A convergence group which is also a pseudo convergence group is compatible with a directed convergence uniformity; this result is clear from the remark following Theorem 3.2. A directed convergence uniformity is a "uniform convergence structure" in the sense of Cook-Fischer [2]. Any convergence structure compatible with a directed convergence uniformity is T_2 whenever it is T_1.

From Theorem 2.1 it follows that every T_1 topology is a weakly uniformizable convergence structure. This may be compared with

THEOREM 4.3. *Every T_2 topology p is a convergence structure compatible with a directed convergence uniformity.*

Proof. Let $\mathfrak{U}_x = \dot{\Delta} \cap (\mathfrak{V}_p(x) \times \mathfrak{V}_p(x))$, and $w = \{\mathfrak{U}_x\colon x \text{ in } S\}$. It is easily seen that w is a weak uniformity compatible with p. But the set w' of finite intersections of members of w is a directed convergence uniformity, and $w' \in [p]$.

Concluding remarks. A meaning has not yet been assigned to the term "convergence uniformity". It would seem appropriate to reserve this name for a weak convergence uniformity satisfying the following conditions: (1) A convergence group is a uniformizable convergence structure; (2) If a topology is uniformizable as a convergence structure, then it is uniformizable in the usual sense; (3) A T_2 uniform convergence space has a (unique?) Hausdorff completion. A definition that meets the first two conditions is the following: w is a convergence uniformity if and only if w is a weak uniformity and $\lambda(q_w)$ is uniformizable. I do not know of a definition that will satisfy all three conditions.

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Some relational systems and the associated topological spaces

by

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The aim of this paper is to investigate relational systems $\langle S, R\rangle$ (S is the field of a binary relation R), and associated algebras:

(1) $$\mathcal{A}(S, R) = \langle \boldsymbol{P}(S), \cup, \cap, -, \mathrm{C}\rangle$$

where $\boldsymbol{P}(S)$ is the set of all subsets of S, $\langle \boldsymbol{P}(S), \cup, \cap, -\rangle$ is the Boolean algebra of subsets of S, and the operation C is defined on the elements of $\boldsymbol{P}(S)$ as follows:

(2) $$\mathrm{C}X = \{y\colon \bigvee x\,(x \in X \wedge xRy)\}.$$

It is easy to see that if the relation R is a quasi-ordering, i.e. if it satisfies two conditions (see [1]):

(3) a. xRx (*reflexivity*),

b. $(xRy \wedge yRz) \rightarrow xRz$ (*transitivity*),

then the algebra $\mathcal{A}(S, R)$ is a topological field of sets (this means that it satisfies the equalities: A. $X \subset \mathrm{C}X$; B. $\mathrm{C}(X \cup Y) = \mathrm{C}X \cup \mathrm{C}Y$; C. $\mathrm{CC}X = \mathrm{C}X$; D. $\mathrm{C}\emptyset = \emptyset$ ($\emptyset$ is the empty set)).

The purpose of these investigations is to characterize the topological fields of sets and related pseudo-Boolean algebras for some special relational systems, e.g. systems satisfying some additional equalities, having a logical meaning (cf. Theorem 1 and Corrolary 3).

This is a continuation of the well-known papers of Tarski and McKinsey [5] and Rasiowa and Sikorski [6].

1. Representation of totally distributive topological spaces. A topological space: $\langle \boldsymbol{P}(S), \cup, \cap, -, \mathrm{C}\rangle$ is *totally distributive* if and only if for every set $X \in \boldsymbol{P}(S)$

(4) $$\mathrm{C}X = \bigcup_{x \in X} \mathrm{C}\{x\}.$$

Hence every finite topological space is totally distributive.

LEMMA 1. *Every totally distributive topological space S is identical with the algebra $\mathcal{A}(S, R)$ for the quasi-ordering relation R defined as follows:*

(5) $$xRy \overset{\mathrm{Df}}{\equiv} y \in \mathrm{C}\{x\}.$$

Proof. Directly from formulas (1)-(5).

