order a sequence of disjoint sets $X_i \subseteq X$ $(i < \omega)$ such that $|X - \bigcup_{i<j} X_i| = \mathfrak{m}$ for every $j < \omega$. In Γ_1 player II wins if $\bigcap_{i<\omega} X_i = 0$ and in Γ_2 if $\bigcup_{i<\omega} X_i = X$. Schreiers' argument proves that in both games, Γ_1 and Γ_2, player II has a winning strategy.

(d) A similar problem stated by S. Ulam [19] is still open.

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A duality property of nerves

by

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1. Our main aim in this paper is to prove the following

1.1. THEOREM. *Let Y be a normal space, and $\mathfrak{U} = \{U_\alpha | \alpha \in \mathfrak{A}\}$ a nbd-finite* (1) *covering of Y by open F_σ-sets. Assume that*

(a) *The order* (2) *of $\mathfrak{U}$ is $\leqslant n$*

and

(b) *For each $k \geqslant 1$, the intersection of every k sets of $\mathfrak{U}$ is $(n-k)$-connected* (3).

Then each canonical map $\varkappa$ of Y into the nerve (4) *$N(\mathfrak{U})$ of $\mathfrak{U}$ has a right homotopy inverse* (5) *$g\colon N(\mathfrak{U}) \to Y$. Moreover, $\varkappa$ and g can be chosen so that $g \circ \varkappa$ is $\mathfrak{U}$-close* (6) *to the identity map of Y.*

In [8], pp. 142-145, Weil derived the above conclusion from the two assumptions: (a') No restriction on the order of $\mathfrak{U}$, and (b') Every finite intersection of sets of $\mathfrak{U}$ is ∞-connected; thus, in 1.1 we strengthen one of his hypotheses and weaken the other. Our proof of 1.1 will be a modification of his; note that the above version does not require the

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(1) $\mathfrak{U}$ is nbd-finite if each point has a neighborhood meeting at most finitely many members of $\mathfrak{U}$.

(2) The largest integer n such that there are $(n+1)$ members of with non-empty intersection, i.e., the dimension of the nerve of $\mathfrak{U}$.

(3) X is k-connected if $\pi_i(X) = 0$ for $0 \leqslant i \leqslant k$; it is ∞-connected if $\pi_i(X) = 0$ for all $i \geqslant 0$; $\pi_0(X) = 0$ denotes that X is path-connected.

(4) We realize the nerve of a covering $\mathfrak{U}$ as a rectilinear polytope in a real vector space spanned by linearly independent vectors in a fixed one-to-one correspondence with the non-empty $U_\alpha \in \mathfrak{U}$. The vertex corresponding to U_α is the unit point on the corresponding vector, and is denoted by p_α. The topology of $N(\mathfrak{U})$ is the CW-topology ([9], p. 223). A continuous $\varkappa\colon Y \to N(\mathfrak{U})$ is called *cannonical* if $\varkappa^{-1}(\mathrm{St}\, p_\alpha) \subset U_\alpha$ for each $\alpha \in \mathfrak{A}$.

(5) That is, $\varkappa \circ g \simeq 1$; equivalently, Y dominates $N(\mathfrak{U})$.

(6) Two maps $f, g\colon X \to Y$ are $\mathfrak{U}$-close if for each $x \in X$ there is a $U_\alpha \in \mathfrak{U}$ containing both $f(x)$ and $g(x)$. Under certain conditions (for example, if each finite intersection of the closures of the U_α is an AR (normal) ([8], p. 142) or if Y is an ANR and the U_α are "sufficiently small" ([5], p. 243)) $\mathfrak{U}$-closed maps are homotopic.

sets of $\mathfrak{U}$ to be even n-connected. Some applications, to derive a generalization of Helly's convex-set theorem, and to some recent work by de Groot, de Vries, van der Walt in [3], will be given.

2. We collect separately some results on mappings of spaces into CW-polytopes which, though more general than required, are worth stating explicitly for future use.

Let L be a real vector space with the finite topology (7). It is known ([4], p. 416, [7], p. 57) that if L has a basis of cardinal $\geqslant 2^{\aleph_0}$, then the addition operation is not continuous. However,

2.1. (a) *Each compact $C \subset L$ is contained in a finite dimensional linear subspace.*

(b) *If X is a k-space* (8) *and $f, g\colon X \to L$ continuous, then $x \to f(x)+g(x)$ is also continuous.*

Proof. It is clear that (b) follows from (a) since addition is continuous on each finite-dimensional linear subspace, so that $x \to f(x)+g(x)$ is continuous on each compact $C \subset X$. To prove (a), assume that C is not contained in any finite-dimensional linear subspace. Choose $x_1, x_2 \in C$, with $x_1 \neq x_2$, and proceed by induction, choosing $x_{n+1} \in C$ to be a point not in the finite-dimensional linear subspace spanned by $x_1, \ldots, x_n$. Then $A = \{x_n \mid n \geqslant 1\}$ is an infinite closed discrete subset of C, since each finite-dimensional linear subspace contains at most finitely many members of A, and therefore C cannot be compact.

