i. e. a simultaneous interpretation of *all* predicate variables as individual predicates. An analogous situation is in Henkin [1]. For a detailed discussion of the already sketched application of part I to logic, see my paper announced in the footnote [3]) of the end of Introduction.

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(For further literature, see notes and footnotes of the text).

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A Note to Rieger's Paper „On Free $\aleph_\xi$-complete Boolean Algebras" [1]).

By

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The subject of this note is a simple proof of Rieger's Theorem 6 [2]).

Let M be an abstract set with cardinal $\mathfrak{m}$ (finite or infinite), and let $C_\mathfrak{m}$ denote the set of all functions f on M, the values of which are the numbers 0 and 1 only. ($C_\mathfrak{m}$ is the so-called *generalized Cantor discontinuum*, i. e. the Cartesian product of $\mathfrak{m}$ spaces, each of which is composed of the numbers 0 and 1 only).

For $a \in M$ let $C_{\mathfrak{m},a}$ denote the set of all $f \in C_\mathfrak{m}$ such that $f(a)=1$. For every (infinite) cardinal $\mathfrak{n}$ let $F_{\mathfrak{m},\mathfrak{n}}$ denote the least $\mathfrak{n}$-additive field of subsets of $C_\mathfrak{m}$ containing all the sets $C_{\mathfrak{m},a}$ $(a \in M)$.

If X is an $\mathfrak{n}$-additive field of subsets of a set $\mathfrak{X}$, and if I is an $\mathfrak{n}$-additive ideal of X, then the $\mathfrak{n}$-complete Boolean algebra X/I is called an $\mathfrak{n}$-*quotient algebra*. In particular, every $\mathfrak{n}$-additive field of sets is also an $\mathfrak{n}$-quotient algebra (the ideal I then contains only the empty set).

Theorem. *$F_{\mathfrak{m},\mathfrak{n}}$ is the free $\mathfrak{n}$-quotient algebra with $\mathfrak{m}$ generators $C_{\mathfrak{m},a}$ $(a \in M)$.*

This means:

For every family $\{A_a\}_{a \in M}$ of elements of any $\mathfrak{n}$-quotient algebra X/I there exists an $\mathfrak{n}$-additive homomorphism h of $F_{\mathfrak{m},\mathfrak{n}}$ into X/I such that $h(C_{\mathfrak{m},a})=A_a$ $(a \in M)$.

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[1]) This volume, pp. 29-46.

[2]) Loc. cit., p. 41.

For every $a \in M$ let $X_a \in \boldsymbol{X}$ be a fixed set such that[3] $[X_a]=A_a$. Let $f=c(x)$ be the characteristic function[4] of the family $\{X_a\}_{a \in M}$, that is, the mapping of $\mathfrak{X}$ into $C_\mathfrak{m}$ which associates with $x \in \mathfrak{X}$ an element $f \in C_\mathfrak{m}$ defined as follows: $f(a)=1$ if and only if $x \in X_a$. The mapping

$$h(F)=[c^{-1}(F)] \quad \text{for} \quad F \in \boldsymbol{F}_{\mathfrak{m},\mathfrak{n}}$$

is an $\mathfrak{n}$-additive homomorphism of $\boldsymbol{F}_{\mathfrak{m},\mathfrak{n}}$ into $\boldsymbol{X}/\boldsymbol{I}$ such that

$$h(C_{\mathfrak{m},a})=[c^{-1}(C_{\mathfrak{m},a})]=[X_a]=A_a, \qquad \text{q. e. d.}$$

Corollary 1 (Rieger's Theorem 6). *The σ-field $\boldsymbol{F}_{\mathfrak{m},\aleph_0}$ is the free Boolean σ-algebra with $\mathfrak{m}$ generators $C_{\mathfrak{m},a}$ $(a \in M)$*[5].

This follows immediately from the fact that every Boolean σ-algebra is isomorphic to an $\aleph_0$-quotient algebra[6].

Corollary 2. *Every $\mathfrak{n}$-quotient algebra $\boldsymbol{X}/\boldsymbol{I}$ with at most $\mathfrak{m}$ generators is isomorphic to an $\mathfrak{n}$-quotient algebra $\boldsymbol{F}_{\mathfrak{m},\mathfrak{n}}/\boldsymbol{J}$, where $\boldsymbol{J}$ is a suitable $\mathfrak{n}$-additive ideal.*

This is a generalization of Rieger's Theorem 4[7].

[3]) For $X \in \boldsymbol{X}$ the symbol $[X]$ will denote the element (coset) of $\boldsymbol{X}/\boldsymbol{I}$ determined by X.

[4]) M. H. Stone, *On Characteristic Functions of Families of Sets*, Fund. Math. **33** (1945), pp. 27-33. See also E. Marczewski, *The characteristic function of sets and some its applications*, Fund. Math. **31** (1938), pp. 207-223.

[5]) Another proof of this fact follows from Theorem VIII in my paper *On an analogy between measures and homomorphisms*, Annales Soc. Pol. Math. **23** (1950), pp. 1-20. That proof is based on Loomis's theorem for Boolean algebras with $\aleph_0$ generators only.

[6]) See L. H. Loomis, *On the representation of σ-complete Boolean algebras*, Bull. Am. Math. Soc. **53** (1947), pp. 757-760, and R. Sikorski, *On the representation of Boolean algebras as fields of sets*, Fund. Math. **35** (1948), pp. 247-258 (Theorem 5.3).

[7]) Loc. cit., p. 39.

Concerning the Cartesian product of Cantor-manifolds.

By

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1. A set of points[1]) is called an *n-dimensional Cantor-manifold*[2]) if it is an n-dimensional compactum and it cannot be disconnected by a subset of dimension $\leqslant n-2$.

It is known[3]) that every n-dimensional Cantor-manifold is n-dimensional in every one of its points and that

(1) *If A and B are n-dimensional Cantor-manifolds and $\dim A \cdot B \geqslant n-1$, then $A+B$ is also an n-dimensional Cantor-manifold.*

We can easily see that if in the formula

$$C=A\times B^{4)} \tag{2}$$

A and B are polytopes[5]) then C is a Cantor-manifold if and only if both polytopes A and B are Cantor-manifolds.

In this paper I shall show, by certain examples, that for arbitrary compacta there exists no relation between the Cantor-manifold property of A, B and C. Namely the following theorem holds:

[1]) It is convenient to assume that all sets of points investigated in this paper are subsets of the Hilbert space.

[2]) P. Urysohn, *Mémoire sur les multiplicités Cantoriennes*, Fund. Math. **7** (1925), p. 124.

[3]) See for instance C. Kuratowski, *Topologie II*, Warszawa-Wrocław 1950, p. 106.

[4]) $A\times B$ denotes the Cartesian product of A and B.

[5]) By a *polytope* we understand a point-set contained in the Hilbert space and having a decomposition in a finite collection of geometrical (rectilinear) simplexes such that every face of each simplex of the collection belongs to the collection. This decomposition of a polytope is called its *triangulation*. Every set homeomorphic to a polytope is called a *curvilinear polytope*.