only that t be a 1:1 mapping with the property that the induced homeomorphism on N' have no periodic points on the support set K in question. It is not even necessary that this homeomorphism of K be onto. A sufficient condition for t to have no finite orbits (i.e. no periodic points) in N' is that t have no finite orbits in N, but this is far from necessary. [1] contains a discussion of this question.

References

[1] R. A. Raimi, *Homeomorphisms and invariant measures for $\beta N - N$*, Duke Math. Journ. 33 (1966), pp. 1-12.

[2] W. Rudin, *Averages of continuous functions on compact spaces*, ibid. 25 (1958), pp. 197-204.

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Remark on Raimi's theorem on translations

by

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The present note gives an alternative proof of Theorem 3 from Ralph A. Raimi's paper *Translation properties of finite partitions of the positive integers* [1] in a changed and refined form. All notations and terminology are preserved. Only, for the sake of simplicity, the set of natural numbers is replaced by the set N of all integers. The author claims that this modifications is completely irrelevant. To make this note self-contained all definitions of the notions used below are reproduced here.

Let t denote the transformation $t(x) = x+1$ in the set N. Let $\mathfrak{A}$ be an arbitrary proper ideal of subsets of N which is *t-invariant* (i.e. $E \in \mathfrak{A}$ iff $tE \in \mathfrak{A}$ for all $E \subset N$). A partition $\mathfrak{V}$ of the set N is an *$\mathfrak{A}$-refinement* of a partition $\mathfrak{M}$ iff each member of $\mathfrak{V}$ is contained mod $\mathfrak{A}$ in some member of $\mathfrak{M}$. A set $E \subset N$ is called *strongly aperiodic* (s.a.) iff for every integer $k \neq 0$ we have

$$\bigcup_{j=0}^{m} t^j(E \triangle t^k E) = N$$

for some m. ($X \triangle Y = X \cup Y \setminus X \cap Y$.)

THEOREM. *If a set $E \subset N$ is s.a., then for any finite partition $\mathfrak{V}$ of N there is a translate $t^k\mathfrak{V}$ which is not an $\mathfrak{A}$-refinement of the partition $\mathfrak{M} = \{E, N \setminus E\}$.*

Proof. Suppose to the contrary that all $t^k\mathfrak{V}$ ($k = 1, 2, \ldots$) for some finite $\mathfrak{V} = \{V_i\}_{i \in I}$ are $\mathfrak{A}$-refinements of $\mathfrak{M}$. Since the set I is finite, there is a partition $I = I_0 \cup (I \setminus I_0)$ and two translations t^p and t^q ($p \neq q$) such that

$$t^p V_i \setminus E \in \mathfrak{A} \quad \text{and} \quad t^q V_i \setminus E \in \mathfrak{A} \quad \text{for} \quad i \in I_0,$$
$$t^p V_i \setminus (N \setminus E) \in \mathfrak{A} \quad \text{and} \quad t^q V_i \setminus (N \setminus E) \in \mathfrak{A} \quad \text{for} \quad i \in I \setminus I_0.$$

Consequently, for $k = p-q$, the set E is t^k-invariant mod $\mathfrak{A}$ (i.e. $E \triangle t^k E \in \mathfrak{A}$). Hence in view of strong aperiodicity of E we get a contradiction with our assumption $N \notin \mathfrak{A}$.

The next lemma shows the existence of s.a. sets.

LEMMA. *The set $E = \{n \in N \colon \cos n > 0\}$ is s.a.*

Proof. Let k be a fixed integer different from 0. We have

(1) $$E \triangle t^k E = \{n \in N\colon \cos n \cdot \cos(n-k) < 0\}.$$

Moreover, the sequence $a_n = \cos n \cdot \cos(n-k)$ is almost periodic (in the sense of H. Bohr) and for some n takes a negative value. The last properties of $\{a_n\}$ and (1) easily imply that E is a s.a. set.

Let us observe finally that a reasoning analogous to the proof of Theorem gives the following:

PROPOSITION. *Let X be a totally disconnected, compact, Hausdorff space and let t be a homeomorphism of X onto itself. Let $\mathcal{A}$ be the Boolean algebra of all clopen subsets of X. The the following conditions are equivalent:*

(i) *The set of all positive iterations of t is equicontinuous.*

(ii) *The same for negative iterations.*

(iii) *For every clopen set $E \subset X$ there is a $k \neq 0$ such that $t^k E = E$.*

(iv) *The algebra $\mathcal{A}$ is a union of a family of t-invariant and finite subalgebras of $\mathcal{A}$.*

Reference

[1] R. A. Raimi, *Translation properties of finite partitions of the positive integers*, Fund. Math., this volume, pp. 253-256.

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Metrically generated probabilistic metric spaces *

by

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1. Introduction. A. Špaček [10] has introduced the concept of a random metric consisting of a set S together with a probability measure μ on the set of all mappings of $S \times S$ into the reals and such that $\mu(M) = 1$, M being the set of all ordinary metrics for S. In [5], Menger, Schweizer and Sklar clarified the relationship between this concept and that of a probabilistic (statistical) metric ([3], [4], [6], [7]) and showed that the condition $\mu(M) = 1$ is extremely restrictive.

In this paper we continue the study of this relationship by investigating the probabilistic metric spaces which are generated (in the sense of definition 1) by a random metric and show that they are indeed of a very special type (Theorems 2 and 4). In part 3, we obtain a representation theorem giving sufficient conditions for a given probabilistic metric space to be generated in this way. We conclude by showing that our representation theorem is a best possible result in this direction.

For explicit definitions the reader is referred either to the paper [7] by Schweizer and Sklar or the paper [11] by Thorp. Also, following a previously established convention, we shall abbreviate "probabilistic metric space" to "PM space".

2. Metrically generated PM spaces. Let S be a set, let $\mathfrak{D}$ be a collection of ordinary metrics for S, and let μ be a measure for $\mathfrak{D}$ (i.e., a non-negative, countably additive set function defined on a σ-algebra of subsets of $\mathfrak{D}$, called *μ-measurable sets*) such that

(A) *for any pair p, q of points in S and any real number x, the set $\{d \in \mathfrak{D};\ d(p,q) < x\}$ is μ-measurable, and*

(B) $\mu(\mathfrak{D}) = 1$.

From the μ-measurability of the sets in (A), it follows that for each pair p, q of points in S, $d(p,q)$ is a numerically valued random variable on $\mathfrak{D}$ whose distribution function F_{pq} is given by

(1) $$F_{pq}(x) = \mu\{d \in \mathfrak{D};\ d(p,q) < x\}.$$

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