La propriété (5) est ainsi établie.

D'après (4) et (5) la famille de tous les ensembles $E(t)$, où $t \in T$ est donc une famille de puissance $2^{\aleph_1}$ d'ensembles linéaires croissants.

L'existence d'une telle famille entraîne tout de suite le théorème suivant:

Théorème. *Si $2^{\aleph_0}=\aleph_1$, l'ensemble de tous les nombres réels est somme de $2^{2^{\aleph_0}}$ ensembles croissants* [3]).

Plus encore: nous savons définir effectivement une famille d'ensembles linéaires croissants pour laquelle on peut démontrer, à l'aide de l'hypothèse du continu, qu'elle est de puissance $2^{2^{\aleph_0}}$.

Or, il est à remarquer que nous savons démontrer sans faire appel à l'hypothèse du continu (en utilisant seulement l'axiome du choix) qu'il existe une famille de puissance $>2^{\aleph_0}$ d'ensembles linéaires croissants [4]).

[3]) J'ai démontré ce théorème par une autre voie dans mon livre *Hypothèse du continu*, Monografie Matematyczne t. IV (Warszawa 1934), p. 120, Proposition C_{64}.

[4]) Voir W. Sierpiński, Fund. Math. **3** (1922), p. 109.

On the imbedding of systems of compacta in simplicial complexes.

By

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1. It is one of the basic facts of geometry that every polytope is decomposable in finite sum of elementar „brics" called simplexes. The importance of such decompositions for the study of topological properties of polytopes suggest the investigation of decompositions of more general spaces into sums of sets having particularly simple homological and homotopical properties.

In the present paper I establish a simple connection between arbitrarily given decomposition of a finite dimensional compactum C in a finite sum of closed sets and a simplicial decomposition of some polytope. It turns out that every such decomposition of C may be obtained by an topological imbedding of C in some polytope P and by intersection of so imbedded set with simplexes of a simplicial decomposition of P (Theorem 1). In the case when C has a decomposition in a finite sum of absolute retracts such that every not empty intersection of those retracts is also an absolute retract, it turns out that all homology and homotopy groups of C are determined by the combinatorial properties of the decomposition.

2. Only metric spaces will be considered. By the cartesian product of two spaces X and Y is meant the space $X \times Y$ consisting of all ordered pairs (x,y), where $x \in X$, $y \in Y$ and where the distance is defined by the formula

$$\varrho\big((x,y),(x',y')\big)=\sqrt{\varrho(x,x')^2+\varrho(y,y')^2}.$$

If the space Y is compact, the space Y^X consisting of all continuous mappings f of X into Y metrized by the formula

$$\varrho(f,g)=\sup_{x\in X}\varrho(f(x),g(x))$$

is complete.

By *retraction* we understand a continuous mapping f of X onto $Y \subset X$ satisfying the condition

$$f(x)=x \quad \text{for every} \quad x \in Y.$$

If a retraction of X onto Y exists then Y is called a *retract* of X. If there exists a continuous mapping $f(x,t)$ (called a *deformation retraction* of X onto Y) of the space $X \times \langle 0,1 \rangle$ (where $\langle 0,1 \rangle$ denotes the interval $0 \leqslant t \leqslant 1$) into X such that $f(x,0) = x$ and $f(x,1)$ is a retraction of X onto Y, then Y is said to be a *deformation retract* of X.

A compactum A is an *absolute retract* whenever a topological image of A in any space X is necessarily a retract of X. A compactum A is an *absolute neighborhood retract* whenever a topological image B of A in any space X is necessarily a retract of some neighborhood of B in X.

By the *Hilbert space* H is meant the space consisting of all sequences of real numbers $x=\{x_n\}$ with $\sum_{n=1}^{\infty} x_n^2$ convergent as points and with the metric

$$\varrho(x,y)=\sqrt{\sum_{n=1}^{\infty}(x_n-y_n)^2}.$$

The numbers x_n are called *coordinates* of the point x. The point with all coordinates vanishing will be denoted by 0.

For the points of the Hilbert space there are defined addition and substraction by the formula

$$\{x_n\} \pm \{y_n\} = \{x_n \pm y_n\}$$

and the multiplication by a real number by the formula

$$t \cdot \{x_n\} = \{t \cdot x_n\}.$$

Let $k \geqslant 0$ be an integral number. The subset C_k of H consisting of all points $\{x_n\}$ with $x_n=0$ for every $n>k$ is congruent with the Euclidean k-dimensional space. Furthermore we define C_{-1} as the empty set and C_ω by the formula

$$C_\omega = \sum_{k=0}^{\infty} C_k.$$

A finite system $x^0,x^1,\ldots,x^k$ of points of the space H is called *linearly independent* if the equation

$$t_0 \cdot x^0 + t_1 \cdot x^1 + \ldots + t_k \cdot x^k = 0,$$

where $t_0,t_1,\ldots,t_k$ are real numbers, with $t_0+t_1+\ldots+t_k=0$, implies $t_0=t_1,\ldots,=t_k=0$.

In particulary linearly independent is every finite system consisting of different points belonging to the sequence $d^0,d^1,\ldots,d^i\ldots$, where

(1) $$d^i=\{\delta_n^i\} \quad \text{with} \quad \delta_n^i=0 \quad \text{for } n \neq i \text{ and } \delta_i^i=1.$$

Let $x^0,x^1,\ldots,x^k$ be a linearly independent system of points of H. By the *k-dimensional* (geometrical) simplex

$$\Delta=\Delta(x^0,x^1,\ldots,x^k)$$

spanned by the *vertices* $x^0,x^1,\ldots,x^k$ we mean the minimal convex subset of H containing the points $x^0,x^1,\ldots,x^k$. By the *(—1)-dimensional simplex* we mean the empty set.

A simplex Δ' will be called a *face* of the simplex $\Delta \neq \Delta'$ if all vertices of Δ' are vertices of Δ. The sum of all faces of the simplex Δ will be called the *boundary* $\dot{\Delta}$ of Δ, and the set $\Delta-\dot{\Delta}$, the *interior* of Δ. Evidently there exists for every k-dimensional simplex Δ a homeomorphic mapping f of Δ on the set

$$Q_k = \underset{x}{E}[x \in C_k;\ \varrho(x,0) \leqslant 1],$$

called *k-dimensional spheric element*. The homeomorphism f maps the boundary $\dot{\Delta}$ of Δ onto the *(k—1)-dimensional sphere*

$$S_{k-1} = \underset{x}{E}[x \in C_k;\ \varrho(x,0)=1]$$

and the interior $\Delta-\dot{\Delta}$ of Δ onto the *interior*

$$I_k = \underset{x}{E}[x \in C_k;\ \varrho(x,0)<1]$$

of Q_k.

