Thus each homeomorphism type in $2^{\mathfrak{X}}$ is the inverse image under U of an isomorphism type in $\mathfrak{S}_2$; which implies that the homeomorphism types are Borel.

The major reason why the author cannot extend his method beyond the closed subspaces of 2^N is that for more general compact metric spaces (even the unit interval $[0,1]$), he does not know how to associate a canonical countable relational structure with the space in such a way that the isomorphism type of the structure determines the homeomorphism type of the space. Maybe a new method is required for this more general question.

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On Borel measurability of orbits

by

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The aim of this note is to extend, using an alternative method, the result of D. Scott [2] concerning groups of permutations to a large class of topological groups. This has also permited us to solve in the whole generality a problem of Freedman partially solved in [2].

Let $\{F_g\}$ be a decomposition of a topological space G, i.e. for every $g \in G$ we have $g \in F_g$ and $F_{g_1} = F_{g_2}$ or $F_{g_1} \cap F_{g_2} = 0$ and F_g is closed. We say that such a decomposition is *open* if $\{g: F_g \cap U \neq 0\}$ is an open set provided U is open. The distance function of G is denoted by ϱ.

LEMMA. *Every open decomposition $\{F_g\}$ of a complete and separable metric space G has a Borel selector, i.e. there exists a Borel set $S \subseteq G$ such that* card $(S \cap F_g) = 1$ *for* $g \in G$.

Proof. Let $r_0, r_1, r_2, \ldots$ be a fixed sequence dense in G. Now we define some functions: $\varphi_n(g) = r_i$ where $i = \min\{k: \varrho(r_k, F_g) < 1/2^n$ and, if $n > 0$, then $\varrho(r_k, \varphi_{n-1}(g)) < 1/2^{n-1}\}$. In view of our assumption all sets $\{g: \varrho(r_k, F_g) < 1/2^n\}$ are open and hence φ_n are Borel functions. Moreover we have the inequality $\varrho(\varphi_n(g), \varphi_{n+1}(g)) < 1/2^n$, and all φ_n are constant on each F_g and $\varrho(\varphi_n(g), F_g) < 1/2^n$. Consequently the limit function $\varphi(g) = \lim \varphi_n(g)$ is Borel, constant on F_g, and $\varphi(g) \in F_g$, which clearly implies that the set $\{g: \varphi(g) = g\}$ is a Borel selector.

THEOREM 1. *If a topological group G admits a complete and separable metrisation and G acts transitively on a metric space X in such a way that, for some $x_0 \in X$, $x_0 g$ is a continuous function of $g \in G$, then X is a Borel space (i.e. X is a Borel set in any metrisable extension of X).*

Proof. Put $F = \{g: x_0 g = x_0\}$. F is a closed subgroup of G. Obviously the decomposition of G given by $F_g = Fg$ $(g \in G)$ is open. Let S be a Borel selector given by Lemma. The continuous mapping of S into X defined by the formula $x_0 s$ $(s \in S)$ is one-to-one and onto (S is a selector!); hence X is a Borel space (cf. [1], p. 396).

Now Scott's Theorem of [2] (see p. 122) clearly is included in Theorem 1. Look there to see several applications of this result, especially Scott's beautiful solution of Kuratowski's problem on the Borelian character of order types.

THEOREM 2 (1). *If G is the group of all autohomeomorphisms of a compact and metrisable space X and $Z \in 2^X$ ($=$ the space of all closed subsets of X with the Vietoris topology) then the set $Z^* = \{Zg\colon\ g \in G\}$ is Borel in 2^X.*

Proof. Since G acts transitively on Z^*, and the usual topology in G (the topology of the uniform convergence) satisfies the conditions of Theorem 1.

Added in proof. Using methods similar to those of this paper, the author has recently solved completely Freedman's problem on homeomorphisms. This solution will be published in the next volume of Fundamenta Mathemataicae.

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(1) Conjectured by D. Freedman and partially proved in [2]; see p. 10.

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