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# Expansive transformation semigroups of endomorphisms


by

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**1. Introduction.** A number of examples of expansive homeomorphisms [1] on compact uniform spaces are actually automorphisms of topological groups: the symbolic flows ([4], 12.24), a homeomorphism on the dyadic solenoid constructed by Williams [7], expansive homeomorphisms of tori ([5], [6]). We formulate and study below notions of a semigroup being expansive when it acts on a uniform space and of a semigroup being regionally expansive when it acts with fixed points on a topological space; these two notions coincide in the case of a semigroup of endomorphisms of a topological group.

In section 4 we generalize the example of Williams by showing how to manufacture an expansive automorphism out of a "positively expansive" endomorphism.

In section 5 we apply the general considerations of sections 2 and 3 in order to characterize completely the expansive automorphisms of finite-dimensional toral groups: they are the automorphisms induced by unimodular matrices whose eigenvalues have modulus different from unity. We show that the toral groups are the only compact connected Lie groups which admit expansive groups of automorphisms, and that a compact connected group $G$ is toral if and only if the power map $x \to x^k$ of $G \to G$ is positively expansive.

**2. Expansive transformation semigroups.** A topological space $T$ provided with an associative binary operation for which $T$ has a bilateral identity 1 is called a *topologized semigroup* and, if $T$ is discrete, a *discrete semigroup*.

A *transformation semigroup* (tsg) is a triple $(X, T, \pi)$ where $X$ is a topological space, $T$ is a topologized semigroup, and $\pi$ is a continuous map of $X \times T$ into $X$ such that, if $\pi^t$ denotes the map $x \to (x, t)\pi$ of $X \to X$ for each $t \in T$, then $\pi^1$ is the identity map of $X$ and $\pi^{tr} = \pi^t \pi^r$ for all $t$,

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$r \in T$. A tsg $(X, T, \pi)$ in which $T$ is a topological group is just a transformation group (tg). The standard terminology and conventions used for a tg in [4] are employed for a tsg $(X, T, \pi)$ to the extent they are meaningful. In particular, $xt$ denotes $(x, t)\pi$ if $x \in X$ and $t \in T$, and a subset $A$ of $X$ is $T$-*invariant* if $AT \subset A$.

A tsg $(X, T, \pi)$ in which $T$ is the discrete additive semigroup $\boldsymbol{N}$ of nonnegative integers is called a *discrete semiflow*. A continuous map $\varphi$ of a topological space $X$ into $X$ determines a unique discrete semiflow $(X, \boldsymbol{N}, \pi)$ such that $\pi^1 = \varphi$ (compare [4], 1.31), and properties of $\varphi$ are often attributed to $(X, \boldsymbol{N}, \pi)$, and *vice versa*.

Let $(X, T, \pi)$ be a tsg where $X$ is a uniform space. An *expansive index of* $X$ (*for* $(X, T, \pi)$) is an index $\alpha$ of the uniformity of $X$ such that $x, y \in X$ and $x \neq y$ implies $(xt, yt) \notin \alpha$ for some $t \in T$. We call $(X, T, \pi)$ *expansive* if there exists such an expansive index for $X$.

Let $X$ be a uniform space. A [continuous map $\varphi$ of $X$ into $X$] [homeomorphism $\varphi$ of $X$ onto $X$] is called [*positively expansive*] [*expansive*] (*on* $X$) if the discrete [semiflow] [flow] $(X, T, \pi)$ such that $\pi^1 = \varphi$ is an expansive tsg.

Special cases of our first theorem have been known for some time (see [1], for example).

THEOREM 1. *Let $(X, T, \pi)$ be a tsg where $X$ is a quasicompact uniform space, let $(X, T, \pi)$ be expansive, and let $T$ be separable. Then $X$ is metrizable.*

Proof. Let $\alpha$ be a closed expansive index of $X$.

We show first that $X$ is separated. Choose a symmetric index $\beta$ of $X$ with $\beta^2 \subset \alpha$. Let $x, y \in X$ with $x \neq y$. There exists $t \in T$ with $(xt, yt) \notin \alpha$. Then $(xt\beta)(\pi^t)^{-1}$, $(yt\beta)(\pi^t)^{-1}$ are disjoint neighborhoods of $x$, $y$.