A relation R is said to be *partially ordering* if it is quasi-ordering and satisfies the following condition:

(6) $$(xRy \wedge yRx) \rightarrow x = y;$$

the partially ordering R is *partially well-ordering* if

(7) $$\bigwedge X\big(X \subset S \wedge X \neq \emptyset \rightarrow \bigvee v\big(v \in X \wedge \bigwedge u(uRv \wedge u \in X \rightarrow u = v)\big)\big).$$

THEOREM 1. *A totally distributive topological space $T = \langle \mathbf{P}(S), \cup, \cap, -, \mathrm{C}\rangle$ is identical with the algebra $\mathcal{A}(S, R)$ for some relation R of partial well-ordering if and only if the following equality* (E) *is true in T:*

(E) $$S = \Big(\mathrm{Int}\, Y \cup \mathrm{C}\Big(\big(\mathrm{Int}(Z \cup \mathrm{Int}\, Y) - \mathrm{Int}\, Y\big) \cup \big(\mathrm{Int}(-Z \cup \mathrm{Int}\, Y) - \mathrm{Int}\, Y\big)\Big)\Big).$$

Proof. First we shall prove that (E) is true in every algebra $\mathcal{A}(S, R)$ for a relation R of partial well-ordering.

According to definition (2) of our topology we have

(8) $$y \in \mathrm{Int}\, Y \equiv \bigwedge x(xRy \rightarrow x \in Y).$$

Let $G = \mathrm{Int}\, Y$ be an open subset of S. Let us consider the set

(9) $$D = \{y\colon y \notin G \wedge \bigwedge x(xRy \wedge x \neq y \rightarrow x \in G)\}.$$

It is easy to prove that

(10) $$G \cup \mathrm{C}D = S.$$

Indeed, if $z \notin G$ let us consider the set $Z_z = \{u\colon uRz \wedge u \notin G\}$. According to (7) there exists such v that $v \in Z_z$ and

$$\bigwedge u(uRv \wedge u \neq v \rightarrow u \notin Z_z).$$

Hence vRz, $v \notin G$ and

$$\bigwedge u(uRv \wedge u \neq v \rightarrow u \in G).$$

Thus $v \in D$ and $z \in \mathrm{C}D$.

Now we shall see that for every Z

(11) $$D \cap Z \subset \big(\mathrm{Int}(Z \cup G) \cap -G\big).$$

Suppose that $z \in D \cap Z$; then by (9) $z \in -G$ and $\bigwedge x\big(xRz \rightarrow (x = z \vee x \in G)\big)$, whence $\bigwedge x(xRz \rightarrow x \in Z \cup G)$ and according to (8) $z \in \mathrm{Int}(Z \cup G)$.

From (11) we infer that

$$D = (D \cap Z) \cup (D \cap -Z) \subset \mathrm{Int}(Z \cup G) - G \cup \mathrm{Int}(-Z \cup G) - G.$$

From this and (10) we obtain (E).

Now we shall show that if R is not partially well-ordering, then (E) is not true in $\mathcal{A}(S, R)$. Then we must suppose that or (6) or (7) is not satisfied.

If (6) is not fulfilled, then there exist two elements x_0 and y_0 such that

(12) $$x_0 \neq y_0 \wedge x_0Ry_0 \wedge y_0Rx_0.$$

Let us put

(13) $$Y = \{x\colon xRx_0 \wedge \sim(x_0Rx)\}, \quad \text{and} \quad Z = \{x_0\}.$$

If $x \in Y$ and zRx, then zRx_0 and $\sim(x_0Rz)$ according to the transitivity of R, and thus $z \in Y$. Hence if $x \in Y$, then $x \in \mathrm{Int}\, Y$ by (8). Then we have

(14) $$Y = \mathrm{Int}\, Y.$$

Now suppose that $x_0 \in \mathrm{C}\big(\mathrm{Int}(Z \cup Y) - Y\big)$. According to (2) and (8) this would mean that for some x

(15) $$xRx_0 \wedge x \notin Y \wedge \bigwedge z\big(zRx \rightarrow (z = x_0 \vee z \in Y)\big).$$

Hence by (13) x_0Rx; then by (12) y_0Rx but $y_0 \neq x_0$ and $y_0 \notin Y$, which contradicts (15). Thus we have proved that