A *simplicial complex* K is a finite sequence of simplexes $\Delta_1,\Delta_2,\ldots,\Delta_m$ such that for every $i,j=1,2,\ldots,m$ the set $\Delta_i\Delta_j$ is the simplex spanned by all common vertices of Δ_i and Δ_j. The vertices of the simplexes $\Delta_1,\Delta_2,\ldots,\Delta_m$ are called the *vertices* of the complex K. The sum of all simplexes $\Delta_1,\Delta_2,\ldots,\Delta_m$ will be denoted by $|K|$. A *polytope* is a set P such that there exists a complex K with $|K|=P$. Such complex K is called a *triangulation* of the polytope P.

3. Lemma. *Let B be a closed subset of a compactum A, such that*

$$\dim(A-B)\leqslant p<\infty,$$

and f_0 a continuous mapping of B in the boundary of an $(2p+1)$-dimensional simplex Δ. Then there exists a continuous extension f of f_0 over A such that f maps the set $A-B$ topologically in the interior of Δ.

Proof. We can consider, instead of mappings in an $(2p+1)$-dimensional simplex Δ the mappings into the $(2p+1)$-dimensional spheric element

$$Q=Q_{2p+1}=\underset{x}{E}[x\in C_{2p+1};\ \varrho(x,0)\leqslant 1].$$

By the known imbedding theorem of Menger-Nöbeling[1]) every compactum of the dimension $\leqslant p$ is homeomorphic to a subset of the interior I_{2p+1} of Q_{2p+1}. We infer that the lemma is true if $B=0$. Hence we can suppose in the sequel that $B\neq 0$. Furthermore we can assume, without diminishing the generality, that the diameter of A is less than $1/2$.

Let us put

(2) $$A_k=\underset{x}{E}\left[x\in A;\ \varrho(x,B)\geqslant\frac{1}{k}\right]\quad\text{for } k=1,2,\ldots$$

Then the sets A_n are compact and

(3) $$A_1=A_2=0;\quad A-B=\sum_{k=1}^{\infty}A_k.$$

Let us denote by $Q^A(f_0)$ the subset of the space Q^A consisting of all continuous mappings f' satisfying both conditions:

(4) $$f'(x)=f_0(x)\quad\text{for every}\quad x\in B,$$

(5) $$\varrho(f'(x),0)\leqslant 1-\frac{1}{k}\quad\text{for every } x\in A_k,\ k=1,2\ldots$$

It is clear that $Q^A(f_0)$ is a closed subset of the complete space Q^A. Hence $Q^A(f_0)$ is also a complete space. Furthermore it is

$$Q^A(f_0)\neq 0.$$

[1]) See, for instance, W. Hurewicz and H. Wallman, *Dimension Theory*, Princeton Math. Series 4, Princeton 1941, p. 56.

For there exists[2]) a mapping $f\in Q^A$ being a extension of f_0 over A and thus satisfying the condition (4). Putting

$$f'(x)=[1-\varrho(x,B)]\cdot f(x)\quad\text{for every}\quad x\in A$$

we obtain a mapping $f'\in Q^A$ such that for every $x\in B$ it is $f'(x)=f(x)=f_0(x)$. Furthermore for every $x\in A_k$ it is $\varrho(x,B)\geqslant\frac{1}{k}$ and consequently

$$\varrho(f'(x),0)\leqslant 1-\varrho(x,B)\leqslant 1-\frac{1}{k}.$$

Hence f' satisfies the inegality (5).

Denote by $\Gamma_{m,n}$, for m and n natural, the subset of the space $Q^A(f_0)$ consisting of all mappings f' satisfying the condition

(6) $$\text{If}\quad x,y\in A_n\quad\text{and}\quad \varrho(x,y)\geqslant\frac{1}{m}\quad\text{then}\quad f'(x)\neq f'(y).$$

Clearly $\Gamma_{m,n}$ is an open subset of $Q^A(f_0)$. Let us prove that $\Gamma_{m,n}$ is dense in $Q^A(f_0)$. It suffices to show that for every mapping $f\in Q^A(f_0)$ and every number ε such that

(7) $$0<\varepsilon<1$$

there exists a mapping $f'\in Q^A(f_0)$ satisfying the inegality

(8) $$\varrho[f'(x),f(x)]\leqslant\varepsilon\quad\text{for every } x\in A$$

which maps the A_n topologically.

Consider the mapping f'' defined by the formula

$$f''(x)=\left(1-\frac{\varepsilon}{6}\right)\cdot f(x)\quad\text{for every } x\in A_n.$$

f'' maps A_n onto a subset of the interior of Q and we have

(9) $$\varrho[f''(x),f(x)]<\tfrac{1}{6}\varepsilon\quad\text{for every } x\in A_n.$$

By the imbedding theorem of Menger and Nöbeling there exists a homeomorphism f''' mapping A_n onto a subset of Q in such a manner that

$$\varrho[f'''(x),f''(x)]<\tfrac{1}{6}\varepsilon\quad\text{for every } x\in A_n.$$

By (9) it follows

(10) $$\varrho[f'''(x),f(x)]<\tfrac{1}{3}\varepsilon\quad\text{for every } x\in A_n.$$

[2]) See, for instance, W. Hurewicz and H. Wallman, l. c. p. 82.

Putting

$$g(x)=f'''(x)-f(x) \quad \text{for every} \quad x \in A_n,$$

we obtain a continuous mapping g of the set A_n onto a subset of the spherical element

$$Q(\tfrac{1}{3}\varepsilon) = \underset{x}{E}[x \in C_{2p+1}; \varrho(x,0) \leqslant \tfrac{1}{3}\varepsilon].$$

We extend the range of g putting

$$g(x)=0 \quad \text{for every} \quad x \in \overline{A-A_{n+1}}.$$

Thus the continuous mapping g is defined in the closed subset $A_n+\overline{A-A_{n+1}}$ of A. There exists [2]) a continuous extension g' of g over A with values belonging to $Q(\frac{1}{3}\varepsilon)$. Putting

$$f^{\mathrm{IV}}(x)=f(x)+g'(x) \quad \text{for every} \quad x \in A$$

we obtain a mapping $f^{\mathrm{IV}} \in Q^A$ such that

(11) $$\varrho[f^{\mathrm{IV}}(x),f(x)] \leqslant \tfrac{1}{3}\varepsilon \quad \text{for every} \quad x \in A.$$

