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Reçu par la Rédaction le 17. 5. 1967

Added in proofreading. Raymond Wong has answered the first part of question 1 in the negative. The proof is very easy. The natural projection p of J^∞ onto J^∞/R is a 2-fold covering map when restricted to $J^\infty/0$. However $I^\infty/\{\text{point}\}$ is simply connected which would contradict the assumption that $J^\infty/R \approx I^\infty$.

Some remarks concerning the mappings of the inverse limit into an absolute neighborhood retract and its applications to cohomotopy groups

by

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If $\{X_\alpha, \pi_\alpha^\beta\}$ is an inverse system (see [2], p. 213) of compact metric spaces and $Y \in \mathrm{ANR}$ (see [1], p. 100), we define a map

$$\Phi\colon [\varprojlim\{X_\alpha, \pi_\alpha^\beta\}, Y] \to \varinjlim\{[X_\alpha, Y], \pi_\alpha^{\beta\#}\},$$

where $[X, Y]$ denotes the set of homotopy classes of maps $X \to Y$. We show that Φ is an isomorphism preserving some structures in the set of homotopy classes: the "dependence" structure, the group structure if Y is a topological group, and the nth cohomotopy group structure if $\dim X_\alpha \leqslant 2n-1$.

The author acknowledges his gratitude to Professors J. Jaworowski and J. Mioduszewski for their valuable remarks and advices.

§ 1. Definitions and notations. Let us denote by 2^N the family of all subsets of a set N. A function $\lambda\colon 2^N \to 2^N$ satisfying conditions:

$$A \subset \lambda(A) \quad \textit{for every set } A \subset N,$$
$$\textit{if} \quad A \subset B \subset N, \quad \textit{then} \quad \lambda(A) \subset \lambda(B),$$
$$\lambda(\lambda(A)) = \lambda(A) \quad \textit{for every set } A \subset N$$

is said to be the *dependence operation* in the set N, and the set N in which a such operation is defined is said to be a *dependence domain* (see [1], p. 66).

Let N_1 and N_2 be two dependence domains with dependence operations λ_1 and λ_2, respectively. A function $f\colon N_1 \to N_2$ satisfying the condition

$$f(\lambda_1(A)) \subset \lambda_2(f(A)) \quad \text{for every set } A \subset N_1$$

will be called a *λ-morphism*. A one-to-one λ-morphism for which the inverse function is a λ-morphism is said to be a *λ-isomorphism* (see [1], p. 66).

Let X and Y be two topological spaces and let $M \subset Y^X$, where Y^X denotes the set of all mappings of X into Y, and $f\colon X \to Y$. We shall say that f is *homotopically dependent* on M provided that there exist maps $\varphi_1, \varphi_2, \ldots, \varphi_k \in M$ and $\vartheta\colon Y^k \to Y$ such that $f \simeq \vartheta\varphi$ where $\varphi\colon X \to Y^k$ is given by the formula

$$\varphi(x) = \big(\varphi_1(x), \varphi_2(x), \ldots, \varphi_k(x)\big)\,.$$

(It is a modification of this notion given in [1], p. 64.) The set of all maps homotopically dependent on M will be denoted by $\omega(M)$, the set of homotopy classes of maps belonging to M by $\boldsymbol{M}$ and the set of homotopy classes of maps belonging to $\omega(M)$ by $\lambda(\boldsymbol{M})$. Analogously as in [1] (see p. 67) one can prove that the operation λ defined is the dependence operation in the set $[X, Y]$.