(16) $$x_0 \notin \mathrm{C}\big(\mathrm{Int}(Z \cup Y) - Y\big).$$

Let us also note that

(17) $$x_0 \notin (-Z \cup Y),$$

and suppose that $x_0 \in \mathrm{C}\big(\mathrm{Int}(-Z \cup Y) - Y\big)$. According to (2) and (8) this would mean that for some x

(18) $$xRx_0 \wedge x \notin Y \wedge \bigwedge z\big(zRx \rightarrow (z \neq x_0 \vee z \in Y)\big).$$

Hence by (13) x_0Rx, but $x_0 \notin Y$, which contradicts (18).

Thus we have proved that

(19) $$x_0 \notin \mathrm{C}\big(\mathrm{Int}(-Z \cup Y) - Y\big).$$

Also $x_0 \notin Y$. Hence (14), (16) and (19) shows that (E) is not satisfied.

If (7) is not satisfied, there exist a set $X \subset S$ and an element x_0 such that

(20) $$x_0 \in X \wedge \bigwedge v\big(v \in X \rightarrow \bigvee u(uRv \wedge u \neq v \wedge u \in X)\big).$$

Hence there exists a sequence $\{x_n\} \subset X$ such that

(21) $$x_{n+1} R x_n \wedge x_{n+1} \neq x_n .$$

We can suppose that (6) is fulfilled and we put

(22) $$Z = \{x_{2n}\colon n \in N\} \quad \text{and} \quad Y = \{y\colon \bigwedge z (zRy \to z \notin Z)\} .$$

First let us notice that $Z \cap Y = \emptyset$ and by the transitivity of R

(23) $$Y = \operatorname{Int} Y,$$

Also we can prove that

(24) $$\operatorname{Int}(Z \cup Y) - Y = \emptyset .$$

Indeed, suppose that $y \in \operatorname{Int}(Z \cup Y)$ and $y \notin Y$, whence $y \in Z$; this means that $y = x_{2n}$ for some n. Then $x_{2n+1} R x_{2n}$, and $x_{2n+1} \neq x_{2n}$ according to (21). Thus by (8) $x_{2n+1} \in Z \cup Y$. If $x_{2n+1} \in Z$, then $x_{2n+1} = x_{2k}$ for some k. If $2n+1 > 2k$, then $x_{2k+1} R x_{2k}$ by (21) and $x_{2n+1} R x_{2k+1}$ by (21) and transitivity. Thus $x_{2k} R x_{2k+1}$, and $x_{2k} = x_{2k+1}$ by (6), which contradicts (21). If $2k > 2n+1$, then $x_{2n+2} R x_{2n+1}$ by (21) and $x_{2k} R x_{2n+2}$ by (21) and transitivity. Hence $x_{2n+1} R x_{2n+2}$ and $x_{2n+1} = x_{2n+2}$, which contradicts (21). Thus $x_{2n+1} \notin Z$, and then $x_{2n+1} \in Y$. This means by (22) that $\bigwedge z (z R x_{2n+1} \to z \notin Z)$, but $x_{2n+2} R x_{2n+1}$ and $X_{2n+2} \in Z$. Hence (24) is proved.

Now we shall prove that

(25) $$x_0 \notin \mathrm{C}\big(\operatorname{Int}(-Z \cup Y) - Y\big) .$$

Suppose the contrary case. This means that there exists a v such that $v R x_0$ and $v \in \operatorname{Int}(-Z \cup Y) - Y$. If $v \notin Y$, then there exists a z such that zRv and $z \in Z$. Hence by (8) $z \in -Z \cup Y$; thus $z \in Y$, but Z and Y are disjoint, and so we have obtained a contradiction. Of course, $x_0 \in Z$ and hence $x_0 \in Y$, thus according to (23), (24), and (25) the formula (E) is not true in $\mathcal{A}(S, R)$.

A relation R is a right-rooted tree iff

$$(xRy \wedge xRz \to (yRz \vee zRy) .$$

THEOREM 2. *Every topological space $\mathcal{T} = \mathcal{A}(S, R)$, for a relation R of partial ordering, can be embedded in a topological space $\mathcal{A}(S', R')$ for some right-rooted tree R'.* (For S finite, S' is also finite.)

