Subdivision rings of a semiring

by

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A semiring R is a non-empty set together with two associative operations called addition and multiplication and denoted by $(+)$ and $(\cdot)$ respectively such that the multiplication is distributive with respect to the addition. By $(R, +)$ $((R, \cdot))$ we denote the additive (multiplicative) semigroup of R. Recall that a (distributive) near-ring is a semiring whose additive semigroup is a group. We first show that the maximal sub-near-rings of any semiring R are just the maximal subgroups of $(R, +)$ whose identity f satisfies $f^2 = f$. Let N_f be such a maximal sub-near-ring and H_e be a maximal subgroup of $(R, \cdot)$ having $e (\neq f)$ as an identity; then exists a subdivision ring of R having f as a zero and e as an identity if and only if $e \in N_f$ and $ne \in H_e \cup \{f\}$ for all positive integers n. Then $(H_e \cap N_f) \cup \{f\}$ is itself a subdivision ring of R if and only if it is additively closed. As an application of these results, we obtain a characterisation of semirings which are union of subdivision rings.

1. Maximal sub-near-rings of a semiring. The additive identity of a near-ring is clearly a multiplicative idempotent. A necessary condition for an element f of a semiring R to be the zero of some sub-near-ring of R is therefore: $f+f = f$ and $f^2 = f$. Conversely, if this condition holds, then f is the zero of some sub-near-ring of R, namely $\{f\}$. Thus we have:

PROPOSITION 1. *A semiring R contains subrings (sub-near-rings) if and only if the subset $F = \{f \in R; f+f = f, f^2 = f\}$ of R is not empty.*

For any additive idempotent f of R, we shall denote by N_f the maximal subgroup of $(R, +)$ having f as an identity. The following lemma precises the multiplicative properties of the N_f.

LEMMA 2. *Let f be an additive idempotent of R. Then:*

(i) *For any $x \in R$, fx and xf are additive idempotents of R.*

(ii) *For any $x \in R$, we have: $N_f x \subseteq N_{fx}$ and $xN_f \subseteq N_{xf}$.*

(iii) *If furthermore $f^2 = f$, then $fx = xf = f$ for any $x \in N_f$.*

Proof. (i) for any $x \in R$, $fx+fx = (f+f)x = fx$ and dually, $xf+$ $+xf = xf$.

(ii) N_f is the set of all $y \in R$ such that $y+f = f+y = y$ and $y+y' = y'+y = f$ for some $y' \in R$ with $y'+f = f+y' = y'$. Then, for all $x \in R$, we have $yx+fx = fx+yx = yx$, $y'x+yx = yx+y'x = fx$, $y'x+fx = fx+ +y'x = y'x$. It follows that $yx \in N_{fx}$. Therefore $N_f x \subseteq N_{fx}$. Dually, $xN_f \subseteq N_{xf}$.

(iii) if $f^2 = f$ and $x \in N_f$, then xf is an additive idempotent by (i) and is in $N_f f \subseteq N_{f^2} = N_f$ by (ii), whence $xf = f$ since a group contains only one idempotent. Dually, $fx = f$.

THEOREM 3. *The maximal sub-near-rings of a semiring R are the maximal subgroups of $(R, +)$ whose identity is a multiplicative idempotent of R.*

Proof. Take $f \in F$, $x, y \in N_f$. By 2, $xy \in xN_f \subseteq N_{xf} = N_f$, which proves that N_f is closed under multiplication. Therefore N_f is a sub-near-ring of R since it is already an additive subgroup. Conversely any sub-near-ring of R is an additive subgroup of R and its zero f is in F, so that it is contained in N_f. Therefore each N_f with $f \in F$ is a maximal sub-near-ring of R and conversely, any maximal sub-near-ring of R has this form.

COROLLARY 4. *Let R be a semiring with a commutative addition. Then the maximal subrings of R are the maximal subgroups of $(R, +)$ whose identity is a multiplicative idempotent.*

2. Subdivision rings of a semiring. We shall first study under what conditions there exists a subdivision ring having as identity and zero two given elements of the semiring R. We keep the notation of Section 1.

