A new extension of countable compactness*

by

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In this paper, we consider a concept — to which we give the name "starcompactness" — which abstracts a property shared by all (T_1) countably compact spaces. Certain properties of starcompact spaces are given; in particular, the relations between starcompactness and other extensions of countable compactness are explored. Some examples which indicate certain of the peculiarities of starcompactness are presented. We have, finally, as an application of these ideas, a simple proof of a generalization of a theorem of G. Aquaro.

§ 1. Preliminaries. We recall, first, the definitions of countable compactness and several of its extensions.

DEFINITION 1. A space X is *countably compact* if every infinite subset of X has a cluster point.

DEFINITION 2. A space X is $\aleph_0$-*compact* if every countable open covering of X has a finite subcover. More generally, if $\mathfrak{m}$ is any infinite cardinal, X is $\mathfrak{m}$-*compact* if every open covering of X whose cardinality does not exceed $\mathfrak{m}$ has a finite subcover.

DEFINITION 3. A space X is *pseudocompact* if each continuous real-valued function on X is bounded.

DEFINITION 4. A space X is *weakly compact* if the only locally finite collections of non-empty open subsets of X are those which are actually finite.

The following relations connecting these concepts are well known. They are stated here to serve as a basis for comparison with similar results on starcompactness to be given in the next section. The proof of Proposition 2, in which no separation is assumed, may be of interest.

PROPOSITION 1. *$\aleph_0$-compactness implies countable compactness. The converse implication holds for T_1 spaces.*

PROPOSITION 2. *$\aleph_0$-compactness implies weak compactness.*

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Proof. Let X be a space which is not weakly compact. Then there is a locally finite collection $\mathcal{A} = \{A_n|\ n \in Z^+\}$ of distinct non-empty open subsets of X. To each $x \in X$ we may correspond a neighborhood U_x of x which meets only finitely many members of $\mathcal{A}$. Let $F_x = \{n \in Z^+|\ U_x \cap A_n \neq \emptyset\}$, and let F be any finite subset of Z^+. Define $V_F = \bigcup\{U_x|\ F_x = F\}$. Then $\mathcal{V} = \{V_F|\ F$ a finite subset of $Z^+\}$ is a countable open covering of X. Clearly any finite subcollection chosen from $\mathcal{V}$ will meet only finitely many members of $\mathcal{A}$. Thus $\mathcal{V}$ has no finite subcover.

Proposition 3. *Weak compactness implies pseudocompactness. The converse holds for completely regular spaces.*

Proposition 4. *A weakly normal pseudocompact space is countably compact.*

(A space is *weakly normal* provided that every pair of disjoint closed subsets, one of which is at most countably infinite, can be separated by open sets.)

§ 2. Starcompactness.

Theorem 1. *Let X be a space which is $\aleph_0$-compact and let $\mathcal{U}$ be any open covering of X. There is a finite set $A \subseteq X$ such that* $\mathrm{St}(A, \mathcal{U}) = X$.

Proof. Suppose that for each finite $A \subseteq X$, $\mathrm{St}(A, \mathcal{U})$ is a proper subset of X. By induction, we construct a countably infinite set $B = \{x_1, \ldots, x_n, \ldots\} \subseteq X$ such that for each integer $n \geqslant 1$, $x_{n+1} \notin V_n = \mathrm{St}(\{x_1, \ldots, x_n\}, \mathcal{U})$. Let $y \in B^-$. There is a $U \in \mathcal{U}$ with $y \in U$, and since U is open, $U \cap B \neq \emptyset$. Let k be such that $x_k \in U$. Then $y \in V_k$. Thus $\mathcal{V} = \{V_n|\ n \geqslant 1\}$ is a countable open covering of B^-. The latter, being a closed subset of an $\aleph_0$-compact space, is, itself, $\aleph_0$-compact. But, by construction of the set B, $\mathcal{V}$ has no finite subcollection which covers even B. This contradiction establishes the theorem.

Definition 5. A space X is *starcompact* if for each open covering $\mathcal{U}$ of X, there is a finite set $A \subseteq X$ such that $\mathrm{St}(A, \mathcal{U}) = X$.

Thus every $\aleph_0$-compact space is starcompact. The example which follows indicates that not every starcompact space is $\aleph_0$-compact.