If $\{X_\alpha, \pi_\alpha^\beta\}$ is an inverse system (see [2], p. 213) over a directed set $(\Gamma, \leqslant)$, then for any space Y the map $\pi_\alpha^\beta\colon X_\beta \to X_\alpha$ (for $\alpha \leqslant \beta$) induces a function $\pi_\alpha^{\beta\#}\colon [X_\alpha, Y] \to [X_\beta, Y]$ given by the formula $\pi_\alpha^{\beta\#}([f]) = [f\pi_\alpha^\beta]$ and then $\{[X_\alpha, Y], \pi_\alpha^{\beta\#}\}$ is a direct system (see [2], p. 212) over the directed set $(\Gamma, \leqslant)$. If $\alpha, \beta \in \Gamma$, then $\beta \geqslant \alpha$ means that $\alpha \leqslant \beta$ and $\alpha \nleqslant \beta$ means that it is not true that $\alpha \leqslant \beta$. The *limit of the inverse system* $\{X_\alpha, \pi_\alpha^\beta\}$ (see [2], p. 215) will be denoted by $\varprojlim\{X_\alpha, \pi_\alpha^\beta\}$ and the *limit of the direct system* $\{X_\alpha, \sigma_\alpha^\beta\}$ by $\varinjlim\{X_\alpha, \sigma_\alpha^\beta\}$. An element of $\varprojlim\{X_\alpha, \pi_\alpha^\beta\}$ (or $\varinjlim\{X_\alpha, \sigma_\alpha^\beta\}$, or the Cartesian product $\underset{\alpha\in\Gamma}{P} X_\alpha$), whose representative in X_α is x_α will be denoted by $\{x_\alpha\}$.

Let λ_α be the dependence operation in the set $[X_\alpha, Y]$ defined as above. For an arbitrary set $B \subset \varinjlim\{[X_\alpha, Y], \pi_\alpha^{\beta\#}\}$ and for an arbitrary element $\alpha \in \Gamma$, let B_α be the subset of $[X_\alpha, Y]$ such that

$$[\varphi_\alpha] \in B_\alpha \Leftrightarrow \{[\varphi_\alpha]\} \in B\,.\ [1]$$

Let $\lambda^\#(B)$ be the subset of $\varinjlim\{[X_\alpha, Y], \pi_\alpha^{\beta\#}\}$ such that

$$\{[\varphi_\alpha]\} \in \lambda^\#(B) \Leftrightarrow \bigvee_{\gamma\in\Gamma}\bigwedge_{\alpha\geqslant\gamma}[\varphi_\alpha] \in \lambda_\alpha(B_\alpha)\,.\ [2]$$

It is easy to see that such defined operation $\lambda^\#$ is a dependence operation in the set $\varinjlim\{[X_\alpha, Y], \pi_\alpha^{\beta\#}\}$.

Let $\{X_\alpha, \pi_\alpha^\beta\}$ be an inverse system of compact metric spaces over a directed set $(\Gamma, \leqslant)$ and $X = \varprojlim\{X_\alpha, \pi_\alpha^\beta\}$. Imbedding each space X_α into the Hilbert cube Q_α we can imbed the space X into the Cartesian product $\underset{\alpha\in\Gamma}{P} Q_\alpha$. Therefore we can assume that $X \subset \underset{\alpha\in\Gamma}{P} Q_\alpha$ and then

[1] „$\Leftrightarrow$" means „if and only if".

[2] „$\bigvee_{\gamma\in\Gamma}$" means „there exists $\gamma \in \Gamma$", and „$\bigwedge_{\alpha\geqslant\gamma}$" means „for each $\alpha \geqslant \gamma$".

$X_\alpha \subset Q_\alpha$. Let us pick a point $x^0 = \{x_\alpha^0\} \in X$ where $x_\alpha^0 \in X_\alpha$. For an arbitrary $\alpha \in \Gamma$, let T_α and $\widetilde{X}_\alpha$ be subspaces of the space $\underset{\alpha\in\Gamma}{P} Q_\alpha$ such that

$$\{x_\beta\} \in T_\alpha \Leftrightarrow [(x_\alpha \in X_\alpha) \wedge (\beta \leqslant \alpha \Rightarrow x_\beta = \pi_\beta^\alpha(x_\alpha)]\,,\ [3]$$

$$\{x_\beta\} \in \widetilde{X}_\alpha \Leftrightarrow [(\{x_\beta\} \in T_\alpha) \wedge (\beta \nleqslant \alpha \Rightarrow x_\beta = x_\beta^0]\,.$$

These definitions imply

(1) $\quad X, X_\alpha \subset T_\alpha\,, \quad \bigcap_{\beta\geqslant\alpha} T_\beta = X\,, \quad \alpha \leqslant \beta \Rightarrow T_\beta \subset T_\alpha \quad$ for each $\alpha, \beta \in \Gamma$.