We shall assume all polytopes taken with the CW-topology; they are not required to be either finite-dimensional or locally finite. Every such polytope P will always be considered embedded as a subspace of a vector space $L(P)$ with finite topology, spanned by independent vectors in 1-1 correspondence with the vertices of P, and with each vertex of P being at the unit point p_a of the corresponding vector.

Call a map $f\colon X \to P$ *locally finite* whenever each $x \in X$ has a nbd V such that $f(V)$ is contained in a finite subpolytope; for example, any canonical map of a normal space into the nerve of a nbd-finite open covering is locally finite, whereas the identity map of a non-locally finite polytope is not.

2.2. THEOREM. *Let $f, g\colon X \to P$ be continuous and such that for each $x \in X$ the points $f(x)$ and $g(x)$ belong to a common open vertex-star. Assume that either* (a) *X is a k-space, or* (b) *both f and g are locally finite maps. Then $f \simeq g$ by a homotopy in which the path of each x lies in an open vertex-star.*

(7) A set is closed if and only if its intersection with each finite-dimensional linear subspace is closed in the Euclidean topology of that subspace.

(8) X is a k-space if a set is open whenever its intersection with each compact subspace is open in that subspace ([2], p. 220; [4], p. 248). A map $h\colon X \to Y$ is therefore continuous if and only if $h|C$ is continuous for each compact C. It is easy to see that ([4], p. 243) every locally compact space, every first-countable space, and every CW-polytope, is a k-space.

Proof. We consider case (a). Let

$$f(x) = \sum_a f_a(x)\cdot p_a\,, \qquad g(x) = \sum_a g_a(x)\cdot p_a\,,$$

where $\{f_a(x)\}$ resp. $\{g_a(x)\}$ are the barycentric coordinates of $f(x)$ resp. $g(x)$. Each f_a, g_a is a continuous real-valued function on X, so for each a,

$$h_a(x) = \min[f_a(x), g_a(x)]$$

is also continuous. For each x, at most finitely many $h_a(x) \neq 0$, for if $(p_{a_1}, \ldots, p_{a_n})$ is the carrier of $f(x)$, then $f_a(x) = 0$ for all $a \neq a_1, \ldots, a_n$. The function $h = \sum_a h_a$ is therefore well-defined and we show that it is continuous on each compact $C \subset X$: Indeed, by 2.1 (a), $f(C)$ and $g(C)$ lie on a finite subpolytope of P, so on C only a fixed finite number of the h_a are not identically zero, and $h|C$, being the sum of a fixed finite number of continuous functions, is therefore continuous. Since X is a k-space, h is consequently continuous on X. Finally, $h(x)$ is never zero: given any x, there is an a such that $f(x), g(x) \in \mathrm{St}(p_a)$; then $f_a(x) > 0$, $g_a(x) > 0$, and $h(x) \geqslant h_a(x) > 0$.

Because of 2.1, the function

$$\lambda(x) = \frac{1}{h(x)} \sum_a h_a(x)\cdot p_a$$

maps X continuously into $L(P)$. Choose any $x \in X$; if $h_{a_i}(x) \neq 0$ for $i = 1, \ldots, n$ and only these indexes, then $\lambda(x)$ belongs to the open simplex $(p_{a_1}, \ldots, p_{a_n})$; since both $f_{a_i}(x)$ and $g_{a_i}(x)$ are not zero, both $f(x)$ and $g(x)$ are carried by simplexes of P having $(p_{a_1}, \ldots, p_{a_n})$ as face. This shows first that in fact λ maps X into P and then that for each x, $g(x)$ and $\lambda(x)$, so well as $\lambda(x)$ and $f(x)$, belong to a common closed simplex of P. Since $X \times I$ is a k-space ([4], p. 263) and, by 2.1, $(x, t) \to tf(x) + (1-t)\lambda(x)$ is a continuous map into P, we find $f \simeq \lambda$ and, similarly, that $\lambda \simeq g$. The proof in case (b) is similar and simpler.

This generalization of the usual theorem, wherein $f(x)$, $g(x)$ are required to be in a common closed simplex for each $x \in X$, is frequently more useful.

