With these facts in mind, let us consider a manifold M made up of two copies K'^* and K''^* of the anchor ring K^* by matching their toral surfaces in such a manner that for every $\xi \epsilon T$ the points $g_1(\xi)$ and $g_{-1}(\xi)$ are identified. It is clear that the mapping g defined as g_1 in A_1 and as g_{-1} in A_{-1} constitutes a homeomorphism mapping the set $S^{(3)}=A_1+A_{-1}$ onto M. But M is a 3-dimensional manifold obtained from two anchor rings K'^* and K''^* by matching their toral surfaces in such a manner that the circumference S_0' of the generating circle of K'^* is matched with the circumference S_0'' of the generating circle of K''^*. It is known [4]) that this condition determines completely the structure of the manifold M. Namely M is an oriented manifold with the Heegaard diagram [5]) consisting of the anchor ring and the system of two circumferences-boundaries of two generating circles. This manifold is homeomorphic to the cartesian product of the circumference and the 2-dimensional sphere [4]).

Thus we may state the following

Theorem. *The third symmetric potency of the circumference is homeomorphic to the cartesian product of the circumference and the 2-dimensional sphere.*

[4]) H. Seifert and W. Threlfall, *Lehrbuch der Topologie*, Leipzig 1934, p. 220.

[5]) P. Heegaard, *Sur l'Analysis Situs*, Bull. Soc. Math. France **44** (1916), p. 161.

A theorem on the structure of homomorphisms.

By

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This paper is a supplement to my paper [1]. Terminology and notation are in this paper the same as in [1].

Let $\boldsymbol{A}$ be a σ-complete Boolean algebra and let $\mathcal{S}$ and $\mathfrak{s}(A)$ denote respectively the set of all prime ideals of $\boldsymbol{A}$ and the set of all prime ideals of $\boldsymbol{A}$ which do not contain an element $A \epsilon \boldsymbol{A}$. As Stone has proved [1]), $S=\mathfrak{s}(A)$ is an isomorphism of $\boldsymbol{A}$ on a field of sets $\boldsymbol{S}=\mathfrak{s}(\boldsymbol{A})$.

Let $\boldsymbol{N}^0$ denote the class of all sets

$$\mathfrak{s}(A)-\sum_{n=1}^{\infty}\mathfrak{s}(A_n)$$

where $A, A_n \epsilon \boldsymbol{A}$, and $A=\sum_{n=1}^{\infty}A_n$.

The class $\boldsymbol{N}$ of all subsets of sets $\sum_{n=1}^{\infty}N_n$ where $N_n \epsilon \boldsymbol{N}^0$ is a σ-ideal of subsets of $\mathcal{S}$.

Let $\boldsymbol{Z}$ denote the class of all sets $Z \subset \mathcal{S}$ which can be represented in the form

$$Z=S-N_1+N_2$$

where $S \epsilon \mathcal{S}$ and $N_1, N_2 \epsilon \boldsymbol{N}$. Obviously $\boldsymbol{N} \subset \boldsymbol{Z}$ and $\boldsymbol{S} \subset \boldsymbol{Z}$.

We have then [2]):

(i) *$\boldsymbol{Z}$ is a σ-field of subsets of $\mathcal{S}$.*

(ii) *The mapping $\bar{\mathfrak{s}}$ defined by the formula*

$$\bar{\mathfrak{s}}(A)=[\mathfrak{s}(A)] \quad \textit{for} \quad A \epsilon \boldsymbol{A}$$

is an isomorphism of $\boldsymbol{A}$ on the σ-quotient algebra $\boldsymbol{Z}/\boldsymbol{N}$.

[1]) Stone [1], p. 106. In general, the field $\boldsymbol{S}$ is not a σ-field.

[2]) The proof of lemmas (i) and (ii) is similar to the proof of theorems 5.2 and 5.1 respectively in my paper [2] (every set $N \epsilon \boldsymbol{N}$ is of first category in Stone's space $\mathcal{S}$). See also Loomis [1], p. 757.

