On the classification of $\sigma(\boldsymbol{B})$-measurable functions defined on an abstract point set

by

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Let X be a non-empty abstract point set, $\boldsymbol{X}$ the system of all its subsets and $\boldsymbol{B} \subset \boldsymbol{X}$ a non-void system of sets. Denote by $\sigma(\boldsymbol{B})$ the σ-algebra generated by the system $\boldsymbol{B}$ and containing the greatest element X. Let $\mathfrak{F}$ be the system of all real-valued functions defined on X and $\mathfrak{M}$ the subsystem of all $\sigma(\boldsymbol{B})$-measurable functions. Denote by $\mathfrak{M}^0$ the system of all real-valued $a(\boldsymbol{B})$-measurable functions (1) on X, $a(\boldsymbol{B})$ being the ring generated by the system $\boldsymbol{B}$ containing the element $X \in a(\boldsymbol{B})$. The elements of the σ-algebra $\sigma(\boldsymbol{B})$ will be called *measurable sets* and the elements of the system $\mathfrak{M}$ *measurable functions.*

The sequential topology λ in the system $\boldsymbol{X}$ is defined by the well-known convergence:

$$\lim A_n = A\,, \qquad \text{whenever} \qquad A = \limsup A_n = \liminf A_n\,,$$

whereby

$$\limsup A_n = \bigcap_{k=1}^{\infty} \bigcup_{n=k}^{\infty} A_n \qquad \text{and} \qquad \liminf A_n = \bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} A_n\,.$$

The sequential topology $\varkappa$ in $\mathfrak{F}$ is defined by the convergence at each point as follows: $\lim f_n = f$ whenever $\lim f_n(x) = f(x)$ for each $x \in X$. In such a way we get two convergence spaces $(\boldsymbol{X}, \lambda)$ and $(\mathfrak{F}, \varkappa)$ the topologies (λ and $\varkappa$) of which are defined by the convergences fulfilling both Fréchet's axioms of convergence (the axiom of the stationary sequence and the axiom of the subsequences of a convergent sequence) and also Urysohn's axiom (if a sequence of elements z_n does not converge to the element z, then there is a subsequence $\{z_{n_i}\}$ no subsequence of which converges to z).

Let L be a covergence space and λ its sequential topology. The closure λA of a subset A (i.e. the set of all limits of sequences of points $x_n \in A$)

(1) $\{V(x)\}$ denotes the set of all $x \in X$ fulfilling the property $V(x)$. The function f is $a(\boldsymbol{B})$-measurable if $\{f(x) > t\} \in a(\boldsymbol{B})$ for each real number t.

need not be closed. Therefore it is possible to form the successive closures. In such a way we get a non-decreasing sequence of closures:

$$\lambda^0 A \subset \lambda^1 A \subset \lambda^2 A \subset \ldots \subset \lambda^\xi A \subset \ldots$$

where $\lambda^0 A = A$, $\lambda^1 A = \lambda A$, $\lambda^{\xi+1} A = \lambda(\lambda^\xi A)$ and $\lambda^\xi A = \bigcup_{\eta<\xi} \lambda^\eta A$ if ξ is a limiting ordinal. From the property of the set of all countable ordinals it follows that there is no sense in forming the closures for uncountable ordinals $\xi > \omega_1$, ω_1 being the first uncountable ordinal. As a matter of fact $\lambda(\lambda^{\omega_1} A) = \lambda^{\omega_1} A$. Thus thes et $\lambda^{\omega_1} A = \bigcup_{\xi<\omega_1} \lambda^\xi A$ is closed, i.e. it fulfills the following property: $\lim x_n = x$ and $x_n \in \lambda^{\omega_1} A$ implies $x \in \lambda^{\omega_1} A$. Evidently $A \subset B$ implies $\lambda A \subset \lambda B$.

LEMMA 1. $\sigma(\boldsymbol{B}) = \lambda^{\omega_1} a(\boldsymbol{B})$.