3. If P is a polytope, we write $\sigma < \tau$ to denote that σ is a *proper* face of τ, and we denote the barycenter of σ by $[\sigma]$. The first barycentric subdivision P' of P consists of all simplexes $([\sigma_1], [\sigma_2], \ldots, [\sigma_s])$ such that $\sigma_1 < \sigma_2 < \ldots < \sigma_s$. For each $\sigma \in P$, the *linked complex* $\mathrm{Lk}(\sigma)$ is that subcomplex of P' consisting of all simplexes $([\sigma_1], [\sigma_2], \ldots, [\sigma_s])$ such that

$\sigma < \sigma_1 < \sigma_2 < \ldots < \sigma_s$ and $\mathrm{Tr}(\sigma)$, the *closed traverse* of σ, is the join $([\sigma], \mathrm{Lk}(\sigma))$, that is, all simplexes $([\sigma], [\sigma_1], [\sigma_2], \ldots, [\sigma_s])$ such that $\sigma < \sigma_1 < \sigma_2 < \ldots < \sigma_s$, together with all faces of such simplexes. If $\dim P \leqslant n$, then clearly $\dim \mathrm{Tr}(\sigma) \leqslant n - \dim\sigma$.

Using the notation in the statement of Theorem 1.1, we now give the

Proof of 1.1. Let $\varkappa\colon Y \to N(\mathfrak{U})$ be the canonical map

$$\varkappa(y) = \sum_{a} \varkappa_a(y)\cdot p_a$$

where $\{\varkappa_a\}$ is a partition of unity subordinated to $\{U_a|\ a \in \mathfrak{A}\}$ and $\varkappa_a^{-1}(0) = Y - U_a$ for each [9] $a \in \mathfrak{A}$. In particular, if $y \in U_{a_1}, \ldots, U_{a_k}$ and only these sets, then $\varkappa(y)$ lies in the *open* simplex $(p_{a_1}, \ldots, p_{a_k})$.

For each $\sigma = (p_{a_1}, \ldots, p_{a_k}) \in N(\mathfrak{U})$, let $E(\sigma) = U_{a_1} \cap \ldots \cap U_{a_k}$. Let N' be the barycentric subdivision of N; we will construct a continuous $g\colon N' \to Y$ such that $g(\mathrm{Tr}(\sigma)) \subset E(\sigma)$ for each $\sigma \in N$.

For each $k \geqslant 0$, let B_k be the subcomplex

$$B_k = \bigcup\{\mathrm{Tr}(\sigma)|\ \dim\sigma \geqslant k\}$$

of N', so that $B_n \subset B_{n-1} \subset \ldots \subset B_0 = N'$ and B_n is discrete. Define $g_n\colon B_n \to Y$ by sending each $[\sigma^n]$ to a point of $E(\sigma^n)$. We proceed by induction, assuming that g_n has been extended to a continuous $g_k\colon B_k \to Y$ such that $g_k(\mathrm{Tr}(\sigma)) \subset E(\sigma)$ whenever $\dim\sigma \geqslant k$. For each σ^{k-1}, observe that $g_k[\mathrm{Lk}(\sigma^{k-1})] \subset E(\sigma^{k-1})$; since $\mathrm{Lk}(\sigma^{k-1})$ is a subcomplex of $\mathrm{Tr}(\sigma^{k-1})$ with $\dim[\mathrm{Tr}(\sigma^{k-1}) - \mathrm{Lk}(\sigma^{k-1})] \leqslant n-k+1$, and since $\pi_i(E(\sigma^{k-1})) = 0$ for $0 \leqslant i \leqslant n-k$, the mapping $g_k|\ \mathrm{Lk}(\sigma^{k-1})$ can therefore be extended ([5], p. 237, [6], p. 241) over $\mathrm{Tr}(\sigma^{k-1})$ with values in $E(\sigma^{k-1})$. Because each intersection $\mathrm{Tr}(\sigma^{k-1}) \cap \mathrm{Tr}(\hat{\sigma}^{k-1}) \subset B_k$, this piecewise extension over each $\mathrm{Tr}(\sigma^{k-1})$ results in a continuous $g_{k-1}\colon B_{k-1} \to Y$ such that $g_{k-1}(\mathrm{Tr}(\sigma)) \subset E(\sigma)$ whenever $\dim\sigma \geqslant k-1$, and completes the inductive step [10]. We set $g = g_0$.