Furthermore we have

(12) $$f^{\mathrm{IV}}(x)=f(x) \quad \text{for every} \quad x \in \overline{A-A_{n+1}}.$$

Now we construct the desired mapping f' by putting

(13) $$f'(x)=(1-\tfrac{2}{3}\varepsilon)\cdot f^{\mathrm{IV}}(x) \quad \text{for every} \quad x \in A_{n+1},$$

(14) $$f'(x)=\{1-\tfrac{2}{3}(n+1)\varepsilon[(n+2)\cdot\varrho(x,B)-1]\}\cdot f^{\mathrm{IV}}(x) \quad \text{for every} \quad x \in \overline{A_{n+2}-A_{n+1}},$$

(15) $$f'(x)=f^{\mathrm{IV}}(x) \quad \text{for every} \quad x \in \overline{A-A_{n+2}}.$$

In order to show that the formulas (13), (14) and (15) define a continuous mapping, let us observe that for $x \in A_{n+1}\cdot\overline{A_{n+2}-A_{n+1}}$ it is $\varrho(x,B)=\dfrac{1}{n+1}$. Consequently the value of f' given by the formulas (13) and (14) are the same. Similarly for $x \in \overline{A_{n+2}-A_{n+1}}\cdot\overline{A-A_{n+2}}$ it is $\varrho(x,B)=\dfrac{1}{n+2}$ and consequently the value of f' defined by (14) is equal to the value defined by (15).

By (13) in the set $A_n \subset A_{n+1}$ the mapping f' is a superposition of the homeomorphism f^{IV} and the homeomorphism

$$h(y)=(1-\tfrac{2}{3}\varepsilon)y$$

mapping Q in itself. Hence f' is over A_n a homeomorphism. By the formulas (13), (14) and (15) the values of $f'(x)$ belong to Q and $f'(x)$ is of the form

$$f'(x)=(1-\alpha(x))\cdot f^{\mathrm{IV}}(x)$$

with $0 \leqslant \alpha(x) \leqslant \frac{2}{3}\varepsilon$. It follows

$$\varrho[f'(x),f^{\mathrm{IV}}(x)]=\alpha(x)\cdot\varrho[f^{\mathrm{IV}}(x),0] \leqslant \tfrac{2}{3}\varepsilon.$$

Combining this with the inequality (11) we conclude that f fulfills the condition (8).

It remains to prove that $f' \in \Gamma_{m,n}$, this is that f' fulfills the conditions (4), (5) and (6). The condition (6) is fulfilled, because f' is a homeomorphism in the set A_n. The condition (4) is a consequence of the formulas (12) and (15), because $x \in B$ implies $x \in \overline{A-A_{n+2}}$ and $f(x)=f_0(x)$.

To prove the inequality (5) we consider the following cases: If $x \in A_k\cdot\overline{A-A_{n+1}}$ with $k \geqslant n+1$, then (12), (14) and (15) given

$$\varrho[f'(x),0] \leqslant \varrho[f^{\mathrm{IV}}(x),0]=\varrho[f(x),0] \leqslant 1-\frac{1}{k}.$$

If however $x \in A_k$ with $k<n+1$, then $x \in A_{n+1}$ and by (13) we have

$$\varrho[f'(x),0]=(1-\tfrac{2}{3}\varepsilon)\cdot\varrho[f^{\mathrm{IV}}(x),0].$$

If $\varrho[f^{\mathrm{IV}}(x),0] \leqslant \frac{1}{2}$, we infer by (3) that

$$\varrho[f'(x),0] \leqslant \tfrac{1}{2} < 1-\frac{1}{k}.$$

If however $\varrho[f^{\mathrm{IV}}(x),0] > \frac{1}{2}$, then by (13) and (11) we have

$$\varrho[f'(x),0]=\varrho[f^{\mathrm{IV}}(x),0]-\tfrac{2}{3}\varepsilon\,\varrho[f^{\mathrm{IV}}(x),0] \leqslant \varrho[f^{\mathrm{IV}}(x),0]-\tfrac{1}{3}\varepsilon \leqslant$$
$$\leqslant \varrho[f^{\mathrm{IV}}(x),0]-\varrho[f^{\mathrm{IV}}(x),f(x)] \leqslant \varrho[f(x),0] \leqslant 1-\frac{1}{k}.$$

Thus the inequality (5) is proved in all cases and consequently $f' \in \Gamma_{m,n}$. Hence the open set $\Gamma_{m,n}$ is dense in the complete space $Q^A(f_0)$. By the known theorem of R. Baire [3]) there exists a mapping $f \in \prod_{m,n=1}^{\infty} \Gamma_{m,n}$. We conclude from the conditions (6), (5) and (3) that f maps $A-B$ topologically in the interior of Q. Hence the lemma is established.

[3]) See, for instance, W. Hurewicz and H. Wallman, l. c. p. 160.

4. By a *decomposition* of a space A we mean a finite sequence $\{A_1, A_2, \dots, A_k\}$ of closed sets whose sum is A.

A decomposition $\{A_1, A_2, \dots, A_k\}$ of a space A is *similar* to a decomposition $\{B_1, B_2, \dots, B_l\}$ of a space B if $k=l$ and for each sequence $i_1, i_2, \dots, i_\nu$ of indices the relations

$$A_{i_1}\cdot A_{i_2}\dots A_{i_\nu}=0 \quad \text{and} \quad B_{i_1}\cdot B_{i_2}\dots B_{i_\nu}=0$$

are equivalent. In particular $A_i=0$ if and only if $B_i=0$. It is clear that the similarity of decompositions is a reflexive, symmetric and transitive relation.

Applying the concept of the *nerve* of a decomposition introduced by P. Alexandroff [4]) we see at once that two decompositions are similar if and only if the correspondence $A_i \to B_i$ induces an isomorphism of their nerves.

