Finally, we note that the first statement of Theorem 4 (concerning isotope maps) is trivial from Theorem 1 if \( K \) has a zero. If \( K \) has a zero define
\[
f(\langle A \rangle) = \bigvee \{ f(A') || A' \leq A \}.
\]
f obviously satisfies all the requirements. If no \( A_0 \) exists, then, of course, \( f(\langle A \rangle) = 0 \).

Non-existence of certain Borel structures

by

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This note conceptually simplifies the proofs and extends the theorems of [1] and puts them in a more general setting.

Let \((X, B)\) be any separable (countably generated and containing singletons) Borel space, where to avoid trivialities \( X \) is assumed to be uncountable. Sets in \( B \) are to be called Borel subsets of \( X \). Throughout, \( B \) is fixed.

**Theorem 1.** For any \( \sigma \)-algebra \( \Sigma \) on \( X \) containing \( B \), the following are equivalent:

(i) Any one-one real \( \Sigma \)-measurable function on \( X \) coincides with a \( B \)-measurable function on an uncountable Borel subset of \( X \).

(ii) Any separable \( \sigma \)-algebra \( S \) on \( X \) with \( B \subseteq S \) coincides with \( B \) on an uncountable Borel subset of \( X \), that is, on some uncountable Borel subset of \( X \) the restrictions of \( B \) and \( S \) coincide.

Proof: Given (i), we can prove (ii) by looking at the Marczewski function associated with any countable generator for \( S \). Conversely, given (ii), we can prove (i) by looking at the separable \( \sigma \)-algebra induced by the given function and \( B \).

**Definition 1.** A \( \sigma \)-algebra \( \Sigma \) on \( X \) containing \( B \) and satisfying any one of the above two equivalent conditions is said to be a \( B \)-Souslin \( \sigma \)-algebra for \( X \) (with due respect to the work done by Souslin).

**Definition 2.** A \( \sigma \)-algebra \( Z \) on \( X \) is said to be \( B \)-mixing if \( Z \) contains \( B \) and any uncountable Borel subset of \( X \) contains an element of \( Z - B \).

From the above definitions and Theorem 1, we have the following theorem, which can be easily proved by contradiction.

**Theorem 2.** Let \( Z \) be any \( B \)-mixing \( \sigma \)-algebra on \( X \). Let \( \Sigma \) be any \( B \)-Souslin \( \sigma \)-algebra containing \( Z \). Then there is no separable \( \sigma \)-algebra on \( X \) containing \( Z \) and contained in \( \Sigma \). Consequently, no separable \( \sigma \)-algebra containing \( Z \) can be a \( B \)-Souslin \( \sigma \)-algebra.

Remark 1. Throughout this paragraph let \( X \) be \( I \) the unit interval, \( B \) its usual Borel \( \sigma \)-algebra, \( Z = A \) the \( \sigma \)-algebra generated by its usual
On uniform universal spaces

by

W. Kulpa (Katowice)

The aim of the paper is to prove (Theorem 2) the existence of a universal space for the class of all uniform spaces whose uniformities have a dimension not greater than \( n \) and have a base of cardinality not greater than \( \gamma \), consisting of coverings of cardinality not greater than \( \tau \), where \( n \) is a finite number, \( \gamma \) and \( \tau \) are infinite cardinal numbers. A theorem of Nagata [6] concerning a universal metrizable space of a given topological dimension may be regarded as a special case of our theorem for \( \gamma = n \).

The condition limiting the cardinalities of the coverings from the base of the uniformities is necessary, because the class of uniform spaces of a given dimension and a fixed cardinality of bases for uniformities, such that each two spaces of the class are not uniformly homeomorphic, does not form a set in general. For example, the class consisting of all discrete spaces (they have uniformities consisting of single-point-set coverings) do not form a set.

The proof of the existence of this universal space is based on Theorem 1, which presents a strengthened form of a factorization theorem from [3].

I wish to express my gratitude to Docent J. Mioduszewski for helpful conversations during the writing of this paper.

§ 1. Preliminaries. A pseudouniformity \( U \) on set \( X \) is a family of coverings of \( X \) such that:

1. \( U \) is directed with respect to star refinement,
2. If \( P \in U \) and \( P \prec P' \), then \( P' \in U \) (\( P \prec P' \) — this means that \( P \) is a refinement of \( P' \)).

A subfamily \( B \) of \( U \) such that each \( P' \in U \) has a refinement \( P \in B \) is said to be a base of \( U \).

If a pseudouniformity \( U \) is such that:

3. for each distinct point \( x' \) and \( x'' \) from \( X \) there exists a \( P \in U \) such that \( x'' \notin st(x', P) \),

then \( U \) is said to be a uniformity.