Proof. Since $\lim A_n = A$ and $\lim B_n = B$ implies $\lim(A_n \cup B_n) = A \cup B$ and since $\lim C_n = C$ if and only if $\lim(X - C_n) = X - C$, it easily follows that each system $\lambda^\zeta a(\boldsymbol{B})$, ζ being any ordinal, is an algebra; so is the system $\lambda^{\omega_1} a(\boldsymbol{B})$ which is identical with $\bigcup_{\xi<\omega_1} \lambda^\xi a(\boldsymbol{B})$. Hence, if A_n are elements of $\lambda^{\omega_1} a(\boldsymbol{B})$, $n = 1, 2, \ldots$, then $\bigcup_{n=1}^{m} A_n \in \lambda^{\omega_1} a(\boldsymbol{B})$ for every $m = 1, 2, \ldots$ Since the system $\lambda^{\omega_1} a(\boldsymbol{B})$ is closed and $\lim_m \bigcup_{n=1}^{m} A_n = \bigcup_1^\infty A_n$, it follows that $\bigcup_1^\infty A_n$ belongs to $\lambda^{\omega_1} a(\boldsymbol{B})$; thus $\lambda^{\omega_1} a(\boldsymbol{B})$ is a σ-algebra. $\boldsymbol{B} \subset \lambda^{\omega_1} a(\boldsymbol{B})$ then implies that $\sigma(\boldsymbol{B}) \subset \lambda^{\omega_1} a(\boldsymbol{B})$.

On the other hand, using the method of trasfinite induction we easily prove that $\lambda^\xi a(\boldsymbol{B}) \subset \sigma(\boldsymbol{B})$ for each $\xi < \omega_1$, so that $\lambda^{\omega_1} a(\boldsymbol{B}) \subset \sigma(\boldsymbol{B})$.

Let (L, λ) be a convergence space. Let A be a subset of L. The set $\lambda^{\xi+1} A - \lambda^\xi A$ is called the $(\xi+1)$th *class* and each point z of it is called a *point of the class* $\xi+1$; it will be denoted by $c(z; A)$ or simply by $c(z)$. Further we define $c(z; A) = 0$ if $z \in A$. If $c(z; A) > 0$, then $c(z)$ is a countable and isolated ordinal such that $z \in \lambda^{c(z)} A - \lambda^{c(z)-1} A$.

LEMMA 2. *If A and B are two elements of $\sigma(\boldsymbol{B})$ and a an ordinal such that* $c(A \cap B; a(\boldsymbol{B})) \geqslant a$, *then either* $c(A; a(\boldsymbol{B})) \geqslant a$ *or* $c(B; a(\boldsymbol{B})) \geqslant a$.

Proof. If we had $c(A) < a$ and $c(B) < a$, then $A \in \lambda^{a'} a(\boldsymbol{B})$ and $B \in \lambda^{a'} a(\boldsymbol{B})$, where $a' = \max(c(A), c(B))$. The product $A \cap B$ would be an element of the algebra $\lambda^{a'} a(\boldsymbol{B})$ and consequently $c(A \cap B) \leqslant a' < a$; this would be a contradiction.

Let f be a $\sigma(\boldsymbol{B})$-measurable function on X. Let $o(f)$ be the least ordinal such that (¹) $c(\{f(x) > t\}; a(\boldsymbol{B})) \leqslant o(f)$ for each real number t; the ordinal $o(f)$ will be called the *order* of the element f. Each characteristic function χ_A of the set $A \in \sigma(\boldsymbol{B})$ belongs to $\mathfrak{M}$ and its order $o(\chi_A) = c(A; a(\boldsymbol{B}))$; it is 0 or an isolated ordinal. Also each simple function $s(x) = \sum_{i=1}^{p} k_i \chi_{A_i}(x)$ where k_i are real numbers *different from one another* and A_i are disjoint elements of $\sigma(\boldsymbol{B})$ such that $\bigcup_1^\infty A_i = X$, has an order $o(s)$, which is 0 or an isolated ordinal. This follows from

LEMMA 3. *The order $o(s)$ of a simple function $s(x) = \sum_{i=1}^{p} k_i \chi_{A_i}(x)$ is equal to* $\max[c(A_1), \ldots, c(A_p)]$.

Proof. Denote by q the natural $\leqslant p$ such that $c(A_q) = \max[c(A_1), \ldots, c(A_p)]$. There exists a semiclosed interval $(a, b\rangle$ containing k_q and no other k_i, $i \neq q$. Then (¹) $\{s(x) > a\} \cap \{s(x) \leqslant b\} = A_q$ so that $c(\{s(x) > a\} \cap \{s(x) \leqslant b\}) = c(A_q)$. In view of Lemma 2 and since $c(\{s(x) \leqslant b\}) = c(\{s(x) > b\})$, it follows that $c(\{s(x) > a\}) \geqslant c(A_q)$ or $c(\{s(x) > b\}) \geqslant c(A_q)$; consequently $o(s) \geqslant c(A_q)$.