A simplicial complex

$$K=\{\Delta_1, \Delta_2, \dots, \Delta_k\}$$

will be called a *simplicial realization* of a decomposition $\{A_1, A_2, \dots, A_k\}$ of a space A if it constitutes a decomposition of the polytope $|K|$ similar to the decomposition $\{A_1, A_2, \dots, A_k\}$ of A and there exists a homeomorphism h mapping A into a subset of $|K|$ in such a manner that

$$h(A_i)=h(A)\cdot\Delta_i \quad \text{for every } i=1,2,\dots,k.$$

5. *Theorem 1.* *For every decomposition of a finite dimensional compactum there exists a simplicial realization.*

Proof. Let $\{A_1, A_2, \dots, A_k\}$ be a decomposition of a compactum A of the dimension $p<\infty$. For every $m=0,1,\dots,(k-1)$ there exists $\alpha(m)=\binom{k}{m}$ of different increasing sequences $i_1, i_2, \dots, i_{k-m}$ with natural terms $\leqslant k$. Let us range the sets of the form

$$A_{i_1}\cdot A_{i_2}\cdot \ldots \cdot A_{i_{k-m}}$$

(not necessarily different) corresponding to such sequences into a (finite) sequence

$$\mathfrak{A}^m=\{A_1^m, A_2^m, \dots, A_{\alpha(m)}^m\}$$

[4]) See P. Alexandroff, *Über den allgemeinen Dimensionsbegriff und seine Beziehungen zur elementaren geometrischen Anschauung*, Math. Ann. **98** (1928), p. 634.

The sequence $\mathfrak{A}^m$ constitutes a decomposition of the set

$$A^m=A_1^m+A_2^m+\dots+A_{\alpha(m)}^m.$$

In particular $\alpha(k-1)=k$; $A^{k-1}=A$ and the sequence $\{A_1^{k-1}, A_2^{k-1}, \dots, A_k^{k-1}\}$ constitutes a permutation of the sequence $\{A_1, A_2, \dots, A_k\}$. We can assume that

$$A_i^{k-1}=A_i \quad \text{for every } i=1,2,\dots,k.$$

We now prove, for $m=0,1,\dots,(k-1)$, the following statement

(16^m) *There exists in the space C_ω a simplicial complex K^m, with all vertices belonging to the sequence $\{d_i\}$, constituting a simplicial realization of the decomposition $\{A_1^m, A_2^m, \dots, A_{\alpha(m)}^m\}$ of the space A^m.*

Evidently (16^{k-1}) is equivalent to our theorem. Let us prove (16^m) by induction.

For $m=0$ we have $\alpha(m)=\binom{k}{k-0}=1$ and $A^1=A_1^1=A_1\cdot A_2\cdot\ldots\cdot A_k$. If this last set is empty, then the statement (16^0) holds if we put K' equal to the complex constituted only by one (-1)-dimensional simplex. If however

$$\dim A^1=p_1\geqslant 0,$$

then the statement (16^0) holds if K^1 is constituted only by one $(2p_1+1)$-dimensional simplex Δ_1^1 with the vertices $d^0, d^1, \dots, d^{2p_1+1}$ because, by the imbedding theorem of Menger-Nöbeling, there exists a homeomorphism h' mapping A' onto a subset of Δ_1'.

Now assume that the statement (16^m) holds for an $m<k-1$. By this hypothesis there exists a simplicial complex

$$K^m=\{\Delta_1^m, \Delta_2^m, \dots, \Delta_{\alpha(m)}^m\}$$

constituting a decomposition of the polytope $|K^m|$ similar to the decomposition $\{A_1^m, A_2^m, \dots, A_{\alpha(m)}^m\}$ of A^m and a homeomorphism h^m mapping A^m onto a subset of the polytope $|K^m|$ in such a manner that

$$(17^m) \qquad h^m(A_i^m)=h^m(A^m)\cdot\Delta_i^m \quad \text{for every } i=1,2,\dots,\alpha(m).$$

Since the decompositions $\{A_1^m, A_2^m, \dots, A_{\alpha(m)}^m\}$ and $\{\Delta_1^m, \Delta_2^m, \dots, \Delta_{\alpha(m)}^m\}$ are similar it follows that for every system of indices $i_1, i_2, \dots, i_\nu$ the vanishing of the set $A_{i_1}^m\cdot A_{i_2}^m\cdot\ldots\cdot A_{i_\nu}^m$ is equivalent to the vanishing of the set $\Delta_{i_1}^m\cdot\Delta_{i_2}^m\cdot\ldots\cdot\Delta_{i_\nu}^m$.

To prove now the statement (16^{m+1}) let us consider the sequence
$$\mathfrak{A}^{m+1}=\{A_1^{m+1},A_2^{m+1},\dots,A_{\alpha(m+1)}^{m+1}\}.$$

For every $\lambda=1,2,\dots,\alpha(m+1)$ the set A_λ^{m+1} can be written in the form
$$A_\lambda^{m+1}=A_{i_{\lambda,1}}\cdot A_{i_{\lambda,2}}\cdot\ldots\cdot A_{i_{\lambda,k-m-1}}\quad\text{with}\quad i_{\lambda,\mu}\neq i_{\lambda,\nu}\quad\text{for}\quad \mu\neq\nu.$$

Let us range all natural numbers $\leqslant k$ not appearing among the numbers $i_{\lambda,1},i_{\lambda,2},\dots,i_{\lambda,k-m-1}$ in the sequence $j_{\lambda,1},j_{\lambda,2},\dots,j_{\lambda,m+1}$. Obviously it is
$$A_\lambda^{m+1}\cdot A^m=\sum_{\nu=1}^{m+1}A_\lambda^{m+1}\cdot A_{j_{\lambda,\nu}}.$$

But every set $A_\lambda^{m+1}\cdot A_{j_{\lambda,\nu}}$ belongs to $\mathfrak{A}^m$ and consequently there exists an index $i(\lambda,\nu)$ such that
$$A_\lambda^{m+1}\cdot A_{j_{\lambda,\nu}}=A_{i(\lambda,\nu)}^m\quad\text{for every}\quad \nu=1,2,\dots,m+1.$$

It follows from (17^m) that
$$h^m(A_{i(\lambda,\nu)})=h^m(A^m)\cdot\Delta_{i(\lambda,\nu)}^m.$$

Let us assign to each set $A_\lambda^{m+1}\in\mathfrak{A}^{m+1}$ the simplex Δ_λ^{m+1} defined in the following manner: If $A_\lambda^{m+1}=0$ and consequently also $A_{i(\lambda,\nu)}^m=0$ for every $\nu=1,2,\dots,m+1$, then we put $\Delta_\lambda^{m+1}=0$. If however $A_\lambda^{m+1}\neq0$, then Δ_λ^{m-1} denotes the simplex having as the set of vertices
$$p_{\lambda,0}^{m+1},p_{\lambda,1}^{m+1},\dots,p_{\lambda,\beta_l}^{m+1}$$
the collection of all vertices of all simplexes $\Delta_{i(\lambda,\nu)}^m$, $\nu=1,2,\dots,m+1$ enlarged by some supplementary vertices which belong to the sequence $\{d^i\}$ but differ from all vertices of the complex K^m and for different values of λ are different. Furthermore we can suppose that the number of supplementary vertices in every simplex Δ_λ^{m+1} is $\geqslant1$ and so large that
$$\beta_\lambda\geqslant2p+1\quad\text{for every}\quad\lambda=1,2,\dots,\alpha(m+1).$$

It follows from our construction that every simplex $\Delta_{i(\lambda,\nu)}^m$, $\nu=1,2,\dots,m+1$ constitutes a face of Δ_λ^{m+1} and that for every other simplex Δ_i^m of K^m the simplexes Δ_i^m and Δ_λ^{m+1} are disjoint.

