Proof of Theorem. Let $R$ be a ring with property $P$. By Lemma 3 we can suppose the existence of divisors of zero. If $R$ contains an element of infinite order, then by Lemma 2 and 4 there exists a number $n \in J$ for which $0 \subset nR \subset R$. But by $R^+ \simeq (nR)^+$ and by property $P$, $R$ is cyclic.

If $R^+$ is a torsion group, then a ring-theoretical direct decomposition $R = \sum_p R_p$ holds, where the ideal $R_p$ is generated by all elements of $p$-power order of $R$. If $R \neq R_p$, then $R$ is a finite cyclic ring. Now let $R$ be a $p$-ring in which $R'$ is generated by all elements of order $p$ of $R$. If $R' \neq R$, then $R$ is cyclic or else of type $p^\infty$ because in both cases $R'$ is cyclic [2]. Finally we assume that $R' = R$. By the existence of divisors of zero, by $pR = 0$, by Lemma 1 and by property $P$ the existence of a left-ideal $L$ of order $p$ of $R$ is necessarily ensured. Now we show the impossibility of $O(R) \geqslant p^3$. It is clear that $Lr$ is a left-ideal in $R$ $(r \in R)$. If there exists an element $0 \neq r \in R$ for which $Lr \neq 0$ and $L \cap Lr = 0$ holds, then for the left-ideal $D = \{L, Lr\}$ it is $R = D$, *i. e.*, $O(R) = p^2$. But if $Lr \subseteq L$ for all $r \in R$, the subring $L$ is a two-sided ideal in $R$. Then $R/L$ has the property $P$ and consequently has no proper left-ideals. By $O(R) \geqslant p^3$ we can assume that $R/L$ is a skew-field, and thus not a zero-ring, but has the property $P$. By $O(R/L) \geqslant p^2$ and by Lemma 5 we have obtained a contradiction, which completes the proof.

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# Errata to the paper "On the $\varepsilon$-theorems"

(Fundamenta Mathematicae 43, p. 156-165)

by

**H. Rasiowa** (Warszawa)

| Page | for | read |
|---|---|---|
| $156_{15}$ | of [10] | of [10] and [13] |
| $156_{11}$ | theories | theories since the non-enumerable case follows immediately from the enumerable one |
| $161_4$ | a consistent | a consistent, enumerable |
| $161_1$ | cf. [8] | cf. [8], or an extension in a Boolean algebra of all subsets of a set, cf. [13]. |
| $162^{10}$ | in algebra $B$ | in algebra $B$ of sets |
| $162^{17}$ | f | of |
| $164^{17}$ | $\varepsilon$-theorem | $\varepsilon$-theorem 5.1 (with open $\alpha$). |