To complete the proof, we shall show that the uniformity of $X$ has a countable base. For each $t \in T$, let

$$\alpha_t = \{(x, y) \mid x, y \in X \;\&\; (xt, yt) \in \alpha\}.$$

Let $S$ be a countable dense subset of $T$, let $\mathfrak{F}$ be the countable class of all finite subsets of $S$, and for each $F \in \mathfrak{F}$ let

$$\alpha_F = \bigcap_{t \in F} \alpha_t ,$$

whence $\alpha_F$ is a closed index of $X$.

Let $\beta$ be an open index of $X$. It is enough to show that $\alpha_F \subset \beta$ for some $F \in \mathfrak{F}$. Assume that $\alpha_F \cap \beta' \neq \emptyset$ for any $F \in \mathfrak{F}$, where $\beta'$ is the complement of $\beta$ in $X \times X$. Then $\{\alpha_F \cap \beta' \mid F \in \mathfrak{F}\}$ is a closed filter-base on $X \times X$ which by the compactness of $X$ has an adherent point $(x, y)$. If $t \in T$ with $(xt, yt) \notin \alpha$, then $\{s \mid s \in T \;\&\; (xs, ys) \notin \alpha\}$ is a nonvacuous open subset of $T$, and there exists $s \in S$ with $(xs, ys) \notin \alpha$ which is impossible since $\{s\} \in \mathfrak{F}$. Thus $t \in T$ implies $(xt, yt) \in \alpha$, but $x \neq y$ since $(x, y) \notin \beta$.



The need in theorem 1 for some sort of countability assumption on $T$ will be shown by the next example.

Let $T$ be a discrete semigroup and let $Y$ be a nonempty topological space. The *shifting transformation semigroup over $Y$ generated by $T$* is by definition the tsg $(X, T, \pi)$ where $X$ is the product space $Y^T$ and $\pi$ is defined by

$$s\big((x, t)\pi\big) = (ts)x \qquad (x \in X;\; t, s \in T);$$

if $T$ is a discrete group the prefix "semi" is omitted. If $Y$ is not a singleton, then $(X, T, \pi)$ is effective. When $T$ is the discrete additive group $\boldsymbol{Z}$ of integers and $Y$ is a finite discrete space having $n$ elements, then $(X, T, \pi)$ is just the symbolic flow on $n$ symbols.

If $Y$ is a discrete uniform space, then the shifting tsg over $Y$ generated by $T$ is expansive with respect to the product uniformity of $X$.

EXAMPLE. (This example is isomorphic to one constructed by R. Ellis (unpublished) in another way.) Let $T$ be a discrete uncountable group, let $D$ be a two point discrete space, let $(Y, T)$ be the shifting tg over $D$ generated by $T$, let $A$ be a nonvacuous countable subset of $T$, let $y$ be the characteristic function of $A$ in $T$, let $X$ be the orbit-closure of $y$ under $T$, and let $(X, T)$ be the restriction of $(Y, T)$ to $X$. Then $X$ is a compact zero-dimensional space and $(X, T)$ is an expansive point-transitive tg.

Suppose that $X$ is metrizable. Then there exists a countable subset $S$ of $T$ such that $yS$ is dense in $X$. Choose $s \in T$ with $As^{-1} \cap S = \emptyset$, choose $t \in T$ with $ts \in A$, and let $x = yt \in X$. There exists $r \in S$ such that $s(yr) = sx$. Then $(rs)y = (ts)y$, $rs \in A$ since $ts \in A$, and $r \in As^{-1} \cap S$.

Thus, $X$ is not metrizable. Note that, by theorem 1, the $t$-transition $x \to xt$ is not expansive on $X$ for any $t \in T$.

Remark. Let $\big((X_i, T) \mid i \in I\big)$ be a family of tsgs with uniform phase spaces, let $X$ be the product of the $X_i$ $(i \in I)$, and let $(X, T)$ be the tsg such that $(x_i \mid i \in I) \in X$ and $t \in T$ implies

$$(x_i \mid i \in I)t = (x_i t \mid i \in I).$$

Then $(X, T)$ is expansive iff $(X_i, T)$ is expansive for all $i \in I$ and $X_i$ is a singleton for all but a finite number of $i \in I$ (compare [2]).