Since all vertices of the simplexes Δ_λ^{m+1} belong to the sequence $\{d_i\}$ the simplexes $\{\Delta_1^{m+1},\Delta_2^{m+1},\dots,\Delta_{\alpha(m+1)}^{m+1}\}$ constitute a simplicial complex K^{m+1}. It remains to prove that K^{m+1} is a simplicial realization of the decomposition $\{A_1^{m+1},A_2^{m+1},\dots,A_{\alpha(m+1)}^{m+1}\}$ of the set A^{m+1}.

We shall first show that the decomposition $\{\Delta_1^{m+1},\Delta_2^{m+1},\dots,\Delta_{\alpha(m+1)}^{m+1}\}$ of the polytope $|K^{m+1}|$ is similar to the decomposition $\{A_1^{m+1},A_2^{m+1},\dots,A_{\alpha(m+1)}^{m+1}\}$ of A^{m+1}.

Let $\lambda_1,\lambda_2,\dots,\lambda_\nu$ be a sequence of natural numbers $\leqslant\alpha(m+1)$. If there exists an index μ such that $1\leqslant\mu\leqslant\nu$ and $A_{\lambda_\mu}^{m+1}=0$ then also $\Delta_{\lambda_\mu}^{m+1}=0$. Consequently both sets $A_{\lambda_1}^{m+1}\cdot A_{\lambda_2}^{m+1}\cdot\ldots\cdot A_{\lambda_\nu}^{m+1}$ and $\Delta_{\lambda_1}^{m+1}\cdot\Delta_{\lambda_2}^{m+1}\cdot\ldots\cdot\Delta_{\lambda_\nu}^{m+1}$ vanish. Hence we can assume that no any of the sets $A_{\lambda_1}^{m+1},A_{\lambda_2}^{m+1},\dots,A_{\lambda_\nu}^{m+1}$ is empty.

If $\lambda_1=\lambda_2=\ldots=\lambda_\nu$, then $A_{\lambda_1}^{m+1}\cdot A_{\lambda_2}^{m+1}\cdot\ldots\cdot A_{\lambda_\nu}^{m+1}=A_{\lambda_1}^{m+1}$ and $\Delta_{\lambda_1}^{m+1}\cdot\Delta_{\lambda_2}^{m+1}\cdot\ldots\cdot\Delta_{\lambda_\nu}^{m+1}=\Delta_{\lambda_1}^{m+1}$. In this case the equivalence of the relations $A_{\lambda_1}^{m+1}\cdot A_{\lambda_2}^{m+1}\cdot\ldots\cdot A_{\lambda_\nu}^{m+1}=0$ and $\Delta_{\lambda_1}^{m+1}\cdot\Delta_{\lambda_2}^{m+1}\cdot\ldots\cdot\Delta_{\lambda_\nu}^{m+1}=0$ is an immediate consequence of the definition of the simplexes Δ_λ^{m+1}.

Let us assume now that in the sequence $\lambda_1,\lambda_2,\dots,\lambda_\nu$ there exist at least two different numbers. Let $A_{\lambda_j}^{m+1}=A_{i_{\lambda_j,1}}A_{i_{\lambda_j,2}}\cdot\ldots\cdot A_{\lambda_j,k-m-1}$ where $i_{\lambda_j,1},i_{\lambda_j,2},\dots,i_{\lambda_j,k-m-1}$ is a system of $k-m-1$ different natural numbers $\leqslant k$. By our hypothesis there exists for every system $i_{\lambda_j,1},i_{\lambda_j,2},\dots,i_{\lambda_j,k-m-1}$ an index $\varkappa_j$ of the form $i_{\lambda_{j'},l}$ with $j'=j$ which does not belong to the system $i_{\lambda_j,1},i_{\lambda_j,2},\dots,i_{\lambda_j,k-m-1}$. The set $A_{\lambda_j}^{m-1}\cdot A_{\varkappa_j}$ belongs to the system $\mathfrak{A}^m$. Let us put

$$A_{\lambda_j}^{m+1}\cdot A_{\varkappa_j}=A_{\mu_j}^m. \tag{18}$$

Hence
$$A_{\lambda_1}^{m+1}\cdot A_{\lambda_2}^{m+1}\cdot\ldots\cdot A_{\lambda_\nu}^{m+1}=A_{\mu_1}^m\cdot A_{\mu_2}^m\cdot\ldots\cdot A_{\mu_\nu}^m.$$

It follows from (18) that the simplex $\Delta_{\mu_j}^m$ constitutes a face of the simplex $\Delta_{\lambda_j}^{m+1}$. If $A_{\lambda_1}^{m+1}\cdot A_{\lambda_2}^{m+1}\cdot\ldots\cdot A_{\lambda_\nu}^{m+1}\neq0$ then $A_{\mu_1}^m\cdot A_{\mu_2}^m\cdot\ldots\cdot A_{\mu_\nu}^m\neq0$ hence, by hypothesis of induction, $\Delta_{\mu_1}^m\cdot\Delta_{\mu_2}^m\cdot\ldots\cdot\Delta_{\mu_\nu}^m\neq0$ and consequently also $\Delta_{\lambda_1}^{m+1}\cdot\Delta_{\lambda_2}^{m+1}\cdot\ldots\cdot\Delta_{\lambda_\nu}^{m+1}\neq0$.