THEOREM 5. *Let e and f be two elements of a semiring R such that $e^2 = e$, $f+f = f$ and $f^2 = f$. Then there exists a subdivision ring of R having e as an identity and f as a zero if and only if the following conditions are satisfied*

(i) *$e \in N_f$ and $e \neq f$;*

(ii) *for all positive integers n, $ne \in H_e \cup \{f\}$.*

Moreover, under these conditions the smallest subdivision ring of R having e as an identity and f as a zero is the set D of all pa_n, where n is a positive integer such that $ne \neq f$, a_n is the inverse of ne in H_e, p is any integer and pa_n is the opposite of $(-p)a_n$ in N_f if $p < 0$, $0a_n = f$.

Proof: If there exists a subdivision ring K of R with identity e and zero f, then $K \subseteq N_f$ since K is a subgroup of $(R, +)$, so that $e \in N_f$. Also, since $K - \{f\}$ is a multiplicative subgroup of R, $K - \{f\} \subseteq H_e$, whence $ne \in H_e$ whenever $ne \neq f$. For the converse, let us suppose that (i) and (ii) hold. Call e' the opposite of e in N_f, whose existence is insured by (i). Observe that $e = a_1$ so that $0e = f$ and $pe = (-p)e'$ if $p < 0$. By

Lemma 2 (iii), $ef = fe = f$ since $e \in N_f$, which clearly implies that $(pe)f = f(pe) = f$ for all integers p.

By (ii) there exists a unique $a_n \in R$ such that $a_n e = ea_n = a_n$ and $a_n(ne) = (ne)a_n = e$ for each positive integer n such that $ne \neq f$. At once we have: $a_n f = a_n(ne)f = ef = f$, and dually, $fa_n = f$.

Now we proceed to show that D is a subdivision ring of R. This will complete the proof since then D will be the smallest subdivision ring of R with identity e and zero f.

First we have: $pa_q+f = (pe)a_q+f = (pe+f)a_q = (pe)a_q = pa_q$ and similarly $f+pa_q = pa_q$ so that f acts as a zero on the elements of D. By definition of a_q, it is easily seen that, for all integers p and all positive integers q such that $qe \neq f$, pa_q is the unique element of R such that $q(pa_q) = pe$ and $e(pa_q) = (pa_q)e = pa_q$. Then considering the following equalities:

$$\begin{aligned}(qq')(pa_q+p'a_{q'}) &= (qq'e)(pa_q+p'a_{q'}) = (qq'e)pa_q+(qq')p'a_{q'} \\ &= pq'(qea_q)+qp'(q'a_{q'}) = pq'e+qp'e = (pq'+qp')e\,,\end{aligned}$$

for all integers p, p' and all positive integers q, q' such that $qe \neq f$, $q'e \neq f$, we obtain $pa_q+p'a_{q'} = (pq'+p'q)a_{qq'}$ provided that $qq'e \neq f$. Now $qq'e \neq f$, or else $f = a_q(qq'e) = (a_q qe)(q'e) = q'e$ which is impossible. Thus we can conclude that D is additively closed. Furthermore it is clear that $(-p)a_q$ is the opposite of pa_q in D so that D is a subgroup of $(R, +)$.

It remains to show that $D - \{f\}$ is a subgroup of $(R, \cdot)$. Since $a_q \in H_e$ whenever defined, we have $e(pa_q) = p(ea_q) = pa_q$ for any $pa_q \in D$. Also, for any two elements pa_q and $p'a_{q'}$ of D, the following equalities hold: $qq'(pa_q)(p'a_{q'}) = pp'(qa_q)(q'a_{q'}) = pp'e$. Therefore $(pa_q)(p'a_{q'}) = pp'a_{qq'}$, since $qq'e \neq f$ results from $qe \neq f$ and $q'e \neq f$. In particular, for any $pa_q \neq f$, $pe \neq f$ (or else $pa_q = pea_q = f$) so that $(pa_q)(qa_p) = pqa_{pq} = e$ if p is positive, $(pa_q)\big((-q)a_{-p}\big) = (-pq)a_{-(pq)} = e$ if p is negative. This shows that pa_q has a right inverse in D, which clearly is also a left inverse. Therefore D is a subdivision ring of R, which completes the proof.

The following lemma precises the structure of $H_e \cap N_f$ when $e \in N_f$.