**3. Designated transformation semigroups.** A *designated transformation semigroup* is an ordered pair $\big((X, T), D\big)$ where $(X, T)$ is a tsg and $D$ is a $T$-invariant subset of $X$, called the *designated set of* $\big((X, T), D\big)$; when $T$ is a topological group the prefix "semi" is omitted. A designated tsg $\big((X, T), D\big)$ is denoted also by $(X, T; D)$ and, in the case that $D$ is a singleton $\{e\}$, by $(X, T; e)$.

A [tsg] [tg] $(G, T, \pi)$ such that $G$ is a topological group and $\pi^t$ is an [endomorphism] [automorphism] of $G$ for each $t \in T$ is called a *transformation* [*semigroup of endomorphisms*] [*group of automorphisms*] *of* $G$.

Let $(G, T)$ be a tsg of endomorphisms of a topological group $G$ with identity element $e$. Then $(G, T; e)$ is a designated tsg. More generally, let $H$ be a $T$-invariant subgroup of $G$, and let $(G/H, T)$ be the partition tsg of $G/H$ induced by $(G, T)$, whence $x \in G$ and $t \in T$ implies $(xH)t = (xt)H$. Then $(G/H, T; H)$ is a designated tsg whose designated set is $\{H\}$.

Let $(X, T; D)$ be a designated tsg. By an *expansive neighborhood of* $D$ *for* $(X, T)$ is meant a neighborhood $U$ of $D$ in $X$ such that $x \in X$ and $x \notin D$ implies $xT \not\subset U$. If an expansive neighborhood of $D$ for $(X, T)$ exists, then we call $(X, T; D)$ *regionally expansive* and say that $(X, T)$ is *regionally expansive at* $D$.

If $(X, T)$ is a tsg where $X$ is a uniform space, if $(X, T)$ is expansive, and if $e \in X$ is left fixed by $T$, then the designated tsg $(X, T; e)$ is regionally expansive.

Remark. Let $(G, T)$ be a tsg of endomorphisms of a topological group $G$ with identity element $e$. Then the following conditions are equivalent:

(1) The designated tsg $(G, T; e)$ is regionally expansive.

(2)-(4) The tsg $(G, T)$ is expansive with respect to the [left] [right] [bilateral] uniformity of $G$.

Consequently, we say that $(G, T)$ is *expansive* in the present instance to mean any one of (1) through (4).

EXAMPLE. Let $T$ be a discrete semigroup. Let $Y$ be any nontrivial finite discrete group and let $(G, T)$ be the shifting tsg over $Y$ generated by $T$. Then $G$ is a compact zero-dimensional group when provided with its product group structure, and $(G, T)$ is an effective expansive tsg of endomorphisms of $G$ (and a tg of automorphisms of $G$ if $T$ is a group).

DEFINITION. Let $(X, T; D)$ and $(Y, T; E)$ be designated tsgs. A continuous map $\varphi$ of $X$ onto $Y$ such that $D\varphi = E$ is called a *homomorphism of* $(X, T; D)$ *onto* $(Y, T; E)$ if $x \in X$ and $t \in T$ implies $xt\varphi = x\varphi t$. A homeomorphism $\varphi$ of a neighborhood $U$ of $D$ onto a neighborhood $V$ of $E$ such that $D\varphi = E$ is called a *homeomorphic local homomorphism of* $(X, T; D)$ *onto* $(Y, T; E)$ if $x \in U$, $t \in T$, and $xt \in U$ implies $xt\varphi = x\varphi t$; when such a homeomorphism exists we say that $(X, T; D)$ is *homeomorphically locally homomorphic to* $(Y, T; E)$.

THEOREM 2. *Let the designated tsg* $(X, T; D)$ *be homeomorphically locally homomorphic to the designated tsg* $(Y, T; E)$. *Then* $(X, T; D)$ *is regionally expansive if* $(Y, T; E)$ *is regionally expansive. The converse holds if* $T$ *is compactly generated, that is, if* $T = \bigcup_{n=1}^{\infty} K^n$ *for some quasicompact subset* $K$ *of* $T$.



Proof. Let $\varphi$ be a homeomorphism of a neighborhood $U$ of $D$ onto a neighborhood $V$ of $E$ such that $\varphi$ is a homeomorphic local homomorphism of $(X, T; D)$ onto $(Y, T; E)$. It is easily seen that if $V_1$ is an expansive neighborhood of $E$ for $(Y, T)$ with $V_1 \subset V$, then $V_1\varphi^{-1}$ is an expansive neighborhood of $D$ for $(X, T)$.