Example 1. Let X consist of the non-negative integers and let the proper open subsets of X be those of the form $\{k \in X|\ 0 \leqslant k \leqslant n\}$ where $n \in X$. It is easily seen that if $\mathcal{U}$ is an open covering of X, then $\mathrm{St}(n, \mathcal{U}) = X$ for each $n \in X$. Thus X is starcompact. On the other hand, the open covering $\mathcal{U} = \big\{\{k \in X|\ 0 \leqslant k \leqslant n\}|\ n \in X\big\}$ has no finite subcover, so that X is not $\aleph_0$-compact. It is to be noted, however, that X is countably compact. In fact, any non-empty subset of X has a cluster point.

An important property of starcompact spaces is given in the following result.

Theorem 2. *Let X be a starcompact space and let $\mathcal{U}$ be a point finite open covering of X (i.e., each point of X belongs to no more than finitely many members of $\mathcal{U}$). Then $\mathcal{U}$ has a finite subcover.*

Proof. Since X is starcompact, there is a finite set $A = \{x_1, \ldots, x_n\} \subseteq X$ such that $X = \mathrm{St}(A, \mathcal{U}) = \mathrm{St}(x_1, \mathcal{U}) \cup \ldots \cup \mathrm{St}(x_n, \mathcal{U})$. But, for each k, $1 \leqslant k \leqslant n$, $\mathrm{St}(x_k, \mathcal{U})$ is the union of finitely many members of $\mathcal{U}$.

Proposition 5. *Starcompactness implies pseudocompactness.*

Proof. Let X be a space which is starcompact and let f be a continuous real-valued function on X. For each integer n, let $U_n = \{x \in X|\ n-1 < f(x) < n+1\}$. Then $\mathcal{U} = \{U_n|\ n$ an integer$\}$ is a point finite open covering of X, so that $\mathcal{U}$ has a finite subcover $\{U_{n_1}, \ldots, U_{n_k}\}$. Let $M = \max_{1 \leqslant j \leqslant k} |n_j| + 1$. For each $x \in X$, there is a j, $1 \leqslant j \leqslant k$, with $x \in U_{n_j}$. Thus $|f(x)| < |n_j| + 1 \leqslant M$.

Proposition 6. *Let X be a starcompact T_1 space. Then X is weakly compact.*

Proof. Suppose that X is not weakly compact. Then there is a countably infinite collection $\mathcal{A} = \{U_n|\ n \geqslant 1\}$ of distinct non-empty open sets which is locally finite. We construct, inductively, a similar collection whose members are pairwise disjoint. Let $x_1 \in U_1$ and let U_{x_1} be a neighborhood of x_1 which meets only finitely many members of $\mathcal{A}$. Let $V_1 = U_1 \cap U_{x_1}$. Let k_2 be the smallest integer such that $U_{x_1} \cap U_j = \emptyset$ for each $j \geqslant k_2$. Choose $x_2 \in U_{k_2}$ and let U_{x_2} be a neighborhood of x_2 which meets only finitely many members of $\mathcal{A}$. Let $V_2 = U_{k_2} \cap U_{x_2}$. Continuing in this manner, we obtain a collection $\mathcal{B} = \{V_n|\ n \geqslant 1\}$ with the desired properties. Moreover, the construction designates in each V_n a point x_n. Let $B = \{x_n|\ n \geqslant 1\}$. For each $x \notin \bigcup \mathcal{B}$, there is a neighborhood U_x of x which meets only finitely many members of $\mathcal{B}$. Let $V_x = U_x \sim B$, V_x is open since X is T_1 and no more than a finite number of points of B are in U_x. Let $\mathcal{V} = \mathcal{B} \cup \{V_x|\ x \notin \bigcup \mathcal{B}\}$. Then $\mathcal{V}$ is an open covering of X. For $x \notin \bigcup \mathcal{B}$, $B \cap \mathrm{St}(x, \mathcal{V}) = \emptyset$. On the other hand, since the members of $\mathcal{B}$ are pairwise disjoint, $\mathrm{St}(x, \mathcal{V})$ contains exactly one element of B for each $x \in \bigcup \mathcal{B}$. Thus $\mathrm{St}(A, \mathcal{V})$ is a proper subset of X for each finite set $A \subseteq X$. Therefore X is not starcompact.

