any elements of B, such that $x \cap y = 0$ and y contains $\aleph_1$ atoms, then there is no σ-homomorphism defined on the Boolean algebra $[u|u \in B, u \subseteq y]$ onto the Boolean algebra $[u|u \in B, u \subseteq x]$.

It is possible that the quotient-algebra Q of B in theorem 9, modulo the σ-ideal of all elements $x \in B$, which are the union of at most $\aleph_0$ atoms, does not admit any σ-homomorphisms. (This would follow from a result of R. Sikorski [12] if the heterogeneous set M which generates B were a Borel-set of real numbers. But, by theorem 4 there is no such M). In this connection note the ingenious construction of B. Jónsson [3] of a Boolean algebra which admits no automorphism except the identity. His algebra is of very high cardinality.

7. The rather ingenious use of well-orderings, employed to prove the fundamental lemma 1, has often been used to derive pseudo-antinomious results about the continuum. It seems to originate with G. Hamel, who devised it to show the existence of a base for the reals.

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On a problem concerning completely regular sets

by

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Słowikowski and Zawadowski have raised the following problem:

A topological space R has the property α if every function defined and continuous on R is bounded. Does the property α always imply the compacticity of any completely regular space R?

We are going to prove that the answer to this question is *negative*. Let $\beta(N)$ be the Čech bicompactification of an infinite isolated point-set N — for instance the set of all naturals. Let $N = \bigcup_{k=1}^{\infty} N_k$ where N_k are infinite subsets of N disjoint from one another. Let us identify in the space

$$\beta(N) - \beta[\bigcup_{k=1}^{\infty} \beta(N_k) - N] \bigcup_{k=1}^{\infty} \beta(N_k)$$

every set $\beta(N_k) - N$ with a new element $a_k \equiv \beta(N_k) - N$, the symbol β indicating the closure in the space $\beta(N)$. In such a way we get a new topological space R. The closure of the set A in R will be denoted by $\bar{A}$.

Some remarkable properties of the space R.

Clearly, the set N is isolated and dense in R.

Further, there is an open basis of R consisting of neighbourhoods which are ambiguous, *i. e.* open and closed in R. We have to prove that in every neighbourhood $O(x)$ of any point $x \in R$ there is an ambiguous neighbourhood $U(x) = \overline{U(x)} \subset O(x)$. As a matter of fact, for $x \in N$ we can put $U(x) = (x)$ and for $x = a_k$ we can choose $U(x) = O(x) \cap [N_k \cup (a_k)]$. Now, let $x \in [N \cup \bigcup_{k=1}^{\infty} (a_k)]$. Then

$$x \in \Big(R - [N \cup \bigcup_{k=1}^{\infty} (a_k)]\Big) \cap \Big(\beta(N) - \beta[\bigcup_{k=1}^{\infty} \beta(N_k) - N]\Big).$$

Since $\beta(N)$ is a normal space there is a set G open in $\beta(N)$ such that $x \in \beta(G) \subset O(x)$ and such that

$$\beta[\bigcup_{k=1}^{\infty} \beta(N_k) - N] \subset \beta(N) - \beta(G).$$

As N is dense in $\beta(N)$, we have $\beta(M)=\beta(G)$, where $M=G\cap N\subset N$. Therefore $\beta(M)\cup\beta(M-N)=\beta(N)$ and $\beta(M)\cap\beta(N-M)=0$, N being a normal [1]) space. From this it follows that the set $\beta(G)$ is ambiguous in $\beta(N)$. Now, we can put $U(x)=\beta(M)\cap R$.

For any infinite subset $K\subset N$ we have $\overline{K}-K\neq 0$. Indeed, there is a point $y\in\beta(K)-K$. Then we have $a_k\in\overline{K}-K$ for $K\subset N_k$ and $y\in\overline{K}-K$ otherwise.

The space R is completely regular.

This follows instantly from the fact that the open basis of R consists of ambiguous neighbourhoods.

The space R has the property α.

Suppose, on the contrary, that $g(x)$, $x\in R$, is a continuous function and $X=\bigcup_{k=1}^{\infty}(x_k)$ a set of points $x_k\in R$ such that $g(x_k)>k$ for $k=1,2,\ldots$ The set X is isolated and closed in R.

Consequently, there is a disjoint system of ambiguous neighbourhoods $U(x_k)$ such that $g(x)>k$ for any $x\in U(x_k)$, $g(x)$ being continuous on R. Let us choose points $n_k\in N\cap U(x_k)$. Since $g(n_k)>k$, we have $\overline{K}-K=0$ where $K=\bigcup_{k=1}^{\infty}(n_k)$; this is a contradiction.

The space R fails to be compact.

Evidently, the set $\bigcup_{k=1}^{\infty}(a_k)$ has no point of accumulation in R.

Note. Since the property α implies the compacticity of any normal space, the space R constructed above cannot be normal. As a matter of fact the sets $\bigcup_{k=1}^{\infty}(a_k)$ and $R-\bigcup_{k=1}^{\infty}\overline{N}_k$ are both closed and disjoint, but they cannot be separated by any two disjoint open sets in R.

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[1]) See E. Čech, *On bicompact spaces*, Annals of Mathematics 38 (1937), p. 833-844.

On completely regular spaces

by

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In the preceding paper J. Novák [1]) has shown the existence of a non-compact, completely regular space X, on which all continuous real functions are bounded. Our purpose is to obtain that result in a more direct way.

LEMMA. *Let N be the set of all natural numbers, and $\mathfrak{N}$ the family of all its infinite subsets. There exists a family $\mathfrak{N}_1\subset\mathfrak{N}$ such that:*

(1) *The family $\mathfrak{N}_1$ is infinite,*

(2) *for every $N_1,N_2\in\mathfrak{N}_1$ the product N_1N_2 is finite,*

(3) *for every $N'\in\mathfrak{N}$ there exists a $N''\in\mathfrak{N}_1$ such that the product $N'N''$ is infinite.*

Proof. Let $N=N_1+N_2+\ldots+N_k+\ldots$, where N_k are infinite and disjoint sets. Let us put the family $\mathfrak{N}-\{N_1,N_2,\ldots,N_k,\ldots\}$ in a transfinite sequence

$$N_\omega,N_{\omega+1},\ldots,N_\alpha,\ldots$$

Hence

$$\mathfrak{N}=\{N_1,N_2,\ldots,N_\omega,N_{\omega+1},\ldots,N_\alpha,\ldots\}.$$

We define the family $\mathfrak{N}_1$ by transfinite induction:

1) $N_1\in\mathfrak{N}_1$,

2) $N_\alpha\in\mathfrak{N}_1$ if and only if for every $N_\beta\in\mathfrak{N}_1$ $(\beta<\alpha)$ the product $N_\alpha N_\beta$ is finite.

It is obvious that the family $\mathfrak{N}_1$, so defined, satisfies the conditions (1)-(3).

Now, let us put $X=N+\mathfrak{N}_1$. The neighbourhoods in X are defined as follows:

1° If $x\in N$ then $O(x)=\{x\}$,

2° if $x\in\mathfrak{N}_1$, i. e. $x=N'\subset N$, then $O(x)=\{x\}+N'-S$ where S is an arbitrary finite subset of N'.

[1]) J. Novák, On a problem concerning completely regular sets, this volume, p. 103-104.