Proof. As S' we take the set of all chains maximal to right and having their starting points:

(26) $$\operatorname{Ch}(X, x) \equiv x \in X \wedge \bigwedge z \Big(z \in X \equiv \big(xRz \wedge \bigwedge y \big(y \in X \to (zRy \vee yRz)\big)\big)\Big)$$

(X is a chain maximal to right with x as the starting element),

(27) $$X \in S' \equiv \bigvee x \big(x \in S \wedge \operatorname{Ch}(X, x)\big) ,$$

(28) $$XR'Y \equiv X, Y \in S' \wedge Y \subset X ;$$

R' is of course a relation of partial ordering. We need to prove that it is a right-rooted tree. This according to (28) means that

(29) $$(X, Y, Z \in S' \wedge Y \subset X \wedge Z \subset X) \to (Y \subset Z \vee Z \subset Y) .$$

Indeed, suppose $\operatorname{Ch}(Y, y)$ and $\operatorname{Ch}(X, x)$ and $\operatorname{Ch}(Z, z)$. From (26) we infer that yRz or zRy. If yRz we can prove that $Z \subset Y$; if zRy, then $Y \subset Z$.

It suffices to consider one of these cases. Suppose zRy; then according to (26) we shall prove that

(30) $$u \in Y \to \big(zRu \wedge \bigwedge w \big(w \in Z \to (uRw \vee wRu)\big)\big) .$$

If $u \in Y$, then yRu according to (26) and zRu by transitivity.

If $w \in Z$ and $u \in Y$, then $w, u \in X$ and by (26) uRw or wRu.

The formula (30) is thus proved and hence $Y \subset Z$ and (29).

The embedding of $\mathcal{A}(S, R)$ in $\mathcal{A}(S; R')$ is given by the following mapping:

(31) $$\varphi(X) = \big\{Y\colon \bigvee x \big(x \in X \wedge \operatorname{Ch}(Y, x)\big)\big\} \quad \text{for any } X \in \boldsymbol{P}(S) .$$

We need some properties of

(32) $$\varphi(X) = \varphi(Y) \to X = Y .$$

If $x \in X$, and $\operatorname{Ch}(U, x)$ for some U, then $U \in \varphi(X) = \varphi(Y)$, whence $x \in Y$, because the starting element of a chain is unique:

(33) $$\operatorname{Ch}(X, x) \wedge \operatorname{Ch}(X, y) \to x = y .$$

Hence we have $X \subset Y$, and conversely $Y \subset X$; thus $X = Y$.

(34) $$\varphi(X \cup Y) = \varphi(X) \cup \varphi(Y) ,$$

(35) $$\varphi(X \cap Y) = \varphi(X) \cap \varphi(Y)$$

are evident. We have to prove that

(36) $$\varphi(S - X) = S' - \varphi(X) .$$

The definition (31) and the uniqueness (33) of the starting point of a chain imply that

$$\begin{aligned} \varphi(S - X) &= \big\{Y\colon \bigvee x \big(x \in S - X \wedge \operatorname{Ch}(Y, x)\big)\big\} \\ &= \big\{Y\colon \bigvee x \operatorname{Ch}(Y, x) \wedge \bigwedge y \big(\operatorname{Ch}(Y, y) \to y \notin X\big)\big\} \\ &= S' - \varphi(X) . \end{aligned}$$

As the last formula we shall prove that

(37) $$\varphi(CX) = C\varphi(X) .$$

According to (31), (27) (18), and (2) this means that

(38) $$\bigvee x, z \big(z \in X \wedge zRx \wedge \mathrm{Ch}(Y, x)\big) \equiv \bigvee Z, z, x \big(\mathrm{Ch}(Y, x) \wedge \mathrm{Ch}(Z, z) \wedge Y \subset Z \wedge z \in X\big) .$$

The implication $\leftarrow$ is easy beacause if $\mathrm{Ch}(Y, x)$, then $x \in Y$ and, according to $Y \subset Z$, $x \in Z$. Hence if $\mathrm{Ch}(Z, z)$ then (26) implies that zRx.