Recall that an open covering $\mathcal{U}$ of a space X is called a *normal open covering* if there exists a sequence $\{\mathcal{U}_n|\ n \in Z^+\}$ of open coverings of X such that $\mathcal{U}$ is refined by $\mathcal{U}_1$, and for each $n \in Z^+$, $\mathcal{U}_n$ is refined by $\{\mathrm{St}(x, \mathcal{U}_{n+1})|\ x \in X\}$.

Definition 6. A space X is called *somewhat normal* (SN) if for each open covering $\mathcal{U}$ of X, $\{\mathrm{St}(x, \mathcal{U})|\ x \in X\}$ is a normal open covering of X.

The foregoing definition was introduced by S. L. Gulden in connection with a generalization of a result of V. L. Klyushin ([3], Theorem 2).

Little is known of this property. SN is implied by almost-2-full normality [1], but it is not known whether SN either implies or is implied by normality. Proposition 7 represents a partial result in this direction. Our interest in SN stems from Proposition 8.

PROPOSITION 7. *Let X be starcompact and normal. Then X is* SN.

Proof. Let $\mathfrak{U}$ be an open covering of X and let $A = \{x_1, \ldots, x_n\} \subseteq X$ be such that $\mathrm{St}(A, \mathfrak{U}) = X$. Then $\mathfrak{V} = \{\mathrm{St}(x_j, \mathfrak{U}) \mid 1 \leqslant j \leqslant n\}$ is a finite open covering of the normal space X; hence ([5], Theorem 4.1 of Chapter V) a normal open covering. Clearly any normal sequence of open coverings whose first member refines $\mathfrak{V}$ will serve to establish the normality of the covering $\{\mathrm{St}(x, \mathfrak{U}) \mid x \in X\}$.

COROLLARY. *If X is $\aleph_0$-compact and normal, then X is* SN.

PROPOSITION 8. *Let X be weakly compact and* SN. *Then X is starcompact.*

Proof. Let $\mathfrak{U}$ be an open covering of X and let $\mathfrak{V} = \{\mathrm{St}(x, \mathfrak{U}) \mid x \in X\}$. Since X is SN, $\mathfrak{V}$ is a normal open covering of X; hence, by Theorem 1.2 of [4], $\mathfrak{V}$ admits a (normal) locally finite open covering $\mathfrak{W}$ as a refinement. But X is weakly compact, so that $\mathfrak{W}$ must be finite. Since $\mathfrak{V}$ is refined by $\mathfrak{W}$, $\mathfrak{V}$ has a finite subcover.

LEMMA 1 (Iséki and Kasahara; [2], Theorem 2). *Let X be a regular space such that each point finite open covering of X has a finite subcover. Then X is countably compact.*

Proof. Suppose that X is not countably compact. Then there is a countably infinite set $B = \{x_n \mid n \in Z^+\} \subseteq X$ with no cluster point. Thus each point of B has a neighborhood containing no other point of B. Let U_1 be such a neighborhood of x_1. Since X is regular, we may find a neighborhood V_1 of x_1 such that $V_1^- \subseteq U_1$. Suppose, now, that we have disjoint neighborhoods $V_1, \ldots, V_n$, of $x_1, \ldots, x_n$, respectively, satisfying $(V_1^- \cup \ldots \cup V_n^-) \cap B = \{x_1, \ldots, x_n\}$. Let U_{n+1} be a neighborhood of x_{n+1} containing no other element of B, and let $W_{n+1} = U_{n+1} \cap \big(X \sim (V_1^- \cup \ldots \cup V_n^-)\big)$. W_{n+1} is a neighborhood of x_{n+1}; hence, there is a neighborhood V_{n+1} of x_{n+1} with $V_{n+1}^- \subseteq W_{n+1}$. By induction, then, we cover B by the disjoint open collection $\mathfrak{V} = \{V_n \mid n \in Z^+\}$. Since each point of $X \sim B$ has a neighborhood which does not meet B, $\mathfrak{W} = \mathfrak{V} \cup \{X \sim B\}$ is an open covering of X. Moreover, no point of X belongs to more than two members of $\mathfrak{W}$. But it is clear that $\mathfrak{W}$ has no finite subcover.

PROPOSITION 9. *A regular starcompact space is countably compact.*

Proof. This follows immediately from Lemma 1 and Theorem 2.