Conversely, let $(X, T; D)$ be regionally expansive and let $K$ be a quasicompact subset of $T$ generating $T$. There exists an expansive neighborhood $U_1$ of $D$ for $(X, T)$ such that $U_1 \cup U_1K \subset U$. Then $V_1 = U_1\varphi$ is a neighborhood of $E$ in $Y$. Let $y \in V_1 - E$. Since $(U_1 - D)\varphi = V_1 - E$, we may choose $x \in U_1 - D$ with $x\varphi = y$. There exists a least positive integer $n$ such that $xK^n \not\subset U_1$. We have $xK^{n-1} \subset U_1$, $xK^n = xK^{n-1}K \subset U_1K \subset U$, and $xK^n \subset U$. Since $\varphi$ is one-to-one, we conclude that $yK^n = x\varphi K^n = xK^n\varphi \not\subset U_1\varphi = V_1$, $yK^n \not\subset V_1$, and $yT \not\subset V_1$.

COROLLARY. *Let* $(G, T)$ *be a tsg of endomorphisms of a topological group* $G$, *let* $H$ *be a discrete* $T$*-invariant subgroup of* $G$, *and let* $(G/H, T)$ *be the partition tsg of* $G/H$ *induced by* $(G, T)$. *Then* $(G, T)$ *is expansive if* $(G/H, T; H)$ *is regionally expansive. The converse holds if* $T$ *is compactly generated.*

Proof. The canonical map $p$ of $G$ onto $G/H$ is a homomorphism of $(G, T; e)$ onto $(G/H, T; H)$, where $e$ is the identity element of $G$. There exists a symmetric open neighborhood $U$ of $e$ with $U^2 \cap H = \{e\}$. Then $p|U$ is a homeomorphic local homomorphism of $(G, T; e)$ onto $(G/H, T; H)$, and we may use the theorem.

**4. The $\varphi$-oidal flows.** Throughout this section, $X$ denotes a uniform space and $\varphi$ denotes a uniformly continuous map of $X$ onto $X$.

Let $Y$ be the set of all bisequences $(x_n | n \in \mathbf{Z})$ over $X$, and provide $Y$ with its product uniformity. We denote the value of any $x \in Y$ at $n \in \mathbf{Z}$ by $x_n$. Let $X^*$ be the subset of $Y$ consisting of all $x \in Y$ such that $x_{n+1}\varphi = x_n$ for every integer $n$, and provide $X^*$ with its subspace uniformity. For each integer $n$, let $\varphi_n$ be the map $x \to x_n$ of $X^*$ onto $X$ and let $\psi_n$ be the map $\varphi_n \times \varphi_n$ of $X^* \times X^*$ onto $X \times X$. Then each $\varphi_n$ is uniformly continuous, and the class of all sets of the form $\alpha\psi_n^{-1}$, where $\alpha$ is an index of $X$ and $n$ is an integer, is a base of the uniformity of $X^*$.

Let $\sigma$ be the map of $X^*$ into itself such that $x \in X^*$ implies

$$x\sigma = (x_n\varphi | n \in \mathbf{Z}) .$$

Then $\sigma\varphi_n = \varphi_n\varphi$ for every integer $n$, $\sigma$ is bijective since

$$x\sigma = (x_{n-1} | n \in \mathbf{Z}) \quad (x \in X^*) ,$$

and $\sigma$ is a unimorphism. The discrete flow $(X^*, \sigma)$ is called the *$\varphi$-oidal flow*; it is the restriction to $X^*$ of the shifting tg over $X$ generated by $\mathbf{Z}$.

THEOREM 3. *Let $\varphi$ be positively expansive on $X$. Then $\sigma$ is expansive on $X^*$.*

Proof. Let $\alpha$ be an expansive index of $X$ for the discrete semiflow defined by $\varphi$. Let $\beta = \alpha\varphi_0^{-1}$, whence $\beta$ is an index of $X^*$. Let $x, y \in X^*$ with $x \neq y$. Choose an integer $j$ with $x_j \neq y_j$. There exists a nonnegative integer $k$ such that $(x_j\varphi^k, y_j\varphi^k) \notin \alpha$. Letting $i = k-j$, we have $(x\sigma^i, y\sigma^i)\psi_0 = (x_j\varphi^k, y_j\varphi^k)$, and $(x\sigma^i, y\sigma^i) \notin \beta$. Hence $\beta$ is an expansive index of $X^*$.