Now, let t be a real number. If $\bigcup^* A_i$ denotes the union of all sets A_i whose indices i fulfill the relation $k_i > t$, then $\{s(x) > t\} = \bigcup^* A_i$. Since $A_i \in \lambda^{c(A_q)} a(\boldsymbol{B})$, $1 \leqslant i \leqslant p$, the set $\bigcup^* A_i$ is an element of the algebra $\lambda^{c(A_q)} a(\boldsymbol{B})$. Consequently $c(\{s(x) > t\}) \leqslant c(A_q)$. Thus $o(s) \leqslant c(A_q)$, so that $o(s) = c(A_q)$.

It is well known that every measurable function g is the limit of a sequence $\{s_n\}_{n=1}^\infty$ of simple functions $s_n(x)$, whereby

$$s_n(x) = \begin{cases} \dfrac{i-1}{2^n} & \text{if} \quad \dfrac{i-1}{2^n} \leqslant f(x) < \dfrac{i}{2^n}, \ i = 0, \pm 1, \ldots, \pm n2^n, \\ n & \text{if} \quad f(x) \geqslant n, \\ -n - \dfrac{1}{2^n} & \text{if} \quad f(x) < -n - \dfrac{1}{2^n}. \end{cases}$$

It is easy to see that $\{g(x) > t\} = \bigcup_{n=1}^\infty \{s_n(x) > t\}$. It follows from Lemma 3 that each simple function s_n has a countable order. Thus each measurable function g has a countable order as well.

THEOREM 1. $\mathfrak{M} = \varkappa^{\omega_1} \mathfrak{M}^0$.

Proof. It is evident that $\varkappa^{\omega_1} \mathfrak{M}^0 \subset \mathfrak{M}$. In order to prove the reverse inclusion it suffices to show that each simple function belongs to $\varkappa^{\omega_1} \mathfrak{M}^0$; as a matter of fact, if g is a measurable function on X and $\{s_n\}$ a sequence of simple functions converging to g, then from the supposition $s_n \in \varkappa^{\omega_1} \mathfrak{M}^0$ and because $\varkappa^{\omega_1} \mathfrak{M}$ is a closed system it follows that $g \in \varkappa^{\omega_1} \mathfrak{M}^0$.

We shall use the method of transfinite induction. Assume that *the system $\varkappa^\xi \mathfrak{M}^0$ contains all simple functions of orders $\leqslant \xi$* where $\xi < a$. If a is a limiting ordinal, then in view of Lemma 3 there is no simple function of order a at all and the system $\varkappa^a \mathfrak{M}^0$ contains—by our supposition—all simple functions of orders $\leqslant a$. Now, suppose that a is isolated and consider a simple function $s(x) = \sum_1^p k_i \chi_{A_i}(x)$ of order a. Then A_i are elements of $\lambda^a a(\boldsymbol{B})$. For every $i \leqslant p$ there exists a sequence $\{A_i^n\}_{n=1}^\infty$ of sets $A_i^n \in \lambda^{a-1} a(\boldsymbol{B})$ converging to the set A_i. For each natural m let B_i^m, $1 \leqslant i \leqslant p$, be the sets defined as follows $B_1^m = A_1^m$; $B_i^m = A_i^m - \bigcup_{j=1}^{i-1} A_j^m$ for $i = 2, \ldots, p-1$ and $B_p^m = X - \bigcup_{j=1}^{p-1} B_j^m = X - \bigcup_{j=1}^{p-1} A_j^m$. The sets B_i^m are disjoint; they are elements of the algebra $\lambda^{a-1} a(\boldsymbol{B})$ and $X = \bigcup_{i=1}^p B_i^m$, so that $c(B_i^m) \leqslant a-1$ for every $i = 1, \ldots, p$. Therefore the function $s_m(x) = \sum_{i=1}^p k_i \chi_{B_i^m}(x)$ is simple and—by Lemma 3—its order is $o(s_m) \leqslant a-1$ for every natural m. By our supposition each function s_m belongs to $\varkappa^{a-1}\mathfrak{M}^0$. Thus it is sufficient to prove that $\lim s_m(x) = s(x)$ for each $x \in X$ since then $s \in \varkappa^a \mathfrak{M}^0$.

