By the definition of the splitting operation, (8.2) implies $\omega_f(X \cap P^*) = X' \cap \omega_f(P^*) = Y' \cap \omega_f(P^*) = \omega_f(Y \cap P^*)$.

Hence

$$X \cap P^* = f\omega_f(X \cap P^*) = f\omega_f(Y \cap P^*) = Y \cap P^*,$$

and thus applying Theorem (3.1) of [6], we infer that X is a strong deformation retract of Y. This completes the proof of Theorem (8.1).

Remark 1. The polyhedron Y is a homogeneously 2-dimensional one.

Remark 2. Let us assume P to be a manifold. Observe that $A^i_j \subset K(X^i_j, \varepsilon)$ for every i, j (see § 2 and (6.10)). For sufficiently small ε there are retractions r^i_j: $A^i_j \to X^i_j$ satisfying the condition $r^i_j(A^i_j - X^i_j) \subset \mathrm{Fr}\, X^i_j$ ([2] p. 139). By Lemma (5.1) all r^i_j are strong deformational retractions and therefore by Lemma (5.2) we get a strong deformational retraction r: $Y \to X$ such that $r(Y - X) \subset \mathrm{Fr}\, X$. Obviously, mapping a union of manifolds onto a Σ-pseudomanifold (see § 7) we obtain the same result for an arbitrary Σ-pseudomanifold.

Using these two remarks we get the following

(8.3) COROLLARY. *Let P be a 2-dimensional Σ-pseudomanifold. If $X \in \mathrm{ANR}$, $X \subset P$, then there exist a homogeneously 2-dimensional subpolyhedron Y of P and a strong deformational retraction r: $Y \to X$ such that $r(Y - X) \subset \mathrm{Fr}\, X$.*

PROBLEM. Can the assumption on P in Theorem (8.1) be replaced by the following weaker one: P is a homogeneously 2-dimensional polyhedron, the set $\dot{P} \cap X$ being of a finite number of components?

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Lifting trees under light open maps *

by

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The purpose of this paper is to prove the nonmetric analog of a theorem due to G. T. Whyburn concerning the liftability of dendrites under light open maps. The proof uses the nonmetric analog of an arc lifting theorem of Whyburn which is due to R. J. Koch.

A continuum is *hereditarily unicoherent* provided the intersection of any two of its subcontinua is connected. A *tree* is a locally connected hereditarily unicoherent continuum. An *arc* is a continuum with exactly two noncutpoints. The closure of a set A will be denoted by A^* and the void set by $\square$.

THEOREM. *Suppose f is a light open map from a compact Hausdorff space X onto a topological space Y. If T is a tree in Y and $a \in f^{-1}(T)$, then there exists a continuum K in X such that $a \in K$ and f maps K topologically onto T.*

Proof. Clearly, it can be assumed that $Y = T$ and $X = f^{-1}(T)$. Let $\mathcal{C}$ be the collection of all continua M in X such that $a \in M$ and f restricted to M is a homeomorphism into T. Then $\{a\} \in \mathcal{C}$ so that $\mathcal{C} \neq \square$. Let $\mathcal{M}$ be a maximal tower in $\mathcal{C}$, let $A = \bigcup \mathcal{M}$, and let $K = A^*$. We show that K is the desired continuum in two parts. First it is shown that f is one-to-one on K and second it is shown that f maps K onto T.

For the first part fix $p \in f(K)$. It suffices to show that $f^{-1}(p) \cap K$ is a single point. Let $\mathcal{U}$ be a basis for the topology of T at p consisting of open connected sets. The proof that $f^{-1}(p) \cap K$ is a single point depends on the following four facts:

(i) $f^{-1}(U) \cap A$ is connected for each $U \in \mathcal{U}$.

(ii) $f^{-1}(p) \cap K \neq \square$.

(iii) $f^{-1}(p) \cap K \subset \liminf\{f^{-1}(U) \cap A\colon U \in \mathcal{U}\}$.

(iv) $\limsup\{f^{-1}(U)\colon U \in \mathcal{U}\} \subset f^{-1}(p)$.

* This paper forms part of the author's doctoral dissertation prepared under the direction of Professor R. J. Koch at Louisiana State University.

Once these are established, (i), (ii), and (iii) imply that

$$\limsup\{f^{-1}(U) \cap A\colon\ U \in \mathfrak{U}\}$$

is connected since the lim sup of a net of connected sets in a compact Hausdorff space is a continuum if the lim inf is nonvoid. (Theorem 2–101, page 101 of [2] generalizes easily to nets yielding the desired result.) This set is obviously contained in $\limsup\{f^{-1}(U)\colon\ U \in \mathfrak{U}\}$. Therefore, by (iv),

$$\limsup\{f^{-1}(U) \cap A\colon\ U \in \mathfrak{U}\}$$

is a connected subset of the totally disconnected set $f^{-1}(p)$ and must be a point. Hence, by (iii), $f^{-1}(p) \cap K$ is a point as was to be shown.

Verification of (i): It is easy to see that for any $U \in \mathfrak{U}$ and $M \in \mathcal{M}$,

$$f^{-1}\big(U \cap f(M)\big) \cap M = f^{-1}(U) \cap M\,.$$

Since $f|M$ is a homeomorphism and $U \cap f(M)$ is connected ([1], Proposition II.4), we have that $f^{-1}(U) \cap M$ is connected for each $M \in \mathcal{M}$. This collection of sets is towered and hence

$$\bigcup\{f^{-1}(U) \cap M\colon\ M \in \mathcal{M}\} = f^{-1}(U) \cap A$$

is connected.

Verification of (ii): This is obvious, as p was chosen in $f(K)$.

Verification of (iii): Fix $x \in f^{-1}(p) \cap K$ and let W be an open set containing x. Then $f^{-1}(U) \cap W$ is an open set containing x for each $U \in \mathfrak{U}$. Therefore

$$\Box \neq [f^{-1}(U) \cap W] \cap A = W \cap [f^{-1}(U) \cap A] \quad \text{for each } U \in \mathfrak{U}$$

so that

$$x \in \liminf\{f^{-1}(U) \cap A\colon\ U \in \mathfrak{U}\}\,.$$

Verification of (iv): Fix $x \in X \backslash f^{-1}(p)$. Then $f(x) \neq p$ implies the existence of disjoint open sets U and V containing p and $f(x)$, respectively. Now, x is a member of the open set $f^{-1}(V)$ and if $U' \in \mathfrak{U}$ with $U' \subset U$ then $f^{-1}(V) \cap f^{-1}(U) = \Box$. Hence,

$$x \notin \limsup\{f^{-1}(U)\colon\ U \in \mathfrak{U}\}\,.$$

Now, for the second part, assume that $f(K)$ is a proper subset of T and fix $y \in T \backslash f(K)$. If e is fixed in $f(K)$, there is a unique arc $[y, e]$ joining y to e in T ([4]; also [1], Proposition I.1). Let $z = \inf\big([y, e] \cap f(K)\big)$ relative to the cutpoint ordering on $[y, e]$ with minimal element y. Then $z \in f(K)$ and we let $b = f^{-1}(z) \cap K$. By Corollary 3 of [3], there exists an arc B in X such that $b \in B$ and f maps B homeomorphically onto $[y, z]$. Letting $K' = K \cup B$, we have that K' is a continuum containing a and f restricted to K' is a homeomorphism into T. This, however, contradicts the maximality of $\mathcal{M}$ and the assumption that $f(K)$ is a proper subset of T is false. This completes the proof of the theorem.

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