If $\Delta_{\lambda_1}^{m+1}\cdot\Delta_{\lambda_2}^{m+1}\cdot\ldots\cdot\Delta_{\lambda_\nu}^{m+1}\neq0$, then there exists a vertex p common to all simplexes $\Delta_{\lambda_j}^{m+1}$, $j=1,2,\dots,\nu$. Such a vertex can not be a „supplementary vertex", because not all numbers $\lambda_1,\lambda_2,\dots,\lambda_\nu$ are identical and the simplexes with different indices have different „supplementary vertices". Hence p is a vertex of the complex K^m. It follows that every simplex $\Delta_{\lambda_j}^{m+1}$, $j=1,2,\ldots,\nu$ contains a face $\Delta_{\mu_j}^m$ with the vertex p. By the construction of $\Delta_{\lambda_j}^{m+1}$ we infer that $A_{\mu_j}^m \subset A_{\lambda_j}^{m+1}$. But $p \in \Delta_{\mu_1}^m \cdot \Delta_{\mu_2}^m \cdot \ldots \cdot \Delta_{\mu_\nu}^m$ implies that $A_{\mu_1}^m \cdot A_{\mu_2}^m \cdot \ldots \cdot A_{\mu_\nu}^m \neq 0$ and consequently also $A_{\lambda_1}^{m+1} \cdot A_{\lambda_2}^{m+1} \cdot \ldots \cdot A_{\lambda_\nu}^{m+1} \neq 0$. Thus we have show that the decomposition $\{A_1^{m+1}, A_2^{m+2}, \ldots, A_{\alpha(m+1)}^{m+1}\}$ of the set A^{m+1} and the decomposition $\{\Delta_1^{m+1}, \Delta_2^{m+1}, \ldots, \Delta_{\alpha(m+1)}^{m+1}\}$ of the polytope $|K^{m+1}|$ are similar.

It remains to show that there exists a homeomorphism h^{m+1} satisfying the statement (16^{m+1}). We define a such homeomorphism by extending the homeomorphism h^m over every of the sets A_λ^{m+1}, $\lambda=1,2,\ldots,\alpha(m+1)$. We can assume that $A_\lambda^{m+1} \neq 0$. Let us denote by B_λ^{m+1} the common party of the set A_λ^{m+1} and the set $\sum_{\mu \neq \lambda} A_\mu^{m+1}$. The homeomorphism h^m maps the set B_λ^{m+1} onto a subset of the boundary of Δ_λ^{m+1}. Since the dimension of Δ_λ^{m+1} is $\geqslant 2p+1$ and the dimension of B_λ^{m+1} is $\leqslant p$ we infer, by the lemma of Nr. **3**, that there exists a homeomorphism h_λ^{m+1} being an extension of h^m over A_λ^{m+1} and mapping $A_\lambda^{m+1} - B_\lambda^{m+1}$ onto a subset of the interior of Δ_λ^{m+1}. Since the interiors of verious simplexes Δ_λ^{m+1} are disjoint the mapping h^{m+1} defined by the formula

$$h^{m+1}(x) = h_\lambda^{m+1}(x) \quad \text{for every } x \in A_\lambda^{m+1}$$

is a homeomorphism of A^{m+1} onto a subset of the polytope $|K^{m+1}|$.

Futhermore, by (17^m), we have

$$\begin{aligned} h^{m+1}(A_\lambda^{m+1}) &= h^{m+1}(A_\lambda^{m+1} - B_\lambda^{m+1}) + h^m(B_\lambda^{m+1}) = \\ &= h^{m+1}(A^{m+1})(\Delta^{m+1} - \dot{\Delta}^{m+1}) + h^m(A^m) \cdot \dot{\Delta}_\lambda^{m+1} = \\ &= h^{m+1}(A^{m+1}) \cdot (\Delta_\lambda^{m+1} - \dot{\Delta}_\lambda^{m+1}) + h^{m+1}(A^{m+1}) \cdot \dot{\Delta}_\lambda^{m+1} = \\ &= h^{m+1}(A^{m+1}) \cdot \Delta_\lambda^{m+1}. \end{aligned}$$

Thus we see that the condition (16^{m+1}) is fulfilled. This completes the proof of our theorem.

6. *Corollary.* *For two similar decompositions of two finite dimensional compacta there exists a complex being their common simplicial realization.*

Proof. {Let $A_1, A_2, \ldots, A_k\}$ be a decomposition of a finite dimensional compactum A and $\{B_1, B_2, \ldots, B_k\}$ a similar decomposition of a finite dimensional compactum B. We can assume that A and B are disjoint subsets of the space H of Hilbert. The sets $\{A_1+B_1, A_2+B_2, \ldots, A_k+B_k\}$ constitute a decomposition of the space $A+B$ similar to the decomposition $\{A_1, A_2, \ldots, A_k\}$ of A. In fact, the relation $A_{i_1} \cdot A_{i_1} \cdot \ldots \cdot A_{i_\nu} = 0$ is equivalent with the relation $B_{i_1} \cdot B_{i_2} \cdot \ldots \cdot B_{i_\nu} = 0$ hence also (by $A \cdot B = 0$) with the relation $(A_{i_1}+B_{i_1}) \cdot (A_{i_2}+B_{i_2}) \cdot \ldots \cdot (A_{i_\nu}+B_{i_\nu}) = 0$.

According to the theorem 1 there exists a simplicial complex $K = \{\Delta_1, \Delta_2, \ldots, \Delta_k\}$ being a simplicial realization of the decomposition $\{A_1+B_1, A_2+B_2, \ldots, A_k+B_k\}$. It means that the decomposition $\{\Delta_1, \Delta_2, \ldots, \Delta_k\}$ of the polytope $|K|$ is similar to the decomposition $\{A_1+B_1, A_2+B_2, \ldots, A_k+B_k\}$ of $A+B$ and there exists a homeomorphism h mapping $A+B$ onto a subset of $|K|$ in such a manner that

$$(19) \qquad h(A_i+B_i) = h(A+B) \cdot \Delta_i \quad \text{for every } i=1,2,\ldots,k.$$

The complex K constitutes the demanded common simplicial realization of the decomposition $\{A_1, A_2, \ldots, A_k\}$ and $\{B_1, B_2, \ldots, B_k\}$. In fact, by the similarity of the decompositions $\{A_1, A_2, \ldots, A_k\}$ and $\{A_1+B_1, A_2+B_2, \ldots, A_k+B_k\}$ it follows that the decompositions $\{A_1, A_2, \ldots, A_k\}$ of A and $\{\Delta_1, \Delta_2, \ldots, \Delta_k\}$ of $|K|$ are similar. The homeomorphism h, considered only in A maps A onto a subset of $|K|$ and from the relations (19) and $A \cdot B = 0$ we find

$$h(A_i) = h(A) \cdot \Delta_i \quad \text{for every } i=1,2,\ldots,k.$$

Hence K is a simplicial realization of the decomposition $\{A_1, A_2, \ldots, A_k\}$. In the same manner we show that K is also a simplicial realization of the decomposition $\{B_1, B_2, \ldots, B_k\}$.