Choose a point $x_0 \in X$. Then there is a natural $q \leqslant p$ such that $x_0 \in A_q$ and for each natural m there is an index i_m such that $1 \leqslant i_m \leqslant p$ and that $x_0 \in B_{i_m}^m$. It follows that $s(x_0) = k_q$ and $s_m(x_0) = k_{i_m}$. Since $\lim_m (A_i^m - \bigcup_{j=1}^{i-1} A_j^m) = A_i - \bigcup_{j=1}^{i-1} A_j = A_i$ for $2 \leqslant i \leqslant p-1$ and $\lim (X - \bigcup_{j=1}^{p-1} A_j) = A_p$, the limits $\lim_m B_i^m = A_i$ do exist for each $i \leqslant p$ and $x_0 \in A_q = \lim_m B_q^m$; consequently $i_m = q$ for all but a finite number of naturals m. Hence $\lim s_m(x_0) = s(x_0)$.

Remark 1. We have just proved that each simple function s of order $a > 0$ is the limit of a sequence of simple functions s_m of order $a-1$.

Lemma 4. *Let f and f_n be measurable functions on X such that $\lim f_n = f$. Then $o(f) \leqslant \sup_n o(f_n) + 3$.*

Proof. Let t be any real number and $\{t_m\}$ a strictly decreasing sequence of real numbers converging to t. Then

$$\{f(x) > t\} = \bigcup_{m=1}^\infty \bigcap_{k=1}^\infty \bigcup_{n=k}^\infty \{f_n(x) > t_m\}.$$

As a matter of fact, if x_0 is a point of X such that $f(x_0) > t$, then $f(x_0) > t_{m_0}$ for a suitable m_0 and consequently the point x_0 belongs to $\limsup \{f_n(x) > t_{m_0}\}$ which is contained in $\bigcup_{m=1}^\infty \bigcap_{k=1}^\infty \bigcup_{n=k}^\infty \{f_n(x) > t_m\}$. On the other hand, if x_1 is a point belonging to the set $\bigcap_{k=1}^\infty \bigcup_{n=k}^\infty \{f_n(x) > t_{m_1}\}$, m_1 being a suitable index, then $f_n(x_1) > t_{m_1}$ for infinitely many n and since $\lim f_n(x_1) = f(x_1)$, we have $f(x_1) \geqslant t_{m_1} > t$.

Since

$$c\big(\{f_n(x) > t_m\}; a(\boldsymbol{B})\big) \leqslant o(f_n) \leqslant \sup o(f_n)$$

for each natural m and n, we have

$$c\Big(\bigcup_{m=1}^\infty \bigcap_{k=1}^\infty \bigcup_{n=k}^\infty \{f_n(x) > t_m\}; a(\boldsymbol{B})\Big) \leqslant \sup o(f_n) + 3$$

for each real t so that

$$o(f) \leqslant \sup o(f_n) + 3.$$

Theorem 2. *If f is a measurable function of the class $c(f; \mathfrak{M}^0) = a\omega_0 + k$, k being a non-negative integer, a an ordinal and ω_0 the least infinite ordinal, then the order $o(f) \leqslant a\omega_0 + 3k$.*

Proof. Suppose that the assertion is true for all $a\omega_0 + k < \zeta$, where $\zeta = \beta\omega_0 + h$, h being a non-negative integer. If $h = 0$, the proof is evident. Let $h \geqslant 1$. If g is a measurable function of the class ζ, then there exists a sequence of functions $g_n \in \varkappa^{\zeta-1}\mathfrak{M}^0$ of classes $\leqslant \zeta - 1 = \beta\omega_0 + h - 1$ such that $\lim g_n = g$. By our supposition each order $o(g_n) \leqslant \beta\omega_0 + 3(h-1)$, $n = 1, 2, \ldots$ According to Lemma 4 the order $o(g) \leqslant \beta\omega_0 + 3h$.

Lemma 5. *Let s be a measurable simple function on X. Then $c(s; \mathfrak{M}^0) \leqslant o(s)$. If the class of the measurable set A is $c\big(A; a(\boldsymbol{B})\big) = a\omega_0 + 1$, then $c(\chi_A; \mathfrak{M}^0) = o(\chi_A) = a\omega_0 + 1$.*

Proof. In the proof of Theorem 1 we have proved the following statement: *The system $\varkappa^\xi\mathfrak{M}^0$ contains all simple functions of order $\leqslant \xi$.* This implies the inequality $c(s; \mathfrak{M}^0) \leqslant o(s)$.

