(\tau, n)$ by

$$\varrho^*(p, p') = \sum \{|p_t - p'_t| \colon t \in T\} \tag{1}$$

where $\operatorname{card} T = \tau$, $p = \{p_t\}$, $p' = \{p'_t\}$ and $p, p' \in R(\tau, n)$.

The space $R(\tau, n)$ contains a dense subset D whose cardinality is τ; namely D is the set of all points whose coordinates are rational numbers.

We define on $R(\tau, n)$ a base for a uniformity consisting of coverings

$$\tilde{P}_i = \{S(p, 2^{-i}):\ p \in D\}, \quad i = 1, 2, \ldots,$$

where $S(p, 2^{-i}) = \{p' \in R(\tau, n):\ \varrho(p', p) < 2^{-i}\}$.

Thus $\operatorname{card}\tilde{P}_i \leqslant \tau$ for $i = 1, 2, \ldots$ and the uniformity on $R(\tau, n)$ defined above has a countable base.

PROPOSITION 9. *Let B be a base for a uniformity U on X such that $\operatorname{card}B \leqslant \gamma$ and let B consists of coverings of cardinality $\leqslant \tau$ and of order $\leqslant n+1$.*

Then for each covering $P \in B$ there exists a uniform map f_P: $(X, U) \to R(\tau, n)$ and there exists a uniform covering $\tilde{P}$ such that $f_P^{-1}(\tilde{P}) \succ P$.

Proof. By assumption, there exists a sequence $\{P_i:\ i = 1, 2, \ldots\}$ of coverings belonging to B such that

(2) $$P = P_1 \underset{*}{<} P_2 \underset{*}{<} P_3 \underset{*}{<} \ldots,$$

(3) $$\operatorname{card}P_i \leqslant \tau,$$

and

(4) $$\operatorname{ord}P_i \leqslant n+1, \quad \text{for} \quad i = 1, 2, \ldots$$

From the well known theorem of the existence of a uniform pseudometric (cf. Isbell [2] or Engelking [1]) it follows that there exists a pseudometric on X satisfying for each $x \in X$

(5) $$\operatorname{st}(x, P_{n+1}) \subset S(x, 2^{-n}) \subset \operatorname{st}(x, P_n), \quad n = 1, 2, \ldots$$

Let ϱ be a pseudometric on X satisfying (5). For each $V \in P = P_1$ we define f_V: $X \to [0, 1]$ by

(6) $$f_V(x) = \varrho(x, X - V).$$

The maps f_V, $V \in P$ induce a map f_P: $X \to R(\tau, n)$ such that

(7) $$\Pi_V[f_P(x)] = f_V(x).$$

The map f_P has the following property:

(8) $$\text{if } \varrho(x, y) < \delta, \text{ then } \varrho^*[f_P(x), f_P(y)] < 2(n+1)\delta.$$

In fact, since $|\varrho(x, A) - \varrho(y, A)| \leqslant \varrho(x, y)$ and each point $x \in X$ belongs at most to $n+1$ elements of P, we have

$$\begin{aligned}\varrho^*[f_P(x), f_P(y)] &= \sum \{|f_V(x) - f_V(y)|:\ V \in P\}\\ &= \sum \{|\varrho(x, X-V) - \varrho(y, X-V)|:\ V \in P,\ x \in V \text{ or } y \in V\}\\ &\leqslant \sum \{\varrho(x, y):\ x \in V \text{ or } y \in V,\ V \in P\} < 2(n+1)\delta.\end{aligned}$$

From (8) it follows that f_P: $(X, U) \to R(\tau, n)$ is a uniform map. Now we show that for the covering $\tilde{P} = \{S(p, \frac{1}{8}):\ p \in D\}$ from the uniformity on $R(\tau, n)$ we have

(9) $$f_P^{-1}(\tilde{P}) \succ P.$$

From (5) and (2) it follows that $S(x, 2^{-n-1}) \subset \operatorname{st}(x, P_{n+1}) \subset V$, where V is an element of P_n and hence $S(x, \frac{1}{4}) \subset V$, where $V \in P$. Thus $f_V(x) = \varrho(x, X - V) \geqslant \frac{1}{4}$ for this V, and hence

(10) $$\varrho^*[f_P(x), f_P(y)] \geqslant \tfrac{1}{4} \quad \text{if} \quad y \notin V,$$

(if $y \notin V$ then $f_V(y) = 0$ and $\varrho^*[f_P(x), f_P(y)] = \sum |f_V(x) - f_V(y)| \geqslant |f_V(x) - f_V(y)| = f_V(x) \geqslant \frac{1}{4}$). This means that

(11) $$f_P^{-1}\{S[f_P(x), \tfrac{1}{4}]\} \subset V.$$

From (11) it follows that

(12) for each $p \in R(\tau, n)$ there exists a $V \in P$ such that $f_P^{-1}[S(p, \frac{1}{8})] \subset V$.

In fact, if $p', p'' \in S(p, \frac{1}{8})$, then $\varrho^*(p', p'') < \frac{1}{4}$ and if $V \in P$ is such that $S(x, \frac{1}{4}) \subset V$ and if $f_P(x) \in S(p, \frac{1}{8})$, then $S(p, \frac{1}{8}) \subset S[f_P(x), \frac{1}{4}]$ and according to (11), we get (12). Since $S(p, \frac{1}{8})$ is an arbitrary element of P, thus the proposition is proved.

We define the uniform space $R(\gamma, \tau, n)$ as the product of γ copies of the uniform space $R(\tau, n)$.

Proposition 9 plays an analogous rôle to Urysohn's lemma.