7. *Lemma.* *If A and $B \subset A$ are absolute retracts, C is a closed subset of A and r is a continuous mapping of the set $C \times \langle 0,1 \rangle$ into A such that*

$$\begin{aligned} &r(x,0) = x, \quad r(x,1) \in B \quad &&\textit{for every } x \in C, \\ &r(x,1) = x &&\textit{for every } x \in B \cdot C, \end{aligned}$$

then there exists a continuous extension $r'(x,t)$ of $r(x,t)$ over the space $A \times \langle 0,1 \rangle$ with the values belonging to A such that:

$$\begin{aligned} &r'(x,0) = x \quad &&\textit{for every } x \in A, \\ &r'(x,1) &&\textit{is a retraction of } A \textit{ onto } B. \end{aligned}$$

Proof. Putting

$$\varphi(x)=x \quad \text{for every } x \in B,$$
$$\varphi(x)=r(x,1) \quad \text{for every } x \in C.$$

we obtain a continuous mapping φ of the closed subset $B+C$ of A into B. Since B is an absolute retract there exists a continuous extension φ' of φ over A with the values belonging to B.

Putting

$$(20)\qquad \begin{aligned} r''(x,0)&=x && \text{for every } x \in A,\\ r''(x,t)&=r(x,t) && \text{for every } (x,t) \in C\times\langle 0,1\rangle,\\ r''(x,1)&=\varphi'(x) && \text{for every } x \in A \end{aligned}$$

we obtain a continuous mapping of the closed subset

$$A\times(0)+C\times\langle 0,1\rangle+A\times(1)$$

of the space $A\times\langle 0,1\rangle$ into A. As A is an absolute retract there exists a continuous extension $r'(x,t)$ of $r''(x,t)$ over $A\times\langle 0,1\rangle$ with the values belonging to A. It follows, by (20) that r' is the demanded extension.

8. ***Theorem 2.*** *If K is a simplicial realization of a regular decomposition of a space A then A is homeomorphic with a deformation retract of the polytope $|K|$.*

Proof. By hypothesis $K=\{\Delta_1,\Delta_2,\dots,\Delta_k\}$ is a simplicial realization of a regular decomposition $\{A_1,A_2,\dots,A_k\}$ of A. Hence there exists a homeomorphism h mapping A into $|K|$ in such a manner that

$$(21)\qquad h(A_i)=h(A)\cdot\Delta_i \quad \text{for every } i=1,2,\dots,k.$$

It suffices to prove that the set

$$B=h(A)$$

is a deformation retract of the polytope $|K|$.

For every $m=0,1,\dots,(k-1)$ there exists $\alpha(m)=\binom{k}{m}$ of different increasing sequences $i_1,i_2,\dots,i_{m-k}$ with natural terms $\leqslant k$. Let $\pi_1,\pi_2,\dots,\pi_{\alpha(m)}$ be these sequences. If

$$\pi_\nu=(i_1,i_2,\dots,i_{m-k}),$$

then we put

$$(22)\qquad A_\nu^m=A_{i_1}\cdot A_{i_2}\cdot\ldots\cdot A_{i_{m-k}};\quad B_\nu^m=h(A_\nu^m),$$

$$(23)\qquad \Delta_\nu^m=\Delta_{i_1}\cdot\Delta_{i_2}\cdot\ldots\cdot\Delta_{i_{m-k}}.$$

The simplexes $\Delta_1^m,\Delta_2^m,\dots,\Delta_{\alpha(m)}^m$ constitute a simplicial complex K^m. We see at once that the sets $\{B_1^m,B_2^m,\dots,B_{\alpha(m)}^m\}$ constitute a regular decomposition of the set $B^m=\sum_{\nu=1}^{\alpha(m)}B_\nu^m$ similar to the decomposition $\{\Delta_1^m,\Delta_2^m,\dots,\Delta_{\alpha(m)}^m\}$ of the polytope $|K^m|$. Furthermore, by (21), (22) and (23) we infer that

$$(24)\qquad B_\nu^m=B^m\cdot\Delta_\nu^m \quad \text{for every } \nu=1,2,\dots,\alpha(m).$$

Hence K^m constitutes a simplicial realization of the regular decomposition $\{B_1^m,B_2^m,\dots,B_{\alpha(m)}^m\}$ of B^m.

We shall now proceede by induction showing for $m=0,1,\dots,k-1$ that there exists a continuous mapping $r^m(x,t)$ of the set $|K^m|\times\langle 0,1\rangle$ into $|K^m|$ such that

$$(25^m)\qquad \begin{aligned} &r^m(x,0)=x && \text{for every } x \in K^m,\\ &r^m(x,1) && \text{is a retraction of } |K^m| \text{ onto } B^m,\\ &r^m(x,t)\in\Delta_\nu^m && \text{for every } x\in\Delta_\nu^m \text{ and } 0\leqslant t\leqslant 1. \end{aligned}$$

In the case $m=0$ the set B^m is identical with the set $B^0=h(A_1\cdot A_2\cdot\ldots\cdot A_k)$ and the complex K^m is constituted by one simplex $\Delta^0=\Delta_1\cdot\Delta_2\cdot\ldots\cdot\Delta_k$. If $B^0=0$ then also $\Delta^0=0$ and our statement is evident. If however $B^0\neq 0$ then B^0 is an absolute retract lying in the simplex Δ^0. By the lemma **7** (where we put $C=0$, $B=B^0$ and $A=\Delta^0$) we infer that there exists a mapping $r^0(x,t)$ satisfying the condition (25^0).