For the converse implication $\rightarrow$ we need the lemma

(39) $$\big(zRx \wedge \mathrm{Ch}(Y, x)\big) \rightarrow \bigvee Z \big(\mathrm{Ch}(Z, z) \wedge Y \subset Z\big) .$$

It may be proved by using the axion of choice. Suppose that all elements x^ξ such that

(40) $$zRx_\xi \wedge x_\xi Rx$$

are enumerated by ordinals $< a$. We define a set U by induction:

$$U_0 = \{z, x\} ;$$

$$U_{\xi+1} = \begin{cases} U_\xi \cup \{x_\xi\} & \text{if} \quad \bigwedge u \big(u \in U_\xi \rightarrow (x_\xi Ru \vee uRx_\xi)\big) ; \\ U_\xi & \text{if not} ; \end{cases}$$

$$U_\lambda = \bigcup_{\xi<\lambda} U_\xi \quad \text{for } \lambda \text{ limit number} .$$

$$U = U_a .$$

Of course $U_\xi \subset U_\zeta$ for $\xi < \zeta$.

We put

$$Z = U \cup Y .$$

We easily verify that:

(41) $$w \in Z \rightarrow zRw ,$$

(42) $$w, v \in Z \rightarrow (wRv \vee vRw) ,$$

(43) $$\big[\bigwedge y \big(y \in Z \rightarrow (wRy \vee yRw)\big) \wedge zRw\big] \rightarrow w \in Z .$$

(41) is evident by (40) and the definition and (42) follows directly from the definition and the monotonicity of $\{U_\xi\}$. Proving (43) we find first that wRx or xRw. If xRw, then $w \in Y$ according to (26). If wRx, then for some ξ we have $w = x_\xi$ and according to the definition $w \in U_{\xi+1}$; thus $w \in U$. In both cases $w \in Z$.

Formulas (41)-(43) mean that $\mathrm{Ch}(Z, z)$. Of course $Y \subset Z$.

Formulas (32), (34), (35), (36) and (37) mean that $\mathcal{A}(S, R)$ is embedded in $\mathcal{A}(S', R')$.

2. Pseudo-Boolean algebras associated with relational systems. For a topological field of sets $\mathcal{C} = \langle \boldsymbol{P}(S), \cap, \cup, -, C\rangle$ we shall consider also the associated pseudo-Boolean algebra $\langle \boldsymbol{B}(S), \cap, \cup, \Rightarrow, \emptyset\rangle$, i.e. the algebra of open sets of the space $\mathcal{C}$:

$$\boldsymbol{B}(S) = \text{the set of all open sets of the space } \mathcal{C};$$

and the operation $\Rightarrow$ is defined as follows:

$$X \Rightarrow Y = \mathrm{Int}(-X \cup Y) .$$

If the topological field of sets $\mathcal{C}$ is associated with a quasi-ordering relational system $\langle S, R\rangle$, i.e. if $\mathcal{C} = \mathcal{A}(S, R)$, then the associated pseudo-Boolean algebra of open sets of $\mathcal{A}(S, R)$ will be denoted by $\mathcal{B}(S, R)$.

First we shall note the following lemma:

LEMMA 2. *Every pseudo-Boolean algebra $\mathcal{B}(S, R)$ for a quasi-ordering R is isomorphic with the pseudo-Boolean algebra $\mathcal{B}(S', R')$ for some relation R' of partial ordering, and $\overline{\overline{S'}} \leqslant \overline{\overline{S}}$.*

Proof. Let $\mathcal{B}(S, R)$ be a pseudo-Boolean algebra and let R be a quasi-ordering relation. Let us define an equivalence relation $\approx$:

(1) $$x \approx y \equiv (xRy \wedge yRx) .$$

First let us note that

(2) $$(X \in B(S) \wedge x \approx y) \rightarrow (x \in X \equiv y \in X) .$$

Hence if we take the relational quotient system $\langle S', R'\rangle$, S' being the set of equivalence classes of $\approx$ in S and

(3) $$[x]R'[y] \equiv xRy ,$$

then R' is a relation of partial ordering, and the pseudo-Boolean algebras $\mathcal{B}(S, R)$ and $\mathcal{B}(S', R)$ are isomorphic according to (2). The function F establishing isomorphism is defined as follows:

$$F(X) = \{[x]\colon x \in X\}, \quad \text{for} \quad X\boldsymbol{B} \in (S) .$$

A right-rooted tree is a *right-rooted stock tree* if it satisfies the condition

$$\bigvee x \in S \bigwedge y \in S\; yRx .$$

Of course every right-rooted tree can be enlarged to a right-rooted stock tree by adding one new end-point.