Let $\iota_\alpha\colon \widetilde{X}_\alpha \to T_\alpha$, $\bar{\pi}_\alpha\colon X \to T_\alpha$, $\bar{\pi}_\alpha^\beta\colon T_\beta \to T_\alpha$ (for $\alpha \leqslant \beta$) be inclusions. The function $h_\alpha\colon \widetilde{X}_\alpha \to X_\alpha$ given by the formula $h_\alpha(\{x_\beta\}) = x_\alpha$ is a homeomorphism. The set T_α is homeomorphical with $\widetilde{X}_\alpha \times \underset{\beta\nleqslant\alpha}{P} Q_\beta$; therefore, it is compact. Let us define the function $r_\alpha\colon T_\alpha \to \widetilde{X}_\alpha$ by the formula

$$r_\alpha(\{x_\beta\}) = \{x'_\beta\} \quad \text{where} \quad x'_\beta = \begin{cases} x_\beta & \text{if} \quad \beta \leqslant \alpha\,, \\ x_\beta^0 & \text{if} \quad \beta \nleqslant \alpha\,. \end{cases}$$

It is obvious that r_α is a deformation retraction. Let

$$\tilde{\pi}_\alpha^\beta = r_\alpha \bar{\pi}_\alpha^\beta \iota_\beta\colon \widetilde{X}_\beta \to \widetilde{X}_\alpha \quad \text{for} \quad \alpha \leqslant \beta\,.$$

Let us observe that $\tilde{\pi}_\alpha^\beta = h_\alpha^{-1}\pi_\alpha^\beta h_\beta$. Therefore, we can identify the sets $\widetilde{X}_\alpha$ with the sets X_α and the maps $\tilde{\pi}_\alpha^\beta$ with the maps π_α^β. Henceforth $\widetilde{X}_\alpha$ will be denoted by X_α and $\tilde{\pi}_\alpha^\beta$ by π_α^β. By this convention we have

(2) $$X_\alpha \subset T_\alpha\,, \quad \iota_\alpha\colon X_\alpha \to T_\alpha\,, \quad r_\alpha\colon T_\alpha \to X_\alpha\,,$$
$$\pi_\alpha^\beta = r_\alpha \bar{\pi}_\alpha^\beta \iota_\beta\colon X_\beta \to X_\alpha \quad \text{for} \quad \alpha \leqslant \beta\,.$$

§ 2. A natural transformation.

Let us prove the following

LEMMA 1. *If $Y \in$ ANR, then for each map $f\colon X \to Y$ there is $\gamma \in \Gamma$ such that for each $\alpha \geqslant \gamma$ there exists a map $f_\alpha\colon T_\alpha \to Y$ such that $f = f_\alpha \bar{\pi}_\alpha$. Moreover, if $\alpha \leqslant \beta$ then $f_\alpha | T_\beta = f_\beta$.*

Proof. Let $\tilde{f}\colon U \to Y$ be an extension of f on some neighborhood U of X in $\underset{\alpha\in\Gamma}{P} Q_\alpha$ (for existence, see [3], Theorem 13.2, p. 333 and Theorem 8.1, p. 325). From (1) and the compactness of T_α it follows that there exists $\gamma \in \Gamma$ such that for $\alpha \geqslant \gamma$ we have $T_\alpha \subset U$. The map $f_\alpha = \tilde{f}|T_\alpha$ satisfies the required conditions.

LEMMA 2. *If $Y \in$ ANR and $f_\alpha\colon T_\alpha \to Y$, $g_\beta\colon T_\beta \to Y$ are maps such that $f_\alpha \bar{\pi}_\alpha \simeq g_\beta \bar{\pi}_\beta$, then there exists $\gamma \in \Gamma$ such that*

$$f_\alpha \iota_\alpha \pi_\alpha^\gamma \simeq g_\beta \iota_\beta \pi_\beta^\gamma\,.$$

[3] „$\wedge$" means „and", and „$a \Rightarrow b$" means „if a then b".