LEMMA 6. *Assume that $e^2 = e$ and $f \in F$ are such that $e \in N_f$ and $e \neq f$. Then:*

(i) $H_e \cap N_f = \{x \in H_e;\, xf = f\} = \{x \in H_e;\, fx = f\}$.

(ii) *$H_e \cap N_f$ is a subgroup of $(R, \cdot)$.*

(iii) *If e' is the opposite of e in N_f, then $x' = xe' = e'x$ is in $H_e \cap N_f$ and is the opposite of x in N_f for all $x \in H_e \cap N_f$.*

(iv) *For any $x, y \in H_e \cap N_f$, $x+y = y+x$.*

Proof. First we prove that $e' \in H_e \cap N_f$. We have the following three relations

(1) $$e+f = f+e = e\,,$$

(2) $$e+e' = e'+e = f\,,$$

(3) $$e'+f = f+e' = e'\,.$$

Multiplying these relations on each side successively by e and e', we obtain, by uniqueness of the opposite in N_f: $e' = ee' = e'e$ and $e = e'e'$ Therefore $e' \in H_e \cap N_f$.

To prove (i) and (iii), assume that $x \in H_e \cap N_f$. By 2 (iii), we have $xf = fx = f$, whence $x \in \{y \in H_e; yf = f\}$ and $x \in \{y \in H_e; fy = f\}$. Conversely, let $x \in H_e$ be such that $xf = f$ (for instance). Multiplying (1), (2) and (3) on the left by x, we obtain: $xe+xf = xf+xe = xe$, $xe+xe' = xe'+xe = xf$, $xe'+xf = xf+xe' = xe'$. Since $xf = f$ and $xe = x$, we conclude that $x \in H_e \cap N_f$ and that xe' is the opposite of x in N_f. Dually, if $x \in H_e$ and $fx = f$, then $x \in H_e \cap N_f$ and $e'x$ is opposite of x in N_f (whence $xe' = e'x$). This completes the proof of (i) and (iii).

Since both H_e and $A_f = \{x \in R; xf = f\}$ are closed under multiplication, it follows from (i) that $H_e \cap N_f = H_e \cap A_f$ is closed under multiplication. For any $x \in H_e \cap N_f$, denote by x^{-1} the inverse of x in H_e; then $x^{-1}f = x^{-1}xf = ef = f$, so that $x^{-1} \in H_e \cap N_f$. Thus $H_e \cap N_f$ is a subgroup of $(R, \cdot)$ and (ii) is proved.

Finally, take $x, y \in H_e \cap N_f$. By distributivity we have:

$$xy+xe+ey+e^2 = (x+e)(y+e) = xy+ey+ex+e^2\,.$$

Adding $e'xy$ and e' to the left and the right respectively, we obtain $xe+ey = ey+xe$; since $x, y \in H_e$, $x+y = y+x$.

Remark. $(H_e \cap N_f) \cup \{f\}$ need not to be closed under addition as it is shown by the following example.

Example 7. Consider the semiring given by the tables:

+	0	a	f	b	c	1
0	0	a	f	b	c	1
a	a	0	b	f	1	c
f	f	b	f	b	c	1
b	b	f	b	f	1	c
c	c	1	c	1	f	b
1	1	c	1	c	b	f

·	0	a	f	b	c	1
0	0	0	0	0	0	0
a	0	0	0	0	a	a
f	0	0	f	f	f	f
b	0	0	f	f	b	b
c	0	a	f	b	1	c
1	0	a	f	b	c	1

We see that $f+f = f$, $f^2 = f$ and $N_f = \{f, b, c, 1\}$; also $1^2 = 1 \in N_f$ and $H_1 = \{1, c\}$, so that $e = 1$ and f satisfy the assumptions of Lemma 6; but $(H_1 \cap N_f) \cup \{f\}$ is not additively closed, since $1+c = b$. Observe

however that $\{1, f\}$ is a (maximal) subdivision ring of R contained in $(H_1 \cap N_f) \cup \{f\}$.