Let now $\boldsymbol{A}_1$ be another σ-complete Boolean algebra and let $\mathcal{S}_1$, $\boldsymbol{S}_1$, $\boldsymbol{N}_1^0$, $\boldsymbol{N}_1$, $\boldsymbol{Z}_1$, $\mathfrak{s}_1$, $\bar{\mathfrak{s}}_1$ have analogous meanings. Then:

(*) *For every σ-homomorphism f of $\boldsymbol{A}$ in $\boldsymbol{A}_1$ there exists a mapping φ of $\mathcal{S}_1$ in $\mathcal{S}$ which induces* [3] *the σ-homomorphism*

$$\bar{f}=\bar{\mathfrak{s}}_1 f \bar{\mathfrak{s}}^{-1}$$

of $\boldsymbol{Z}/\boldsymbol{N}$ in $\boldsymbol{Z}_1/\boldsymbol{N}_1$, i. e.:

$$\varphi^{-1}(Z)\in\boldsymbol{Z}_1 \quad \textit{and} \quad \bar{f}([Z])=[\varphi^{-1}(Z)] \quad \textit{for every} \quad Z\in\boldsymbol{Z}.$$

Theorem (*) explains the structure of σ-homomorphisms between arbitrary σ-complete Boolean algebras. It shows [4] namely that every σ-homomorphism f may be considered as a homomorphism induced by a mapping φ. The study of σ-homomorphisms of $\boldsymbol{A}$ in $\boldsymbol{A}_1$ can be reduced to the study of some mappings of $\mathcal{S}_1$ in $\mathcal{S}$.

Proof. The homomorphism

$$g=\mathfrak{s}_1 f \mathfrak{s}^{-1}$$

maps the field $\boldsymbol{S}$ in the field $\boldsymbol{S}_1$. Since every two-valued measure on $\boldsymbol{S}$ is trivial [5], there exists [6] a mapping φ which induces the homomorphism g, i. e.:

$$\varphi^{-1}(S)\in\boldsymbol{S}_1\,{}^{7)} \quad \text{and} \quad g(S)=\varphi^{-1}(S) \quad \text{for every} \quad S\in\boldsymbol{S}.$$

If $N\in\boldsymbol{N}^0$, i. e. $N=\mathfrak{s}(A)-\sum_{n=1}^{\infty}\mathfrak{s}(A_n)$ where $A=\sum_{n=1}^{\infty}A_n$, then

$$\varphi^{-1}(N)=\varphi^{-1}(\mathfrak{s}(A))-\sum_{n=1}^{\infty}\varphi^{-1}(\mathfrak{s}(A_n))=\mathfrak{s}_1(f(A))-\sum_{n=1}^{\infty}\mathfrak{s}_1(f(A_n))\in\boldsymbol{N}_1^0$$

since $f(A)=\sum_{n=1}^{\infty}f(A_n)$. Consequently

(iii) $\varphi^{-1}(N)\in\boldsymbol{N}_1$ for every $N\in\boldsymbol{N}$.

[3] Sikorski [1], p. 7.

[4] See an analogous remark on homomorphisms in my paper [1], pp. 11 and 12.

[5] Stone [1], p. 106.

[6] Sikorski [1], p. 10.

[7] If the sets $\mathcal{S}$ and $\mathcal{S}_1$ are considered as topological spaces with Stone's definition of neighbourhoods, the condition: $\varphi(S)\in\boldsymbol{S}_1$ for $S\in\boldsymbol{S}$ means that φ is a continuous mapping of $\mathcal{S}_1$ in $\mathcal{S}$.

This fact implies that

(iv) $\varphi^{-1}(Z)\in\boldsymbol{Z}_1$ for every $Z\in\boldsymbol{Z}$.

It follows from (iii) and (iv) that

(v) the formula $\bar{g}([Z])=[\varphi^{-1}(Z)]$ for $Z\in\boldsymbol{Z}$ defines a σ-homomorphism $\bar{g}$ of $\boldsymbol{Z}/\boldsymbol{N}$ in $\boldsymbol{Z}_1/\boldsymbol{N}_1$.

We have:

(vi) $\bar{g}([S])=\bar{f}([S])$ for every $S\in\boldsymbol{S}$.

Let Z be any set belonging to the field $\boldsymbol{Z}$. By the definition of $\boldsymbol{Z}$, there exists a set $S\in\boldsymbol{S}$ such that $[Z]=[S]$. Therefore

(vii) $\bar{f}([Z])=\bar{f}([S])=\bar{g}([S])=g([Z])=[\varphi^{-1}(Z)]$

on account of (v) and (vi).

Theorem (*) follows immediately from (iv) and (vii).

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