Proof. By hypothesis there exists a map $h\colon X\times\langle 0,1\rangle\to Y$ such that

$$h(x,0)=f_\alpha\overline{\pi}_\alpha(x)\quad\text{and}\quad h(x,1)=g_\beta\overline{\pi}_\beta(x)\quad\text{for}\quad x\in X.$$

Let us define the function $F\colon X\times\langle 0,1\rangle\cup T_\alpha\times(0)\cup T_\beta\times(1)\to Y$ by the formula

$$F(x,t)=\begin{cases} h(x,t) & \text{for}\quad x\in X \text{ and } 0\leqslant t\leqslant 1,\\ f_\alpha(x) & \text{for}\quad x\in T_\alpha \text{ and }\quad t=0,\\ g_\beta(x) & \text{for}\quad x\in T_\beta \text{ and }\quad t=1.\end{cases}$$

It is obvious that F is well defined and continuous. Let $\widetilde{F}\colon V\to Y$ be an extension of F on some neighborhood V of $X\times\langle 0,1\rangle\cup T_\alpha\times(0)\cup T_\beta\times(1)$ in $\underset{\alpha\in\Gamma}{P}Q_\alpha\times\langle 0,1\rangle$ (for existence, see [3], Theorem 13.2, p. 333 and Theorem 8.1, p. 325). From (1) and the compactness of T_α^1 (for $\alpha\in\Gamma$) it follows that there exists $\gamma\in\Gamma$ such that $\gamma\geqslant\alpha$, $\gamma\geqslant\beta$ and $T_\gamma\times\langle 0,1\rangle\subset V$. Let $H=\widetilde{F}|T_\gamma\times\langle 0,1\rangle$. Then $H\colon T_\gamma\times\langle 0,1\rangle\to Y$ and $H(x,0)=f_\alpha\overline{\pi}_\alpha^\gamma(x)$ and $H(x,1)=g_\beta\overline{\pi}_\beta^\gamma(x)$ for $x\in T_\gamma$. Therefore $f_\alpha\overline{\pi}_\alpha^\gamma\simeq g_\beta\overline{\pi}_\beta^\gamma$. Hence, since the maps $\iota_\alpha r_\alpha$ and $\iota_\beta r_\beta$ are homotopic with the identity maps, we have

$$f_\alpha\iota_\alpha r_\alpha\overline{\pi}_\alpha^\gamma\iota_\gamma\simeq g_\beta\iota_\beta r_\beta\overline{\pi}_\beta^\gamma\iota_\gamma.$$

Hence, from (2) $f_\alpha\iota_\alpha\pi_\alpha^\gamma\simeq g_\beta\iota_\beta\pi_\beta^\gamma$ and the lemma is proved.

Let us define the function

$$\Phi\colon [\varprojlim\{X_\alpha,\pi_\alpha^\beta\},Y]\to\varinjlim\{[X_\alpha,Y],\pi_\alpha^{\beta\#}\}$$

by the formula

$$\Phi([f])=\{[f_\alpha\iota_\alpha]\}$$

where $f_\alpha\colon T_\alpha\to Y$ are mappings associated to f by lemma 1. It is well defined, because if $f\simeq g$ and $f=f_\alpha\overline{\pi}_\alpha$, $g=g_\beta\overline{\pi}_\beta$, then by lemma 2 $f_\alpha\iota_\alpha\pi_\alpha^\gamma\simeq g_\beta\iota_\beta\pi_\beta^\gamma$, hence $\pi_\alpha^{\gamma\#}[f_\alpha\iota_\alpha]=\pi_\beta^{\gamma\#}[g_\beta\iota_\beta]$, therefore $\{[f_\alpha\iota_\alpha]\}=\{[g_\beta\iota_\beta]\}$. The function Φ will be called a *natural transformation.*

THEOREM. *The natural transformation*

$$\Phi\colon [X,Y]\to\varinjlim\{[X_\alpha,Y],\pi_\alpha^{\beta\#}\}$$

is a one-to-one function.