Theorem 8. *Let e and f be two elements of a semiring R such that $f+f = f$, $f^2 = f$, $e^2 = e$, $e \neq f$ and $e \in N_f$. Then the following conditions are equivalent:*

(i) *$(H_e \cap N_f) \cup \{f\}$ is a subdivision ring of R;*

(ii) *$(H_e \cap N_f) \cup \{f\}$ is additively closed;*

(iii) *for all $x \in H_e \cap N_f$ such that $e+x \neq f$:*

(4) $$e = a(e+x) = (e+x)a' \text{ for some } a, a' \in R\,.$$

Under any of these conditions, $(H_e \cap N_f) \cup \{f\}$ is a maximal subdivision ring of R with identity e and zero f.

Proof. By Lemma 6, (ii) implies (i). Clearly, (i) implies (iii) (take $a = a' = (e+x)^{-1}$ in H_e). To prove that (iii) implies (ii), take $x, y \in (H_e \cap N_f) \cup \{f\}$; we may assume that $x \neq f$, $y \neq f$, $x+y \neq f$. Set $yx^{-1} = c$. Clearly, $e+c \neq f$ (or else $x+y = (e+c)x = f$) and $c \in H_e \cap N_f$ by Lemma 6 (ii). By (iii) there exists $a, a' \in R$ such that $e = a(e+x) = (e+x)a'$. We may suppose that $ea = ae = a$ and $ea' = a'e = a'$; if this is not the case, replace a and a' by eae and $ea'e$ respectively, and (4) still holds since $e(e+c) = (e+c)e = e+c$. Then $e+c \in H_e \cap N_f$ so that $x+y = (e+c)x \in H_e \cap N_f$, which completes the proof since the last assertion is trivial.

Observe that the conditions in Theorem 8 hold in case H_e is additively closed. Also, in case R is a near-ring, we must take $f = 0$ and then $H_e \subset N_f$.

3. Semirings which are union of division rings. The following characterization of division rings results naturally from Lemma 6.

Theorem 9. *Let R be a semiring with zero. Then R is a division ring if and only if $R - \{0\}$ is a multiplicative group whose identity 1 satisfies: $1+x = y+1 = 0$ for some $x, y \in R$.*

Proof. Clearly the condition is necessary. For the converse, we have to prove that $(R, +)$ is an abelian group. By the condition on 1, $1 \in N_0$. By Lemma 6 (i), $H_1 \cap N_0 = \{x \in H_1; x0 = 0\} = H_1$, since $x0 = 0$ for all $x \in R$; hence $N_0 = R$. Finally it follows from Lemma 6 (iv) that the addition of R is commutative.

Lemma 10. *If K and K' are two subdivision rings of a semiring R with e and e' (f and f') as identities (zeros) respectively, we have:*

(i) *$K \cap K' = \emptyset$ if $f \neq f'$;*

(ii) *$K \cap K' = \{f\}$ if $f = f'$, $e \neq e'$;*

(iii) *$K \cap K'$ is a subdivision ring of R if $f = f'$ and $e = e'$.*

The proof is straightforward.

LEMMA 11. *If R is a semiring which is the union of its subdivision rings, then any additive idempotent of R is also a multiplicative idempotent. Furthermore, for all $f \in F$, the maximal subgroup of $(R, \cdot)$ having f as identity is reduced to one element.*

Proof. Let f be an additive idempotent of R. Then f belongs to some subdivision ring K of R, which implies that f is the zero of K. Therefore f is a multiplicative idempotent. Let now $f \in F$ and take $x \in H_f$, x is an additive idempotent of R; by the first part of the proof, x is a multiplicative idempotent of R, whence $x = f$ since $x \in H_f$. Therefore H_f is reduced to one element.

LEMMA 12. *If R is a semiring which is the union of its subdivision rings, then R is a union of multiplicative subgroups of the form H_e where $e \in E$, each of them being contained in a maximal sub-near-ring of R.*