The maps $g\circ\varkappa$ and $1\colon Y \to Y$ are $\mathfrak{U}$-close. For if $y \in Y$ belongs to $U_{a_1}, \ldots, U_{a_k}$ and only these sets, then $\varkappa(y)$ lies in the open simplex $\sigma = (p_{a_1}, \ldots, p_{a_k})$; if $\sigma' = ([\sigma_1], \ldots, [\sigma_s])$ is the carrier of $\varkappa(y)$ in N', then σ_1 must be some face of σ, and for any vertex p_a of σ_1, both y and $g\circ\varkappa(y)$ belong to U_a.

The map $\varkappa\circ g \simeq 1$. Let $p \in N$; if $\sigma' = ([p_{a_0}, \ldots, p_{a_i}], \ldots)$ is the carrier of p, then $g(p) \in U_{a_0} \cap \ldots \cap U_{a_i}$; and, if $U_{a_0}, \ldots, U_{a_i}, \ldots, U_{a_k}$ are all those sets containing $g(p)$, then $\varkappa\circ g(p)$ lies in the *open* simplex $(p_{a_0}, \ldots, p_{a_i}, \ldots \ldots, p_{a_k})$. In particular, p and $\varkappa\circ g(p)$ both belong to $\mathrm{St}(p_{a_0})$. Since N is a k-space, 2.2 shows that $\varkappa\circ g \simeq 1$. This completes the proof.

It is simple to generalize 1.1 to the case where the nerve is locally finite-dimensional (that is, each $\mathrm{St}(p_a)$ is finite-dimensional); in this case, the conclusion holds if each $E(\sigma)$ is required to be $[\dim\mathrm{Tr}(\sigma)-1]$-connected, so that the required connectedness of the intersection of k sets $U_{a_1}, \ldots, U_{a_k}$ depends on the maximal number of sets that can be adjoined to $U_{a_1}, \ldots, U_{a_k}$ and still have a non-empty intersection. Thus, for example, we find

3.1. THEOREM. *Let Y be a locally equiconnected metric space* [11], *and λ an equiconnecting function defined on the* nbd$W$ *of the diagonal in $Y\times Y$. Let $\mathfrak{U} = \{U_a|\ a \in \mathfrak{A}\}$ be any* nbd-*finite open covering of Y such that each $U_a \times U_a \subset W$. If $N(\mathfrak{U})$ is locally finite-dimensional, and if each $E(\sigma)$ is $[\dim\mathrm{Tr}(\sigma)-1]$-connected, then Y belongs to the homotopy type of $N(\mathfrak{U})$, and $\varkappa\colon Y \to N(\mathfrak{U})$ is a homotopy equivalence.*

Proof. In this case, $\lambda[g\circ\varkappa(y), y, t]$ provides the homotopy $g\circ\varkappa \simeq 1$. Note that, if $\dim N(\mathfrak{U}) = n$, then no U_a is required to be even n-connected.

4. We now establish a generalization of Helly's convex-set theorem in the following form:

4.1. THEOREM. *Let $\mathfrak{U} = \{U_1, \ldots, U_k\}$, $k \geqslant n+2$, be open sets in E^n, such that each $(k-1)$ of them have a non-empty intersection. If the intersection of every j of them is $(k-j-2)$-connected, then $U_1 \cap \ldots \cap U_k \neq \emptyset$.*

Proof. Assume $U_1 \cap \ldots \cap U_k = \emptyset$. Then $N(\mathfrak{U})$ is homeomorphic to $\mathrm{Fr}(\sigma^{k-1})$ consequently $H_{k-2}(N) = Z$. It follows from 1.1 that $\bigcup_1^k U_i$ dominates N, so $H_{k-2}(\bigcup_1^k U_i)$ must have $H_{k-2}(N)$ as a direct summand. This is impossible, since $\bigcup_1^k U_i$ is open in E^n so that $H_l(\bigcup_1^k U_i) = 0$ for all $l \geqslant n$.

The above version of the theorem indicates that, for finite families of k open sets in E^n, $k \geqslant n+2$, the Helly property is dependent on the $(k-3)$-connectedness (rather than convexity) of each set [12], and on the connectedness of the higher intersections not decreasing too rapidly. It will be shown elsewhere that 4.1 is also true for families of closed sets in E^n.

5. In a recent paper ([3], pp. 607-609), it has been shown that, if $\mathfrak{U} = \{U_i|\ i = 1, \ldots, n\}$ is any finite open covering of E^2 by convex sets,

(9) Any two canonical $\varkappa, \hat{\varkappa}\colon Y \to N(\mathfrak{U})$ are homotopic, since they are locally finite maps and, for each y, both $\varkappa(y)$ and $\hat{\varkappa}(y)$ belong to a common closed simplex.