THEOREM 3. *Every pseudo-Boolean algebra $\mathcal{B}(S, R)$ for a quasi-ordering R can be embedded in a pseudo-Boolean algebra $\mathcal{B}(S'', R'')$ for some right-rooted stock tree R''.* (For a finite S, S'' is also finite.)

Proof. According to Lemma 2, R may be considered as a relation of partial ordering. Hence according to Theorem 2 the topological space $\mathcal{A}(S, R)$ can be embedded in a topological space $\mathcal{A}(S', R')$ for some right-rooted tree R'. Hence the same isomorphic function φ establishes the embedding of $\mathfrak{B}(S, R)$ in $\mathfrak{B}(S', R')$. Then we enlarge R' to R'' and S' to S'' as follows:

$$u \in S'' \equiv (u \in S \vee u = x_0),$$
$$y R'' u \equiv (y R' u \vee u = x_0),$$

where x_0 is a new element different from all $u \in S$. The new embedding is given by the function φ':

$$\varphi'(X) = \begin{cases} \varphi(X) & \text{if} \quad X \in \boldsymbol{B}(S) \text{ and } X \neq S, \\ S'' & \text{if} \quad X = S. \end{cases}$$

3. Applications to modal and intuitionistic logic.

Corollary 1. *The set of equalities true in all pseudo-Boolean algebras is identical with the set of equalities true in all pseudo-Boolean algebras* $\mathfrak{B}(S, R)$ *for a finite right-rooted stock tree* $\langle S, R\rangle$.

Proof. According to Mc. Kinsey and A. Tarski [6] this set of equalities is identical with the set of equalities true in all finite pseudo-Boolean algebras. And according to Theorem 3 finite pseudo-Boolean algebras can be embedded in finite trees.

Corollary 1 may be considered as a formulation of Jaśkowski's [3] theorem on the characterization of intuitionistic propositional calculus by his matrices, for Jaśkowski's matrices may be shown to be of the form $\mathfrak{B}(S, R)$ for finite trees R (see [2]).

Corollary 2. *If we add to the system of modal logic* S 4 *the axiom*

$$\text{(G)} \qquad \Big(\big((Z \Rightarrow \Box Y) \Rightarrow \Box Y\big) \wedge \big((\sim Z \Rightarrow \Box Y) \Rightarrow \Box Y\big)\Big) \Rightarrow \Box Y$$

($\Box$ = the necessity sign and $\Rightarrow$ = the strict implication), *we obtain a system* G *stronger that* S 4, *not contained in* S 5; *but interpreting intuitionistic connectives in the usual way, we obtain in* G *as theorems only the intuitionistic tautologies as in the case of* S 4.

Proof. According to Tarski's topological interpretation of modal logic, the matrices for G are topological spaces satisfying equality (E). (See Theorems 3.4 and 3.6 of McKinsey and Tarski [5].) Hence if Ψ is a theorem of G, then $\Psi = S$ is true in every topological space satisfying (E).

Suppose that Ψ is not intuitionistic; then according to Corrolary 1 there is a finite right-rooted stock tree $\langle S, R\rangle$ such that $\Psi \neq S$ in $\mathfrak{B}(S, R)$. But $\langle S, R\rangle$, being finite, is partially well-ordered, and according to Theorem 1 (E) is true in the topological space $\mathcal{A}(S, R)$. We obtain a contradiction.

The formula (G) is not a theorem of S 5 becausse S 5 is true in the space S of two points a, b such that $Ca = Cb = \{a, b\} = S$. The relation R defined in (5) is not a relation of partial ordering and hence (E) is not true in S.

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