Proof. Clearly R is the union of its multiplicative groups, so that R is the union of its maximal subgroups $(H_e)_{e \in E}$. Let now e be a multiplicative idempotent. If e is also an additive idempotent, then $e \in F$ and, by Lemma 11, H_e is reduced to e; in particular H_e is contained in N_f, which is a maximal sub-near-ring of R, by Theorem 3. Assume now that e is not an additive idempotent. Since e belongs to some subdivision ring of R, $e \in N_f$ for some maximal sub-near-ring N_f of R. To show that $H_e \subseteq N_f$, let $x \in H_e$; then x belongs to some subdivision ring K of R. Let e' and f' be the zero and the identity of K, respectively. First, $K - \{f'\} \subseteq H_{e'}$; also $x \neq f'$, since H_e contains a unique multiplicative idempotent e which is not an additive idempotent, so that no element of H_e can be equal to $f' \in F$. Thus $x \in K - \{f'\}$ so that $x \in H_e \cap H_{e'}$. It follows that $e = e'$. Since $K \subseteq N_{f'}$, $e = e' \in N_f \cap N_{f'}$ so that $f = f'$ and $x \in N_f$. Therefore $H_e \subseteq N_f$ which completes the proof.

THEOREM 13. *Let R be a semiring which is the union of its subdivision rings, and E be the set of all multiplicative idempotents of R. Then $F (\subseteq E)$ is the set of all additive idempotents and, if $(K_i)_{i \in I}$ is a family of subdivision rings of R such that $\bigcup_{i \in I} K_i = R$, then there exists a partition $(I_f)_{f \in F}$ of I such that $\bigcup_{i \in I_f} K_i = N_f$ for every $f \in F$. Furthermore, there exists a subpartition $(I_e)_{e \in E-F}$ of the partition $(I_f)_{f \in F}$ such that, for any $e \in E - F$, $\bigcup_{i \in I_e} K_i$ is the maximal subdivision ring of R with identity e and zero f, where f is determined by $e \in N_f$. The set of elements of the maximal subdivision ring equals $H_e \cup \{f\}$.*

Proof. It follows from Lemma 11 that, if R is the union of its subdivision rings, then F is the set of all additive idempotents of R.

Let now $(K_i)_{i \in I}$ be a family of subdivision rings of R such that $\bigcup_{i \in I} K_i = R$. For all $f \in F$, set $I_f = \{i \in I;\ f \text{ is the zero of } K_i\}$. Clearly,

$(I_f)_{f \in F}$ is a partition of I. Let $f \in F$; then, for all $i \in I_f$, $K_i \subseteq N_f$ by Lemma 12, whence $\bigcup_{i \in I_f} K_i \subseteq N_f$. Conversely, if $x \in N_f$, then $x \in K_i$ for some $i \in I$; again by Lemma 12, since $x \in K_i \cap N_f$, we have $K_i \leqslant N_f$ so that $i \in I_f$. Therefore $\bigcup_{i \in I_f} K_i = N_f$.

For all $e \in E - F$, let $I_e = \{i \in I;\ e \text{ is the identity of } K_i\}$. If $e \in N_f$, by Lemma 12, then $K_i \subseteq N_f$ for all $i \in I_e$, so that f is the zero of K_i and $I_e \subseteq I_f$; thus $(I_e)_{e \in E}$ is a subpartition of $(I_f)_{f \in F}$, where f is determined by $e \in N_f$. Clearly, if $e \in E - F$, $e \in N_f$ for some $f \in F$, then $H_e \cup \{f\} = \bigcup_{i \in I_e} K_i$. Finally, to prove that $\bigcup_{i \in I_e} K_i$ is the maximal subdivision ring of R with identity e and zero f, it is enough to show that $H_e \cup \{f\}$ is a subdivision ring of R. By Theorem 8, we only have to prove that, for all $x \in H_e$ such that $e + x \neq f$, $e = a(e+x) = (e+x)a'$ for some $a, a' \in R$. For any $x \in H_e$ such that $e + x \neq f$, $x \in K_k$ for some $k \in I_e$; then we may take $a = a' = (e+x)^{-1}$, then inverse of $(e+x)$ in K_k.

DEFINITION 14. Let R be a semiring and $(K_i)_{i \in I}$ be a family of subdivision rings of R. We say that R is *F-disjoint union* of the family $(K_i)_{i \in I}$ if $\bigcup_{i \in I} K_i = R$ and the intersection of any two K_i is empty or reduced to one element of F.