Now, let $A \in \sigma(\boldsymbol{B})$ and let $c(A) = a\omega_0 + 1$. Then also $o(\chi_A) = a\omega_0 + 1$. In view of the first part of this proof the inequality $c(\chi_A; \mathfrak{M}^0) \leqslant a\omega_0 + 1$ holds true. If we had $c(\chi_A; \mathfrak{M}^0) < a\omega_0 + 1$, then $c(\chi_A; \mathfrak{M}^0) = 0$ or $c(\chi_A; \mathfrak{M}^0) < \eta$ for a suitable ordinal $\eta < a\omega_0$, each class being 0 or an isolated ordinal. By Theorem 2 we should have $o(\chi_A) \leqslant a\omega_0$; but this would contradict the fact that $o(\chi_A) = a\omega_0 + 1$.

Theorem 3. *If ϱ is the least ordinal such that the ϱ-th class $\lambda^\varrho a(\boldsymbol{B}) - \lambda^{\varrho-1} a(\boldsymbol{B})$ is empty and if τ is the least ordinal such that the τ-th class $\varkappa^\tau\mathfrak{M}^0 - \varkappa^{\tau-1}\mathfrak{M}^0$ is empty, then the following inequality holds true: $\tau \leqslant \varrho + 1$. If $\varrho = a\omega_0 + 1$, then also $\varrho \leqslant \tau$.*

Proof. Suppose, on the contrary, that $\varrho + 2 \leqslant \tau$. Then there is a function $f \in \varkappa^{\varrho+1}\mathfrak{M}^0 - \varkappa^\varrho\mathfrak{M}^0$ and, consequently, there is a sequence of simple functions of classes $\geqslant \varrho$ which converges to f. Denote by s one of these simple functions. From Lemma 5 it follows that $c(s; \mathfrak{M}^0) \leqslant o(s)$. By Lemma 3 there is a set A_0 of the class $c\big(A_0; a(\boldsymbol{B})\big) = o(s)$. Thus $\varrho \leqslant c(s; \mathfrak{M}^0) \leqslant o(s) = c\big(A_0; a(\boldsymbol{B})\big)$. Therefore $\varrho \leqslant c\big(A_0; a(\boldsymbol{B})\big)$. This is a contradiction.

Now, suppose that $\varrho = a\omega_0 + 1$. Since τ is an isolated ordinal, we may denote it by $\beta\omega_0+k_0$ where k_0 is a suitable natural. Assume, on the contrary, that $\tau<\varrho$. Then $\beta\omega_0+3k_0<\alpha\omega_0$ and consequently there is a set A_0 of class $c\big(A_0;\, a(\boldsymbol{B})\big)=\beta\omega_0+3k_0$. Denote $c(\chi_{A_0};\, \mathfrak{M}^0)=\gamma\omega_0+h$; here h is a non-negative integer, each class being 0 or an isolated ordinal. Evidently $\gamma\omega_0+h<\beta\omega_0+k_0$. From Theorem 2 it follows that $o(\chi_{A_0})\leqslant\gamma\omega_0+3h<\beta\omega_0+3k_0$. However $o(\chi_{A_0})=c\big(A_0;\, a(\boldsymbol{B})\big)$. Thus we have got a contradictory result $c(A_0)<\beta\omega_0+3k_0$.

Remark 2. If $\varrho=\alpha\omega_0+k$, $1<k<\omega_0$, is the least ordinal such that the ϱ-th class $\lambda^{\varrho}a(\boldsymbol{B})-\lambda^{\varrho-1}a(\boldsymbol{B})$ is empty, then from Lemma 5 it follows that each class $\varkappa^{\xi+1}\mathfrak{M}^0-\varkappa^{\xi}\mathfrak{M}^0$, $\xi\leqslant\alpha\omega_0$, is non-emtpy. In this case the number of all other non-empty classes $\varkappa^{\eta+1}\mathfrak{M}^0-\varkappa^{\eta}\mathfrak{M}^0$, $\eta>\alpha\omega_0$, is at most finite; this follows immediately from Theorem 3.

Remark 3. Let X be the set of all real numbers and $\boldsymbol{P}$ the system of all semiclosed intervals $\langle a,b)\subset X$ where $-\infty\leqslant a<b\leqslant\infty$. Denote by $\mathfrak{L}$ the system of all real-valued continuous functions on X. Then $\sigma(\boldsymbol{P})$ is the system of all linear Borel sets, $\mathfrak{M}^0$ the system of all $a(\boldsymbol{P})$-measurable and $\mathfrak{M}$ the system of all B-measurable real functions, $\varkappa^{\omega_1}\mathfrak{L}$ is the system of all Baire functions.