PROPOSITION 10. *Each uniform space with uniformity of dimension $\leqslant n$ and of double weight $\leqslant (\gamma, \tau)$ is uniformly embedded into $R(\gamma, \tau, n)$.*

Proof. The uniform maps f_P: $(X, U) \to R(\tau, n)$, P belonging to a base for U, of cardinality $\leqslant \gamma$, induce a uniform map f: $(X, U) \to R(\gamma, \tau, n)$. The condition $f_P^{-1}(\tilde{P}) \succ P$ for some $\tilde{P}$ belonging to the uniformity on $R(\tau, n)$, ensure that f is a uniform embedding.

THEOREM 2. *There exists a universal uniform space with a uniformity of dimension equal to n and double weight equal to (γ, τ), i.e. such that each space with a uniformity of dimension $\leqslant n$ and double weight $\leqslant (\gamma, \tau)$ may be uniformly embedded into that space.*

Proof. Let S be a set of spaces with uniformities of dimension $\leqslant n$ and double weight $\leqslant (\gamma, \tau)$ such that every uniform space having these properties is uniformly homeomorphic to a space from S. The existence of such a set S follows, e.g., from Proposition 10. Let X be the sum of spaces belonging to S. Let us assume that the spaces are disjoint. The uniformity U on X is such that a covering P belongs to U iff the covering induced by P on each space contained in the sum is a covering from the uniformity on that space. It is easy to see that $\dim U \leqslant n$. By Proposition 10, each space from S is uniformly embedded in $R(\gamma, \tau, n)$. These embeddings induce a uniform map $f\colon (X, U) \to R(\gamma, \tau, n)$ such that if $Y \in S$, then $f|Y\colon (Y, U_Y) \to R(\gamma, \tau, n)$ is the embedding mentioned before. Applying Theorem 1 to the map $f\colon (X, U) \to R(\gamma, \tau, n)$, we get a factorization $(X, U) \xrightarrow[\text{onto}]{g} (Z, W) \xrightarrow{h} R(\gamma, \tau, n)$ into uniform maps, where $\dim W \leqslant \dim U \leqslant n$ and $\operatorname{dweight} W \leqslant (\gamma, \tau)$. The map $g|Y\colon (Y, U_Y) \to (Z, W)$, being a inner factor of a uniform embedding, is also a uniform embedding.

§ 4. Topological applications. If a completely regular space X is of dimension $\leqslant n$ (dimension always means here the covering dimension), then $\dim \beta X \leqslant n$ (where βX is a Čech-Stone compactification). If $\operatorname{weight} X \leqslant \tau$, then X is topologically embedable into a Tychonov cube I^τ. From Mardešić's theorem [4] we conclude that there exists a compactification αX of X such that $\dim \alpha X \leqslant n$ and $\operatorname{weight} \alpha X \leqslant \tau$. Thus X has a uniformity U inducing the same topology on X and such that $\dim U \leqslant n$ and $\operatorname{dweight} U \leqslant (\tau, \aleph_0)$. From Theorem 2 and Proposition 8 follows

COROLLARY 1 (Pasynkov [8]). *There exists a compact Hausdorff space of dimension $= n$ and weight $= \tau$, such that every completely regular space of dimension $\leqslant n$ and weight $\leqslant \tau$ is topologically embedded into this space.*

COROLLARY 2 (Nagata [6]). *There exists a metric space of dimension $= n$ and weight $= \tau$ such that every metric space of dimension $\leqslant n$ and weight $\leqslant \tau$ may be topologically embedded into that space.*

Proof. To prove this it suffices to know two facts:

1° every metric space of dimension $\leqslant n$ and weight $\leqslant \tau$ has a uniformity U inducing the topology on X such that $\dim U \leqslant n$ and $\operatorname{dweight} U \leqslant (\aleph_0, \tau)$.

2° (Theorem V.1 in Nagata [5], p. 126), if there exists a uniformity U on X such that $\dim U \leqslant n$ and $\operatorname{dweight} U \leqslant (\aleph_0, \tau)$, τ arbitrary, then $\dim X \leqslant n$.

We know that if $\operatorname{dweight} U \leqslant (\gamma, \tau)$, then the weight of the topology induced by U is $\leqslant \max(\gamma, \tau)$.

From the above facts it follows that if X is a metric space, then $\dim X \leqslant n$ and $\operatorname{weight} X \leqslant \tau$, iff there exists a uniformity U inducing the topology on X, such that $\dim U \leqslant n$ and $\operatorname{dweight} U \leqslant (\aleph_0, \tau)$.

Now it is obvious that Theorem 2 implies Corollary 2.

References

[1] R. Engelking, *Outline of general topology*, Warszawa 1968.

[2] I. R. Isbell, *Uniform spaces*, Providence 1964.

[3] W. Kulpa, *Factorization and inverse expansion theorems for uniformities*, Colloq. Math. 21 (1970), pp. 217–227.

[4] S. Mardešić, *On covering dimension and inverse limits of compact spaces*, Illinois J. Math. 4 (1960), pp. 278–291.

[5] J. Nagata, *On a universal n-dimensional set for metric spaces*, J. Reine Angew. Math. 204 (1960), pp. 132–138.

[6] — *Modern dimension theory*, Amsterdam 1965.

[7] Б. А. Пасынков, *Об универсальных бикомпактах данного веса и данной размерности*, ДАН СССР 154 (1964), pp. 1042–1043.

[8] — *Об универсальных бикомпактах и метрических пространствах данной размерности*, Fund. Math. 60 (1967), pp. 285–309.

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