PROPOSITION 15. *Let R be a semiring which is an F-disjoint union of a family $(K_i)_{i \in I}$ of subdivision rings of R. Then* $\operatorname{Card} I = \operatorname{Card}(E - F)$ *and $\{K_i;\ i \in I\} = \{H_e \cup \{f\};\ e \in E - F,\ e \in N_f\}$.*

Proof. By Theorem 13, there exists a partition $(I_e)_{e \in E-F}$ of I such that $\bigcup_{i \in I_e} K_i = H_e \cup \{f\}$ (where $e \in N_f$). To prove the result, it is enough to show that, for all $e \in E - F$, I_e has only one element. Let us suppose on the contrary that there exist $j, k \in I_e$ such that $j \neq k$. Since R is an F-disjoint union of the family $(K_i)_{i \in I}$, $K_i \cap K_j$ has at most one element; but this is impossible since by 10, $K_j \cap K_k$ is a division ring.

The next theorem is the main result of this section.

THEOREM 16. *Let R be a semiring, F be the set of its additive idempotents and E be the set of its multiplicative idempotents. Then the following are equivalent:*

(i) *R is the union of its subdivision rings;*

(ii) *R is F-disjoint union of a family of subdivision rings of R;*

(iii) *$(R, \cdot)$ is a union of groups, $F \subseteq E$, and there exists a mapping φ of $E - F$ onto F such that $H_e \cup \{\varphi(e)\}$ is a subdivision ring of R for all $e \in E - F$;*

(iv) *$(R, \cdot)$ is a union of groups, $E \subseteq F$, and there exists a mapping φ of $E - F$ onto F such that, whenever $f = \varphi(e)$, $e \in N_f$, $xf = f$ for all $x \in H_e$ and $H_e \cup \{f\}$ is additively closed.*

If furthermore any of these conditions holds, the mapping φ *of* (iii) *and* (iv) *is unique.*

Proof. Obviously, (ii) implies (i). By Lemma 11 and Definition 14, (i) implies (iii). By Theorem 8, (iii) is equivalent to (iv). It remains to show that (iii) implies (ii). By Lemma 11, $F \subseteq E$ implies that H_f is reduced to f for any $f \in F$. Since $(R, \cdot)$ is a union of groups, we have

$$R = \bigcup_{e \in E} H_e = \bigcup_{e \in E-F} H_e \cup \bigcup_{f \in F} H_f = \Big(\bigcup_{e \in E-F} H_e\Big) \cup F = \bigcup_{e \in E-F} \big(H_e \cup \varphi(e)\big)$$

and this is obviously an F-disjoint union of subdivision rings of R, which completes the proof.

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An example of a monostratiform λ-dendroid

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A metric compact continuum is said to be a *dendroid* if it is hereditarily unicoherent and arcwise connected. It follows that it is hereditarily decomposable (see [2], (47), p. 239). A hereditarily unicoherent and hereditarily decomposable continuum is called a *λ-dendroid*. Note that every subcontinuum of a λ-dendroid is also a λ-dendroid.

It is proved in [3], Corollary 2, p. 29, that for every λ-dendroid X there exists a unique decomposition $\mathfrak{D}$ of X (called the *canonical decomposition*):

$$X = \bigcup \{S_d \colon d \in \Delta(X)\}$$

such that

(i) $\mathfrak{D}$ is upper semicontinuous,

(ii) the elements S_d of $\mathfrak{D}$ are continua,

(iii) the hyperspace $\Delta(X)$ of $\mathfrak{D}$ is a dendroid,

(iv) $\mathfrak{D}$ is the finest possible decomposition among all decompositions satisfying (i), (ii) and (iii).

The elements S_d of $\mathfrak{D}$ are called *strata* of X. The question arises whether there exists a λ-dendroid X with trivial canonical decomposition, i.e. such that X has only one stratum.

The purpose of this paper is to give the affirmative answer to the above question.

Call a λ-dendroid to be *monostratiform* if it consists of only one stratum. Thus the hyperspace of the canonical decomposition of a monostratiform λ-dendroid is a point. It follows from [3], Theorem 7, p. 29 that:

(1) A λ-dendroid X is monostratiform if and only if every monotone mapping onto a dendroid is trivial, i.e. the whole X goes onto a point.

(See also [4], Corollaries 1 and 2, p. 933).

Construction. The decription of the example is based upon the description of Lelek's example of a dendroid with 1-dimenional set of end points (see [9], § 9, p. 314).