First, let us prove that $\mathfrak{M}^0\subset\varkappa^3\mathfrak{L}$. Suppose that $g\in\mathfrak{M}_0$. Then $g(x)=\lim r_m(x)$ for each $x\in X$, r_m being simple functions. Since $\{a\leqslant g(x)<b\}=\lim\{a-1/n<g(x)\leqslant b-1/n\}\in\lambda a(\boldsymbol{P})$, we may suppose, with respect to Lemma 3 that the order $o(r_m)\leqslant 1$ for every m. According to Remark 1, each simple function of order α is the limit of a sequence of simple functions of order $\alpha-1$. Put here $\alpha=1$ and notice that each element of the algebra $a(\boldsymbol{P})$ is a finite disjoint union of semiclosed intervals so that each simple function of order 0 is the limit of a sequence of continuous functions. It follows that $r_m\in\varkappa^2\mathfrak{L}$ for each m so that $\mathfrak{M}^0\subset\varkappa^3\mathfrak{L}$.

Now prove that $\mathfrak{L}\subset\varkappa^3\mathfrak{M}^0$. Let $f\in\mathfrak{L}$ and let $\lim s_n(x)=f(x)$, for each $x\in X$, s_n being simple functions. Since each set $\{a\leqslant f(x)<b\}$ is a G_δ-set belonging to $\lambda^2 a(\boldsymbol{P})$, we may suppose, in view of Lemma 3, that each order $o(s_n)\leqslant 2$ so that, by Lemma 5, also $c(s_n,\mathfrak{M}^0)\leqslant 2$. Consequently $f\in\varkappa^3\mathfrak{M}^0$ and hence $\mathfrak{L}\subset\varkappa^3\mathfrak{M}^0$.

Using the method of induction we easily prove that $\varkappa^n\mathfrak{M}^0\subset\varkappa^{n+3}\mathfrak{L}$ and $\varkappa^n\mathfrak{L}\subset\varkappa^{n+3}\mathfrak{M}^0$, $1\leqslant n<\omega_0$. Then we can conclude that $\varkappa^{\omega_0}\mathfrak{M}^0=\varkappa^{\omega_0}\mathfrak{L}$. Thus we have proved the following statement:

Each ξ-th class of Baire functions is identical with the ξ-th class of B-measurable functions, $\omega_0<\xi<\omega_1$.

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О диадических бикомпактах

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Введение

В этой работе дается внутренняя характеристика диадических бикомпактов, т.е. бикомпактов X, являющихся непрерывными образами так называемых обобщенных канторовых дисконтинуумов D^τ (под D^τ, где τ — бесконечное кардинальное число, понимается, как известно, топологическое произведение τ бикомпактов D_λ, каждый из которых состоит из конечного числа изолированных точек [1]).

Важность класса диадических бикомпактов подтверждается, например, следующими фактами:

1°. Класс [2] диадических бикомпактов есть наименьший класс, удовлетворяющий следующим условиям

а) он содержит все бикомпакты, состоящие из конечного числа точек;

б) вместе с данными бикомпактами X_α он содержит их топологическое произведение $\prod\limits_{\alpha} X_\alpha$;

в) вместе с данным бикомпактом X он содержит и всякий бикомпакт Y, являющийся непрерывным образом бикомпакта X.

2°. Класс диадических бикомпактов совпадает с классом всех бикомпактов, являющихся непрерывными образами бикомпактных топологических групп. В частности, пространство всякой бикомпактной топологической группы есть диадический бикомпакт (теорема Ивановского-Кузьминова [4] [3]).

3°. Всякий диадический бикомпакт, удовлетворяющий 1-ой аксиоме счетности, метризуем (теорема А. С. Есенина-Вольпина [3]).

Из последнего предложения легко следует, что

4°. Всякий упорядоченный диадический бикомпакт гомеоморфен ограниченному множеству действительных чисел.

Интересные свойства диадических бикомпактов установлены Э. Марчевским (Шпильрайном) [6], Н. А. Шаниным [8] и другими исследователями.

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(1) Без ограничения общности можно предполагать, что каждый множитель D_λ этого произведения состоит из двух точек.

(2) Можно ограничить его требованием, чтобы вес или мощность рассматриваемых бикомпактов не превосходил данного кардинального числа.

(3) Цифры в квадратных скобках означают ссылки на литературу, помещенную в конце статьи.