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(1) A space X is *almost-2-fully normal* provided that each open covering $\mathfrak{U}$ of X is refined by an open covering $\mathfrak{V}$ with the following property; if $x, y \in \mathrm{St}(z, \mathfrak{V})$, then there is a $U \in \mathfrak{U}$ with $x, y \in U$.

§ 3. Subspaces and products. We have seen, in Example 1, a space which is starcompact but not $\aleph_0$-compact. This space has the property that it is homeomorphic to each of its closed subspaces. Thus, each of its closed subspaces is starcompact. The following example, communicated to the author by J. H. Weston, shows that, in general, this need not be true.

EXAMPLE 2. Let ω_1 denote the first uncountable ordinal. For any ordinal a, let $W(a)$ denote the well-ordered set of all ordinals which precede a and provide $W(a)$ with the interval topology. Let X be the topological sum of countably many copies of the space $W(\omega_1+1)$ and let Y be the quotient space of X in which all equal countable ordinals are identified. Y can be considered as $W(\omega_1) \cup \{a_n \mid n \in Z^+\}$ where the basic neighborhoods of a point of $W(\omega_1)$ are the open intervals of $W(\omega_1)$ to which it belongs, while the basic neighborhoods of a point a_n are the sets $\{a_n\} \cup \{\beta \in W(\omega_1) \mid \beta \geqslant a\}$ for $a \in W(\omega_1)$. Clearly, Y is a T_1 space but not a Hausdorff space.

Y is starcompact.

Proof. Let $\mathfrak{U}$ be an open covering of Y and for each $n \in Z^+$, let $U_n \in \mathfrak{U}$ be such that $a_n \in U_n$. Then there is an ordinal $a_n \in W(\omega_1)$ such that

$$\{a_n\} \cup \{\beta \in W(\omega_1) \mid \beta \geqslant a_n\} \subseteq U_n .$$

Since no sequence is cofinal in $W(\omega_1)$, there is an $a \in W(\omega_1)$ with $a_n \leqslant a$ for all $n \in Z^+$. Thus $a \in \bigcap \{U_n \mid n \in Z^+\}$ so that

$$\{a_n \mid n \in Z^+\} \cup \{\beta \in W(\omega_1) \mid \beta \geqslant a\} \subseteq \mathrm{St}(a, \mathfrak{U}) .$$

The subspace $\{\gamma \in W(\omega_1) \mid \gamma \leqslant a\}$ of Y is compact; hence there are sets $V_1, \ldots, V_k \in \mathfrak{U}$ such that

$$\{\gamma \in W(\omega_1) \mid \gamma \leqslant a\} \subseteq V_1 \cup \ldots \cup V_k .$$

Let A be any set consisting of a and one point from each V_j; $1 \leqslant j \leqslant k$. Then A is finite and $\mathrm{St}(A, \mathfrak{U}) = Y$.

Y is not countably compact.

Proof. $\{a_n \mid n \in Z^+\}$ has no cluster point.

Y has a closed subspace which is not starcompact in its relative topology.

Proof. $\{a_n \mid n \in Z^+\}$ is discrete in its relative topology and closed in Y.

THEOREM 3. *Let X be a T_1 space. Then the following are equivalent:*

(i) *X is $\aleph_0$-compact;*

(ii) *X is countably compact;*

(iii) *Each closed subspace of X is starcompact;*

(iv) *Each closed subspace of X is weakly compact;*

(v) *Each closed subspace of X is pseudocompact.*

Proof. For T_1 spaces, it is clear that (i) $\Leftrightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (iv) $\Rightarrow$ (v).

(v) $\Rightarrow$ (ii). Suppose that $A = \{x_n|\ n \in Z^+\}$ is an infinite subset of X that has no cluster point. Then, since X is T_1, A is closed in X and discrete in its relative topology. Thus if we set $f(x_n) = n$ for each $n \in Z^+$, f is an unbounded, continuous, real-valued function on A.

While the product of an $\aleph_0$-compact space with a compact space is again $\aleph_0$-compact, it is not the case that the product of a starcompact space with a compact space is always starcompact.