(10) The property that $g(\mathrm{Tr}\,\sigma) \subset E(\sigma)$ for each σ is the only one used in the balance of the proof. In the case of ∞-connectedness, its construction is by induction on the k-skeleton of N' itself, starting by sending each $[\sigma]$ to a point of $E(\sigma)$, and extending over each simplex so that always $g([\sigma_1], [\sigma_2], \ldots, [\sigma_s]) \subset E(\sigma_1)$.

(11) Y is locally equiconnected ([4], p. 334) if there is a neighborhood W of the diagonal in $Y\times Y$ and a continuous $\lambda\colon W\times I \to Y$ such that $\lambda(a, b, 0) = a$, $\lambda(a, b, 1) = b$, $\lambda(a, a, t) = a$ for all $(a, b) \in W$, $t \in I$.

(12) Of course, an $(n-1)$-connected open $U \subset E^n$ is necessarily ∞-connected.

then each continuous $f\colon E^2 \to E^2$ has a $\mathfrak{U}$-fixed point ([13]), that is, there exists some $x \in E^2$ such that x and $f(x)$ lie in a common U_i. The question of extending this result to E^n, $n > 2$, was raised; by using a completely different method, we can prove, more generally,

5.1. THEOREM. *Let Y be a normal space with $\pi_i(Y) = 0$ for $i \leqslant n$, and let $\mathfrak{U} = \{U_i \mid i = 1, \ldots, l\}$ be any finite open covering of Y by F_σ-sets, having order $\leqslant n$. If for all $k \geqslant 1$ the intersections of each k sets is $(n-k)$-connected, then every continuous $f\colon Y \to Y$ has a $\mathfrak{U}$-fixed point.*

Proof. The nerve $N(\mathfrak{U})$ is a finite polytope, $\dim N \leqslant n$. According to 1.1, Y dominates N, so that $\pi_i(N) = 0$ for $0 \leqslant i \leqslant n$; hence N is ([5], p. 239) a compact AR and therefore ([1], p. 161) has the fixed-point property. Now let $f\colon Y \to Y$ be given; then $\varkappa f g(p) = p$ for some $p \in N$, and $g(p)$ is a $\mathfrak{U}$-fixed point of f: for, $g\varkappa[fg(p)]$ and $fg(p)$ lie in a common U_i, and $g\varkappa[fg(p)] = g(p)$. The proof is complete.

In [3], p. 612, an example is given of a covering $\mathfrak{U}$ of E^2 by four connected open sets and a continuous $f\colon E^2 \to E^2$ that does not have a $\mathfrak{U}$-fixed point. Because necessarily $\dim N(\mathfrak{U}) \leqslant 3$ for any such covering, it follows from 5.1 that if each U_i is 2-connected, each $U_i \cap U_j$ is 1-connected (which is not the case in the example of [3]) and if each $U_i \cap U_j \cap U_k$ is 0-connected, then every $f\colon E^2 \to E^2$ will have a $\mathfrak{U}$-fixed point. Note that if $\dim N(\mathfrak{U}) \leqslant 2$, then the requirements on the four U_i can be relaxed, so that a general answer to the question 2 in [3], p. 612, would appear to be fairly complicated.

As a further consequence of 5.1, we have the following

5.2. COROLLARY. *Let Y be compact, $\pi_i(Y) = 0$ for all $i \geqslant 0$. Assume that Y has a cofinal family of finite open coverings ([14]) by F_σ-sets such that in each covering, each finite intersection of its sets is ∞-connected. Then Y has the fixed-point property.*

Proof. If an $f\colon Y \to Y$ did not have a fixed point, then it is evident that there is a finite open covering $\mathfrak{U}$ such that f has no $\mathfrak{U}$-fixed point. If $\mathfrak{V}$ is a member of the cofinal family that refines $\mathfrak{U}$, then f has no $\mathfrak{V}$-fixed point. This contradicts 5.1.

Note that 5.2 leads to still another proof of Tychonov's fixed point theorem. Note also that if Y is any compact 1-dimensional LC^0 space such that $\pi_1(Y) = 0$, then because finite open coverings by path-connected sets, of order $\leqslant 1$, are cofinal in the family of all coverings, it follows in the same way, using 5.1, that Y has the fixed-point property.

([13]) Called an "almost-fixed point" in [3].

([14]) A family $\{\mathfrak{V}_\alpha / \alpha \in \mathfrak{A}\}$ of open coverings is *cofinal* if for each open covering $\mathfrak{U}$ there is some $\mathfrak{V}_\alpha$ refining $\mathfrak{U}$. Since Y is compact, each $\mathfrak{V}$ can be assumed to be a finite covering.

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