We observe that if $\varphi$ is itself unimorphic, then for each integer $n$ the map $\varphi_n$ is a unimorphism of $X^*$ onto $X$ and hence is an isomorphism of the discrete flow $(X^*, \sigma)$ with the discrete flow $(X, \varphi)$.

Suppose now that the uniformity of $X$ is the [left] [right] [bilateral] uniformity defined by a structure of a topological group on $X$ and that $\varphi$ is an endomorphism for this structure. Then $X^*$ is a topological subgroup of the product group $Y$, the uniformity of $X^*$ is just the [left] [right] [bilateral] uniformity of the topological group $X^*$, and $\sigma$ is an automorphism of $X^*$.

Thus, if we are given a positively expansive continuous surjective endomorphism $\varphi$ of a topological group $G$ then the $\varphi$-oidal flow $(G^*, \sigma)$ is an expansive tg of automorphisms of $G^*$.

EXAMPLE. Let $C$ be the circle group, let $p$ be a prime number, let $\varphi$ be the endomorphism $z \to z^p$ of $C$, and let $(C^*, \sigma)$ be the $\varphi$-oidal flow. Then $C^*$ is isomorphic with the $p$-adic solenoid and so is a compact connected abelian group. Now the automorphism $x \to px$ of $\boldsymbol{R}$ is clearly positively expansive, and it follows from theorem 2 that $\varphi$ is positively expansive. Hence $\sigma$ is an expansive automorphism of $C^*$. If in this example we take $p = 2$, then we obtain, essentially, the example of Williams mentioned in the introduction.

## 5. Expansive endomorphisms of toral groups.

Let $G$ be a compact group. A homeomorphic group automorphism of $G$ is called simply an automorphism of $G$. Let $\mathrm{Aut}(G)$ be the group of all automorphisms of $G$, and provide $\mathrm{Aut}(G)$ with its compact-open topology. Then $\mathrm{Aut}(G)$ is a separated topological group which acts continuously on $G$.

Since a tg$(X, T)$ with both $X$ and $T$ compact is uniformly equicontinuous, we have:

LEMMA 1. *Let $(G, T)$ be a tg of automorphisms of an infinite compact group $G$ and let $\mathrm{Aut}(G)$ be compact. Then $(G, T)$ is not expansive.*

Remark. Let $n$ be a positive integer. In [5], [6] Reddy shows that each $n \times n$ unimodular matrix (integral entries and determinant $\pm 1$) having $n$ distinct real eigenvalues different from 1 defines an expansive homeomorphism of the $n$-dimensional torus; by exhibiting such matrices for $n = 2$ and $n = 3$ he deduces, using a special case of our remark at the end of section 2, the existence of an expansive homeomorphism of the $n$-dimensional torus for any $n > 1$. Now these homeomorphisms are all automorphisms of toral groups. Our next theorem contains all these facts as special cases of the general results of section 3.



LEMMA 2. *Let $u$ be an automorphism of a separated finite-dimensional real topological vector space $E$. Then a necessary and sufficient condition for $u$ to be [positively expansive] [expansive] on $E$ is that $[|\lambda| > 1]$ $[|\lambda| \neq 1]$ for each complex characteristic root $\lambda$ of $u$.*

Proof. See [3].

THEOREM 4. *Let $G$ be the $n$-dimensional toral group $\boldsymbol{R}^n/\boldsymbol{Z}^n$, where $n > 1$, and let $\mathfrak{U}$ be the set of all $n \times n$ unimodular matrices whose complex eigenvalues all have modulus different from unity. With respect to the usual basis of $\boldsymbol{R}^n$ each $M \in \mathfrak{U}$ defines an automorphism of $\boldsymbol{R}^n$ leaving $\boldsymbol{Z}^n$ invariant and hence induces an automorphism $f(M)$ of $G$. Then $f$ is a one-to-one map of $\mathfrak{U}$ onto the set of all expansive automorphisms of $G$.*

Proof. Use lemma 2 and the corollary to theorem 2.