Example 3. Let Y be as in Example 2. Let N denote the space of positive integers with the discrete topology and $N^* = N \cup \{\infty\}$, the one-point compactification of N. We shall exhibit an open covering $\mathfrak{U}$ of $Y \times N^*$ with the property that, if B is a finite subset of $Y \times N^*$, then $\mathrm{St}(B, \mathfrak{U})$ is a proper subset of $Y \times N^*$. To this end, for each $n \in Z^+$, let $S_n \subseteq Y$ be defined by

$$S_n = \{a_n\} \cup \{\alpha \in W(\omega_1)|\ n \leqslant \alpha\}.$$

Let $T_n \subseteq N^*$ be the set $\{\infty\} \cup \{k \in N|\ n < k\}$. Let

$$\mathfrak{U} = \{S_n \times T_n|\ n \in Z^+\} \cup \big\{Y \times \{n\}|\ n \in N\big\}.$$

$\mathfrak{U}$ is an open covering of $Y \times N^*$. No member of $\mathfrak{U}$ is superfluous — the only set in $\mathfrak{U}$ to which (a_n, ∞) belongs is $S_n \times T_n$, and the only set in $\mathfrak{U}$ to which (a_n, n) belongs is $Y \times \{n\}$. Suppose, now, that B is a finite subset of $Y \times N^*$. Let k be a positive integer such that $B \cap (Y \times \{k\}) = \emptyset$. Since $Y \times \{k\}$ is the only member of $\mathfrak{U}$ to which (a_k, k) belongs, $(a_k, k) \notin \mathrm{St}(B, \mathfrak{U})$. Thus if B is finite, $\mathrm{St}(B, \mathfrak{U})$ is a proper subset of $Y \times N^*$. We see, then, that $Y \times N^*$ is not starcompact.

As in the previous instance, this example leads directly to another set of conditions, each of which is equivalent to $\aleph_0$-compactness. Here, no separation need be assumed.

Theorem 4. *Let X be a space. Then the following conditions are equivalent:*

(i) *X is $\aleph_0$-compact;*

(ii) *$X \times K$ is starcompact for each compact space K;*

(iii) *$X \times N^*$ is starcompact.*

Proof. It is clear that (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii).

(iii) $\Rightarrow$ (i). Suppose that X is not $\aleph_0$-compact. Then there is an open covering $\mathfrak{U} = \{U_n|\ n \in Z^+\}$ of X with no finite subcovering. For each $n \in Z^+$, let

$$V_n = \bigcup \{U_i|\ 1 \leqslant i \leqslant n\} \quad \text{and} \quad T_n = \{\infty\} \cup \{k \in N|\ n < k\}.$$

T_n is an open subset of N^*. Thus

$$\mathfrak{V} = \{V_n \times T_n|\ n \in Z^+\} \cup \big\{X \times \{n\}|\ n \in N\big\}$$

is an open covering of $X \times N^*$. Suppose that B is a finite subset of $X \times N^*$. Let $n \in Z^+$ be chosen so large that $B \cap (X \times \{k \in N|\ k \geqslant n\}) = \emptyset$. If it were the case that for each $k \geqslant n$, $V_{k+1} \sim V_k = \emptyset$, then, since $X = V_n \cup \cup (\bigcup \{V_{k+1} \sim V_k|\ k \geqslant n\})$, $\{U_1, \ldots, U_n\}$ would constitute a finite subcovering chosen from $\mathfrak{U}$. Thus there is a $k \geqslant n$ for which $V_{k+1} \sim V_k \neq \emptyset$. Let $x \in V_{k+1} \sim V_k$. If $(x, k) \in V_j \times T_j$ for some $j \in Z^+$, then $k \in T_j$ so that $j < k$. But then $x \in V_j \subseteq V_k$ so that $x \notin V_{k+1} \sim V_k$, contradicting the choice of x. Thus the only set of $\mathfrak{V}$ to which (x, k) belongs is $X \times \{k\}$. Since $B \cap (X \times \{k\}) = \emptyset$, $(x, k) \notin \mathrm{St}(B, \mathfrak{V})$. We have shown that if B is a finite subset of $X \times N^*$, $\mathrm{St}(B, \mathfrak{V})$ is a proper subset of X. Thus $X \times N^*$ is not starcompact.

§ 4. Starcompactness and compactness. In order to present the results of this section, we require the following definitions.