Proof. For any map $\varphi_\alpha\colon X_\alpha\to Y$ we have $\Phi([\varphi_\alpha r_\alpha\overline{\pi}_\alpha])=\{[\varphi_\alpha]\}$, therefore Φ is onto. Now, let there be given two maps $f,g\colon X\to Y$ such that $\Phi([f])=\Phi([g])$. Let $f=f_\alpha\overline{\pi}_\alpha$ and $g=g_\alpha\overline{\pi}_\alpha$ where f_α and g_α are mappings associated to f by lemma 1. From the hypothesis, $\{[f_\alpha\iota_\alpha]\}=\{[g_\alpha\iota_\alpha]\}$, therefore there exists $\beta\geqslant\alpha$ such that $f_\alpha\iota_\alpha\pi_\alpha^\beta\simeq g_\alpha\iota_\alpha\pi_\alpha^\beta$. Hence by (2)

$$f_\alpha\iota_\alpha r_\alpha\overline{\pi}_\alpha^\beta\iota_\beta\simeq g_\alpha\iota_\alpha r_\alpha\overline{\pi}_\alpha^\beta\iota_\beta,$$

thus $f_\alpha\overline{\pi}_\alpha^\beta\iota_\beta r_\beta\simeq g_\alpha\overline{\pi}_\alpha^\beta\iota_\beta r_\beta$, and nextly

$$f=f_\alpha\overline{\pi}_\alpha=f_\alpha\overline{\pi}_\alpha^\beta\overline{\pi}_\beta\simeq g_\alpha\overline{\pi}_\alpha^\beta\overline{\pi}_\beta=g_\alpha\overline{\pi}_\alpha=g,$$

and the proof is concluded.

§ 3. Some properties of the natural transformation. Let λ, λ_α (for $\alpha\in\Gamma$) and $\lambda^\#$ be the dependence operations in the sets $[X,Y]$, $[X_\alpha,Y]$ and $\varinjlim\{[X_\alpha,Y],\pi_\alpha^{\beta\#}\}$, respectively, defined as in § 1. Then the following theorem is true.

THEOREM 1. *The natural transformation*

$$\Phi\colon [X,Y]\to\varinjlim\{[X_\alpha,Y],\pi_\alpha^{\beta\#}\}$$

is a λ-isomorphism.

Proof. It suffices to prove that for each set $A\subset[X,Y]$ we have $\Phi(\lambda(A))=\lambda^\#(\Phi(A))$. For, suppose that $[f]\in\lambda(A)$ and let M be the set of all representatives of the homotopy classes belonging to A, and M_α be the set of all representatives of the homotopy classes belonging to $(\Phi(A))_\alpha$ (see the definition of B_α in § 1). Hence, we have $A=\boldsymbol{M}$ and $(\Phi(A))_\alpha=\boldsymbol{M}_\alpha$. Thus $f\in\omega(M)$, therefore $f\simeq\vartheta\varphi$, where $\vartheta\colon Y^k\to Y$, $\varphi\colon X\to Y^k$, $\varphi(x)=(\varphi_1(x),\ldots,\varphi_k(x))$ and $\varphi_i\in M$ (for $i=1,2,\ldots,k$). By lemma 1 there exists $\gamma\in\Gamma$ such that for $\alpha\geqslant\gamma$ we have $f=f_\alpha\overline{\pi}_\alpha$ and $\varphi_i=\varphi_{i\alpha}\overline{\pi}_\alpha$ (for $i=1,2,\ldots,k$). Let us define the map $\varphi_\alpha\colon T_\alpha\to Y^k$ by the formula

$$\varphi_\alpha(x)=(\varphi_{1\alpha}(x),\varphi_{2\alpha}(x),\ldots,\varphi_{k\alpha}(x)).$$

Let $g_\alpha=\vartheta\varphi_\alpha\iota_\alpha\colon X_\alpha\to Y$. Since $f\simeq\vartheta\varphi$, we have $g_\alpha\simeq f_\alpha\iota_\alpha$. Let us observe that

$$\varphi_\alpha\iota_\alpha(x)=(\varphi_{1\alpha}\iota_\alpha(x),\ldots,\varphi_{k\alpha}\iota_\alpha(x))\quad\text{and}\quad\{[\varphi_{i\alpha}\iota_\alpha]\}=\Phi([\varphi_{i\alpha}\overline{\pi}_\alpha])=\Phi([\varphi_i])$$