COROLLARY. *There exists an expansive automorphism of $G$.*

THEOREM 5. *Let $G$ be a compact connected Lie group of dimension $n \geqslant 1$. Then the following statements are equivalent:*

(1) *There exists an expansive tg $(G, T)$ of automorphisms of $G$.*

(2) *$G$ is toral and $n > 1$.*

Proof. That (2) implies (1) follows from the preceding corollary. Assume (1). We show (2). Let $G'$ be the commutator subgroup of $G$. Then $G'$ is $T$-invariant, and the restriction of $(G, T)$ to $G'$ is expansive. Then $G'$ is a closed semisimple analytic subgroup of $G$, and a compact connected semisimple Lie group has compact automorphism group. It follows from lemma 1 that $G'$ is the trivial group. Hence $G$ is abelian and, *a fortiori*, toral. Since the circle group has exactly two automorphisms, lemma 1 implies that $n \neq 1$.

EXAMPLE. We show that the compactness hypothesis of theorem 5 cannot be removed. Let $G$ be the subgroup of $GL(3, \boldsymbol{R})$ consisting of all matrices of the form

$$\begin{Vmatrix} 1 & x_1 & x_3 \\ 0 & 1 & x_2 \\ 0 & 0 & 1 \end{Vmatrix}$$

(we denote such a matrix simply by $[x_1, x_2, x_3]$). Then $G$ is a noncompact connected simply-connected nonabelian nilpotent three-dimensional Lie group. Let $\lambda$ be any real number with $|\lambda| > 1$, and let $\varphi$ be the map of $G$ into $G$ such that $x_1, x_2, x_3$ real imply

$$[x_1, x_2, x_3]\varphi = [\lambda x_1, \lambda x_2, \lambda^2 x_3].$$

It may be verified directly that $\varphi$ is a positively expansive automorphism of $G$.

Remark. T. S. Wu [8] has improved theorem 5 by showing that a compact connected finite-dimensional group admitting an expansive automorphism is necessarily abelian. The infinite-dimensional case remains unsettled; the remark at the end of section 2 seems to account for some of the difficulty in this case.

THEOREM 6. *Let $G$ be a locally compact group, let $k$ be an integer with $|k| \geqslant 2$, and let $\varphi$ be the map $x \to x^k$ of $G$ into $G$. Then:*

(1) *If the discrete semiflow $(G, \varphi)$ is regionally expansive at the identity element $e$ of $G$, then $G$ is a Lie group.*

(2) *If $G$ is a compact connected group, then $\varphi$ is positively expansive on $G$ if and only if $G$ is toral.*

Proof. (1) Let $U$ be an expansive neighborhood of $e$ for $(G, \varphi)$. It is enough to show that $U$ contains no nontrivial subgroup of $G$. Let $H$ be a subgroup of $G$ with $H \subset U$. If there is some $x \in H$ with $x \neq e$, then $x \in U$, $x\varphi^i \notin U$ for some positive integer $i$, and $x\varphi^i = x^{k^i} \in H \subset U$, which is a contradiction. Hence $H = \{e\}$.

(2) If $G$ is toral of dimension $n$, then that $\varphi$ is positively expansive follows from theorem 2 and the fact that the automorphism $x \to kx$ of $\boldsymbol{R}^n$ is positively expansive.

Assume that $G$ is compact connected and that $\varphi$ is positively expansive on $G$. By (1), $G$ is a Lie group. Suppose that $G$ is nonabelian, and let $U$ be a neighborhood of $e$. We shall show that $U$ cannot be expansive for $(G, \varphi)$.

Let $W$ be a neighborhood of $e$ with $W^2 \subset U$. Since $G$ is compact, there exists a symmetric neighborhood $V$ of $e$ such that $z^{-1}Wz \subset W$ for all $z \in G$. Then $z^{-1}a^{-1}za \in U$ for all $a \in V$, $z \in G$.

There exists a noncentral element $x$ of $G$. Since $V$ generates $G$, there exists $a \in V$ such that $ax \neq xa$. Let $y = a^{-1}xa$. Then $x \neq y$, but $(x\varphi^i)^{-1}(y\varphi^i) = (x\varphi^i)^{-1}a^{-1}(x\varphi^i)a \in U$ for each nonnegative integer $i$.

QUESTION. Call a tg $(X, T, \pi)$, where $X$ is a uniform space, *properly expansive* if it is effective and expansive and if $\pi^t$ is not expansive on $X$ for any $t \in T$. In section 2 we constructed a properly expansive tg of automorphisms of a compact zero-dimensional group. Does there exist a properly expansive tg of automorphisms of a compact connected group?

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