Definition 7. A space is *paracompact* provided that each of its open coverings admits, as a refinement, a locally finite open covering. If $\mathfrak{m}$ is an infinite cardinal, we say that X is $\mathfrak{m}$-*paracompact* if each open covering $\mathfrak{U}$ of X with $|\mathfrak{U}| \leqslant \mathfrak{m}$ admits, as a refinement, a locally finite open covering.

Definition 8. A space X is *metacompact* if each open covering of X admits, as a refinement, a point finite open covering. Again, for infinite cardinal $\mathfrak{m}$, X is $\mathfrak{m}$-*metacompact* provided that the above requirement is satisfied by each open covering $\mathfrak{U}$ of X with $|\mathfrak{U}| \leqslant \mathfrak{m}$.

It is clear that weak compactness is precisely the condition which, together with paracompactness, yields compactness. One is tempted to modify the definition of weak compactness in an obvious way to obtain a condition (stronger than weak compactness) which, in the presence of metacompactness, gives compactness. This, in fact, is not feasible, since in any infinite Hausdorff space there is an infinite collection of non-empty, disjoint, open sets. The results which follow indicate that in this context, starcompactness is a suitable modification of weak compactness.

Theorem 5. *Let X be a topological space and $\mathfrak{m}$, an infinite cardinal. Then the following conditions are equivalent:*

(i) *X is $\mathfrak{m}$-compact;*

(ii) *X is starcompact and $\mathfrak{m}$-paracompact;*

(iii) *X is starcompact and $\mathfrak{m}$-metacompact.*

Proof. Since, for infinite $\mathfrak{m}$, $\mathfrak{m}$-compactness implies both $\aleph_0$-compactness and $\mathfrak{m}$-paracompactness, it is clear that (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii).

(iii) $\Rightarrow$ (i). Let $\mathfrak{U}$ be an open covering of X with $|\mathfrak{U}| \leqslant \mathfrak{m}$. There is a point finite open covering $\mathfrak{V}$ of X such that $\mathfrak{V}$ refines $\mathfrak{U}$. By Theorem 2, $\mathfrak{V}$ has a finite subcover $\{V_1, \ldots, V_k\}$. For each j, $1 \leqslant j \leqslant k$, let $U_j \in \mathfrak{U}$ be such that $V_j \subseteq U_j$. Then $\{U_1, \ldots, U_k\}$ is a finite subcover extracted from $\mathfrak{U}$.

COROLLARY. *The following conditions on the space X are equivalent:*

(i) *X is compact;*

(ii) *X is starcompact and paracompact;*

(iii) *X is starcompact and metacompact.*

Theorem 6 and its corollary constitute generalizations of results of G. Aquaro [1]. Exploitation of the properties of starcompact spaces leads to a remarkably brief proof of the theorem.

THEOREM 6. *Let X be a space which is $\mathfrak{m}$-compact and let $\mathfrak{U}$ be a point-$\mathfrak{m}$ open covering of X (i.e., a covering such that no point of X belongs to more than $\mathfrak{m}$ of its members). Then $\mathfrak{U}$ has a finite subcover.*

Proof. As noted above, $\mathfrak{m}$-compactness implies starcompactness. Thus there is a finite set $A = \{x_1, \ldots, x_n\} \subseteq X$ such that $X = \mathrm{St}(A, \mathfrak{U}) = \mathrm{St}(x_1, \mathfrak{U}) \cup \ldots \cup \mathrm{St}(x_n, \mathfrak{U})$. Since for each k, $1 \leqslant k \leqslant n$, $\mathrm{St}(x_k, \mathfrak{U})$ is the union of no more than $\mathfrak{m}$ members of $\mathfrak{U}$, $\mathfrak{U}$ has a subcover consisting of a most $\mathfrak{m}$ sets. However, X is $\mathfrak{m}$-compact, so that a finite subcover can be extracted from this subcover.

COROLLARY. *Let X be a space and $\mathfrak{m}$, an infinite cardinal. Then the following conditions are equivalent:*

(i) *X is compact;*

(ii) *X is $\mathfrak{m}$-compact and each open covering of X admits, as a refinement, an open, point $\mathfrak{m}$-covering.*

QUESTION. *Does there exist a Hausdorff starcompact space which is not countably compact?*

Added in proof: The above question has been answered by Mr. Raymond S. Houston who has given an elegant proof of the following result:

THEOREM. *Every starcompact Hausdorff space is countably compact.*

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