Finally, we note that the first statement of Theorem 4 (concerning isotone maps) is trivial from Theorem 1 if K has a zero. If K has a zero define

$$f(\langle A\rangle) = \bigvee \left(f_\lambda(A_{(\lambda)}) \mid \lambda \in \Lambda\right).$$

f obviously satisfies all the requirements. If no $A_{(\lambda)}$ exists, then, of course, $f(\langle A\rangle) = 0$.

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Non-existence of certain Borel structures

by

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This note conceptually simplifies the proofs and extends the theorems of [1] and puts them in a more general setting.

Let $(X, \boldsymbol{B})$ be any separable (countably generated and containing singletons) Borel space, where to avoid trivialities X is assumed to be uncountable. Sets in $\boldsymbol{B}$ are to be called Borel subsets of X. Throughout, $\boldsymbol{B}$ is fixed.

THEOREM 1. *For any σ-algebra $\boldsymbol{\Sigma}$ on X containing $\boldsymbol{B}$, the following are equivalent:*

(i) *Any one-one real $\boldsymbol{\Sigma}$-measurable function on X coincides with a $\boldsymbol{B}$-measurable function on an uncountable Borel subset of X.*

(ii) *Any separable σ-algebra $\boldsymbol{S}$ on X with $\boldsymbol{B} \subset \boldsymbol{S} \subset \boldsymbol{\Sigma}$ coincides with $\boldsymbol{B}$ on an uncountable Borel subset of X, that is, on some uncountable Borel subset of X the restrictions of $\boldsymbol{B}$ and $\boldsymbol{S}$ coincide.*

Proof: Given (i), we can prove (ii) by looking at the Marczewski function associated with any countable generator for $\boldsymbol{S}$. Conversely, given (ii), we can prove (i) by looking at the separable σ-algebra induced by the given function and $\boldsymbol{B}$.

DEFINITION 1. A σ-algebra $\boldsymbol{\Sigma}$ on X containing $\boldsymbol{B}$ and satisfying any one of the above two equivalent conditions is said to be a $\boldsymbol{B}$-*Souslin σ-algebra for X* (with due respect to the work done by Souslin).

DEFINITION 2. A σ-algebra $\boldsymbol{Z}$ on X is said to be $\boldsymbol{B}$-*mixing* if $\boldsymbol{Z}$ contains $\boldsymbol{B}$ and any uncountable Borel subset of X contains an element of $\boldsymbol{Z}-\boldsymbol{B}$.

From the above definitions and Theorem 1, we have the following theorem, which can be easily proved by contradiction.

THEOREM 2. *Let $\boldsymbol{Z}$ be any $\boldsymbol{B}$-mixing σ-algebra on X. Let $\boldsymbol{\Sigma}$ be any $\boldsymbol{B}$-Souslin σ-algebra containing $\boldsymbol{Z}$. Then there is no separable σ-algebra on X containing $\boldsymbol{Z}$ and contained in $\boldsymbol{\Sigma}$. Consequently, no separable σ-algebra containing $\boldsymbol{Z}$ can be a $\boldsymbol{B}$-Souslin σ-algebra.*

Remark 1. Throughout this paragraph let X be I the unit interval, $\boldsymbol{B}$ its usual Borel σ-algebra, $\boldsymbol{Z} = \boldsymbol{A}$ the σ-algebra generated by its usual analytic sets, and Σ the class of Lebesgue-measurable sets or sets with the Baire property. From well-known facts it is easy to verify that the conditions of the above theorem are satisfied. Consequently, Theorem 1 of [1] follows from the above theorem. It also follows that there is no separable σ-algebra on I containing $\boldsymbol{A}$ and contained in $\boldsymbol{O}$, the class of sets with the Baire property. We believe that Theorem 2 says something more in the following sense: Fix any analytic non-Borel set A in I and let $\boldsymbol{A}_0$ be the σ-algebra on I generated by $\boldsymbol{B}$ and all the Borel isomorphs of A. Then $\boldsymbol{A}_0$ is also $\boldsymbol{B}$-mixing and hence the preceeding two special cases of Theorem 2 are still valid with $\boldsymbol{A}$ replaced by $\boldsymbol{A}_0$. However, we do not known whether $\boldsymbol{A}_0$ is properly contained in $\boldsymbol{A}$. We do not know whether any two analytic non-Borel subsets of I are Borel isomorphic.

The following theorem is a direct consequence of Theorem 2.

THEOREM 3. *Assume the hypothesis of Theorem 2. Let U be any subset of $X\times X$ such that the vertical sections of U generate $\boldsymbol{Z}$. Then $U\notin \boldsymbol{C}\times\Sigma$. Here $\boldsymbol{C}$ is the class of all subsets of X.*

Clearly, Theorem 2 of [1] is a simple special case of the above theorem.

Remark 2. Assume the setup of Remark 1. If $\boldsymbol{C}$ is a $\boldsymbol{B}$-Souslin σ-algebra, then there is no separable σ-algebra containing $\boldsymbol{A}$. In fact, there is no such algebra containing $\boldsymbol{A}_0$ in that case. Thus, in particular, if one assumes the axiom of determinateness, then there is no separable σ-algebra containing $\boldsymbol{A}_0$ on I. However, we do not know whether, conversely, the non-existence of a separable σ-algebra containing $\boldsymbol{A}$ implies that $\boldsymbol{C}$ is a $\boldsymbol{B}$-Souslin σ-algebra.

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Note added in proof: Regarding non-isomorphic analytic sets see A. Maitra and C. Ryll-Nardzewski in Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phy. 18 (1970) pp. 177–178.

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On uniform universal spaces

by

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The aim of the paper is to prove (Theorem 2) the existence of a universal space for the class of all uniform spaces whose uniformities have a dimension not greater than n and have a base of cardinality not greater than γ, consisting of coverings of cardinality not greater than τ, where n is a finite number, γ and τ are infinite cardinal numbers. A theorem of Nagata [6] concerning a universal metrizable space of a given topological dimension may be regarded as a special case of our theorem for $\gamma=\aleph_0$.

The condition limiting the cardinalities of the coverings from the base of the uniformities is necessary, because the class of uniform spaces of a given dimension and a fixed cardinality of bases for uniformities, such that each two spaces of the class are not uniformly homeomorphic, does not form a set in general. For example, the class consisting of all discrete spaces (they have uniformities consisting of single-point-set coverings) do not form a set.

The proof of the existence of this universal space is based on Theorem 1, which presents a strenghtened form of a factorization theorem from [3].

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§ 1. Preliminaries. A *pseudouniformity* U on set X is a family of coverings of X such that:

(1) U is directed with respect to star refinement,

(2) if $P\in U$ and $P\succ P'$, then $P'\in U$ ($P\succ P'$ — this means that P is a refinement of P').

A subfamily B of U such that each $P'\in U$ has a refinement $P\in B$ is said to be a *base* of U.

If a pseudouniformity U is such that:

(3) for each distinct point x' and x'' from X there exists a $P\in U$ such that $x''\notin \operatorname{st}(x', P)$,

then U is said to be a *uniformity*.