and since $\varphi_i\in M$, then $[\varphi_i]\in A$, therefore $\{[\varphi_{i\alpha}\iota_\alpha]\}\in\Phi(A)$. Hence $[\varphi_{i\alpha}\iota_\alpha]\in(\Phi(A))_\alpha$, thus $\varphi_{i\alpha}\iota_\alpha\in M_\alpha$. Therefore $[g_\alpha]\in\lambda_\alpha((\Phi(A))_\alpha)$, and since $g_\alpha\simeq f_\alpha\iota_\alpha$, we have $[f_\alpha\iota_\alpha]\in\lambda_\alpha((\Phi(A))_\alpha)$, thus $\{[f_\alpha\iota_\alpha]\}\in\lambda^\#(\Phi(A))$. Then we have $\Phi([f])=\{[f_\alpha\iota_\alpha]\}\in\lambda^\#(\Phi(A))$, therefore

$$\Phi(\lambda(A))\subset\lambda^\#(\Phi(A)).$$

Now, suppose that $\{[\varphi_\alpha]\}\in\lambda^\#(\Phi(A))$. Then $\varphi_\alpha\in\omega(M_\alpha)$. It follows that there exist maps $\vartheta\colon Y^k\to Y$ and $\psi\colon X_\alpha\to Y^k$ such that $\psi(x)=(\psi_1(x),\ldots,\psi_k(x))$, $\psi_i\in M_\alpha$ (for $i=1,2,\ldots,k$) and $\varphi_\alpha\simeq\vartheta\psi$. If we set $\chi_i=\psi_i r_\alpha\overline{\pi}_\alpha\colon X\to Y$ (for $i=1,2,\ldots,k$) and define $\chi\colon X\to Y^k$ by the formula $\chi(x)=(\chi_1(x),\ldots,\chi_k(x))$; then

$$\vartheta\chi=\vartheta\psi r_\alpha\overline{\pi}_\alpha\simeq\varphi_\alpha r_\alpha\overline{\pi}_\alpha.$$

Since $\psi_i \in M_a$, we have $\{[\psi_i]\} \in \Phi(A)$. On the other hand,

$$\{[\psi_i]\} = \{[\psi_i r_a \iota_a]\} = \Phi([\psi_i r_a \overline{\pi}_a]) = \Phi([\chi_i]) .$$

Hence $\chi_i \in M$, therefore $[\varphi_a r_a \overline{\pi}_a] \in \lambda(A)$, but since $\Phi([\varphi_a r_a \overline{\pi}_a]) = \{[\varphi_a]\}$, we have $\{[\varphi_a]\} \in \Phi(\lambda(A))$. Therefore $\lambda^{\#}(\Phi(A)) \subset \Phi(\lambda(A))$. Thus $\Phi(\lambda(A)) = \lambda^{\#}(\Phi(A))$. This completes the proof of theorem 1.

Now, let Y be a topological group. In the sets $[\varprojlim \{X_a, \pi_a^\beta\}, Y]$ and $\varinjlim \{[X_a, Y], \pi_a^{\beta\#}\}$ there is given the group operation as usual. Then it is easy to see that the natural transformation is a homomorphism, and since it is one-to-one function, therefore it is an isomorphism. Hence, we obtain the next

THEOREM 2. *If Y is a topological group, then the natural transformation Φ is an isomorphism.*

EXAMPLE. Let S_i (for $i = 1, 2, \ldots$) be a circle considered as the set of all complex numbers z with $|z| = 1$. Let π_i^j: $S_j \to S_i$ (for $i \leqslant j$) be a map given by the formula $\pi_i^j(z) = z^{p^{j-i}}$, where p is a fixed natural number. The space $X = \varprojlim \{S_i, \pi_i^j\}$ is called the *p-adic solenoid* (see [2], p. 230). Applying theorem 2 we can easily calculate the first cohomotopy group $\pi^1(X)$ of the p-adic solenoid, for it is isomorphic with the group $\varinjlim \{\pi^1(S_i), \pi_i^{j\#}\}$. If $\pi^1(S_i)$ is considered as the group of integers, then $\pi_i^{j\#}$: $\pi^1(S_i) \to \pi^1(S_j)$ (for $i \leqslant j$) is given by the formula $\pi_i^{j\#}(c_i) = p^{j-i} \cdot c_i$, where $c_i \in \pi^1(S_i)$. Let $G(p)$ be the group of all rational numbers of the form m/p^i, where $m = 0, \pm 1, \pm 2, \ldots$, $i = 1, 2, \ldots$ It is easy to see that the group $\varinjlim \{\pi^1(S_i), \pi_i^{j\#}\}$ is isomorphic with the group $G(p)$, namely the function Ψ: $\varinjlim \{\pi^1(S_i), \pi_i^{j\#}\} \to G(p)$ given by the formula $\Psi(\{c_i\}) = c_i/p^i$, where $c_i \in \pi^1(S_i)$, is an isomorphism.

THEOREM 3. *If for each $a \in \Gamma$ $\dim X_a \leqslant 2n-1$, then the natural transformation*

$$\Phi\colon \pi^n(\varprojlim \{X_a, \pi_a^\beta\}) \to \varinjlim \{\pi^n(X_a), \pi_a^{\beta\#}\}$$

is an isomorphism.

Proof. Take two arbitrary maps f, g: $X \to S^n$, where $X = \varprojlim \{X_a, \pi_a^\beta\}$ and S^n is n-dimensional sphere. Since $\dim X_a \leqslant 2n-1$ and X_a are compact, then $\dim X \leqslant 2n-1$. Let F: $X \times \langle 0, 1\rangle \to S^n \times S^n$ be a normalizing homotopy for f and g, and let h: $X \to S^n \vee S^n = (S^n \times (s)) \cup ((s) \times S^n)$ be a normalization of them (see [4], p. 210). Then $F(x, 0) = (f(x), g(x))$ and $F(x, 1) = h(x)$ for $x \in X$. Let there be given the map Ω: $S^n \vee S^n \to S^n$ defined by the formula $\Omega(y, s) = \Omega(s, y) = y$. Then $[f] + [g] = [\Omega h]$ (see [4], p. 210). Take $\gamma \in \Gamma$ such that for each $a \geqslant \gamma$ there exist f_a, g_a: $T_a \to S^n$ and h_a: $T_a \to S^n \vee S^n$ such that $f = f_a \overline{\pi}_a$, $g = g_a \overline{\pi}_a$, $h = h_a \overline{\pi}_a$ and, moreover, if $a \leqslant \beta$ then $f_a | T_\beta = f_\beta$, $g_a | T_\beta = g_\beta$, $h_a | T_\beta = h_\beta$ (see lemma 1). For each $a \geqslant \gamma$, let us define the function

$$F_a\colon X \times \langle 0, 1\rangle \cup T_a \times (0) \cup T_a \times (1) \to S^n \times S^n$$

by the formula

$$F_a(x, t) = \begin{cases} F(x, t) & \text{for} \quad x \in X \text{ and } 0 \leqslant t \leqslant 1, \\ (f_a(x), g_a(x)) & \text{for} \quad x \in T_a \text{ and } t = 0, \\ h_a(x) & \text{for} \quad x \in T_a \text{ and } t = 1; \end{cases}$$

then if $a \leqslant \beta$ then $F_a | X \times \langle 0, 1\rangle \cup T_\beta \times (0) \cup T_\beta \times (1) = F_\beta$. Let $\widetilde{F}$: $V \to S^n \times S^n$ be an extension of F on some neighborhood V in $\mathop{P}\limits_{a \in \Gamma} Q_a \times \langle 0, 1\rangle$. Take $\beta \geqslant a$ such that $X_\beta \times \langle 0, 1\rangle \subset T_\beta \times \langle 0, 1\rangle \subset V$. Setting $H = \widetilde{F} | X_\beta \times \langle 0, 1\rangle$ we obtain a normalizing homotopy H: $X_\beta \times \langle 0, 1\rangle \to S^n \times S^n$ for maps $f_\beta \iota_\beta$ and $g_\beta \iota_\beta$ and then $h_\beta \iota_\beta$ is a normalization of them. Hence, by the definition of the natural transformation, we conclude that Φ is a homomorphism, and since it is a one-to-one function, then it is an isomorphism.

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Reçu par la Rédaction le 26. 5. 1967