Now let us assume that for an $m<k-1$ there exists a continuous mapping $r^m(x,t)$ of $|K^m|\times\langle 0,1\rangle$ into $|K^m|$ satisfying the condition (25^m). Now let us consider two not empty different simplexes Δ_μ^{m+1} and Δ_ν^{m+1} of the complex K^{m+1}. It is clear that there exists two simplexes $\Delta_{\mu'}^m$ and $\Delta_{\nu'}^m$ of the complex K^m such that

$$\Delta_\mu^{m+1}\cdot\Delta_\nu^{m+1}=\Delta_{\mu'}^m\cdot\Delta_{\nu'}^m.$$

Hence the mapping $r^m(x,t)$ is defined for every $x\in\Delta_\mu^{m+1}\cdot\Delta_\nu^{m+1}$ and $0\leqslant t\leqslant 1$ and, by (25^m) its values belong to the set $\Delta_{\mu'}^m\cdot\Delta_{\nu'}^m=\Delta_\mu^{m+1}\cdot\Delta_\nu^{m+1}$. Furthermore the set

$$B_\mu^{m+1}=B^{m+1}\cdot\Delta_\mu^{m+1}$$

is an absolute retract. Applying the lemma **7** we infer that the mapping $r^m(x,t)$ may be extended over the set $\Delta_\mu^{m+1}\times\langle 0,1\rangle$ in such a manner that its values belong to Δ_μ^{m+1} and that it constitutes a deformation retraction of Δ_μ^{m+1} onto the B_μ^{m+1}. Extending $r^m(x,t)$ in such a manner over every set Δ_μ^{m+1}, $\mu=1,2,\dots,a(m+1)$, we obtain a mapping $r^{m+1}(x,t)$ satisfying the condition (25^{m+1}).

Thus we see that a mapping $r^m(x,t)$ satisfying the condition (25^m) can be constructed for every $m=0,1,\dots,k-1$. In particular the mapping $r^{k-1}(x,t)$ constitutes a deformation retraction of the polytope $|K^{k-1}|=|K|$ onto the set $B^{k-1}=B$. Thus our theorem is established.

9. Corollary 1. *If A and B are finite dimensional compacta having similar regular decompositions then there exists a polytope P containing two deformation retracts homeomorphic respectively with A and B.*

Proof. By the corollary of Nr. **6** there exists a simplicial complex K being common simplicial realization of given similar regular decompositions of A and B. By the last theorem A and B are homeomorphic with some deformation retracts of the polytope $|K|$.

Following Hurewicz two spaces X and Y are said to be of the same *homotopy type* provided there exist two mappings $f\in X^Y$ and $g\in Y^X$ such that there exists a continuous mapping $\varphi(x,t)$ of $X\times\langle 0,1\rangle$ into X and a continuous mapping $\psi(y,t)$ of $Y\times\langle 0,1\rangle$ into Y such that

$$(26)\qquad \begin{aligned}&\varphi(x,0)=x \quad\text{and}\quad \varphi(x,1)=fg(x) \quad\text{for every } x\in X\\ &\psi(y,0)=y \quad\text{and}\quad \psi(y,1)=gf(y) \quad\text{for every } y\in Y.\end{aligned}$$

It is known [5]) that two absolute neighborhood retracts of the same homotopy type have isomorphic homology and homotopy groups.

We assert that *if Y is a deformation retract of a compactum X then X and Y are of the same homotopy type.*

In fact, if Y is a deformation retract of X then there exists a continuous mapping $\varphi(x,t)$ of $X\times\langle 0,1\rangle$ into X such that

$$\varphi(x,0)=x \quad\text{for every } x\in X$$

[5]) See W. Hurewicz, *Beiträge zur Topologie der Deformationen III, Klassen und Homologietypen von Abbildungen.* Proceedings Akademie te Amsterdam **39** (1936), p. 125.

and that the mapping

$$g(x)=\varphi(x,1)$$

is a retraction of X onto Y. If we put

$$f(y)=y \quad\text{for every } y\in Y$$

and

$$\psi(y,t)=y \quad\text{for every } (y,t)\in Y\times\langle 0,1\rangle,$$

we see at once that

$$f\in X^Y,\quad g\in Y^X$$

$$fg(x)=g(x)=\varphi(x,1) \quad\text{for every } x\in X$$

and

$$gf(y)=y=\psi(y,1) \qquad\text{for every } y\in Y.$$

Hence the conditions (26) are fulfilled, that is X and Y have the same homotopy type.

It enables us, with reference to the corollary 1, to formulate the following

Corollary 2. *Finite dimensional spaces admitting similar regular decompositions have necessarily the same homotopy type.*

It follows that all finite dimensional spaces admitting similar regular decompositions have isomorphic homology and homotopy groups. Hence the topological structure of finite dimensional spaces admitting regular decompositions is in high degree determined by the combinatorial scheme of their regular decompositions.

Furthermore it follows from our proof that for every finite dimensional space with a regular decomposition there exists a polytope having the same homotopy type. Let us observe now that the notion of nerve enables us to construct such a polytope.

Let $\mathfrak{A}=\{A_1,A_2,\dots,A_k\}$ be regular decomposition of a finite-dimensional space A such, that $A_i\neq 0$ for every $i=1,2,\dots,k$. Let K denote the simplicial complex constituted by all simplexes spanned by vertices $d^{i_1},d^{i_2},\dots,d^{i_m}$ belonging to the sequence (1) with $1\leqslant i_\nu\leqslant k$ for every $\nu=1,2,\dots,m$ and $A_{i_1}\cdot A_{i_2}\cdot\ldots\cdot A_{i_m}\neq 0$. It is clear that K is a geometric realization of the nerve of $\mathfrak{A}$. Let K' denote the barycentric subdivision [6]) of K. If we denote by B_i the sum of all simplexes of K' containing the vertex d^i then we obtain a regular decompo-

[6]) *Barycentric subdivision* of K is the simplicial subdivision of K whose vertices are the vertices of K and in addition the barycenters of all cells of K.

sition $\mathfrak{B}=\{B_1,B_2,...,B_k\}$ of the polytope $|K|$. This decomposition is similar to the decomposition $\mathfrak{A}$ of the space A, because for every system $i_1,i_2,...,i_m$ of indices the relation $B_{i_1}\cdot B_{i_2}\cdot...\cdot B_{i_m}\neq 0$ holds if and only if all vertices $a^{i_1},a^{i_2},...,a^{i_m}$ belong to one of the simplexes of K [7]), that is if $A_{i_1}\cdot A_{i_2}\cdot...\cdot A_{i_m}\neq 0$.

By corollary 2 we infer that the space A and the polytope $|K|$ have the same homotopy groups.

Thus we have the following

Corollary 3. *If the simplicial complex K is a geometric realization of the nerve of a regular decomposition of a finite dimensional space A then the space A and the polytope $|K|$ have the same homotopy type.*

Problem. *Remain the statements of the corollaries* 1, 2 *and* 3 *true if we omit the hypothesis of the finite dimension?*

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[7]) See, for instance, P. Alexandroff and H. Hopf, *Topologie I* Berlin, Springer 1935, p. 148.

Sur la méthode de généralisation de Laurent Schwartz et sur la convergence faible.

Par

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Nous montrons dans la note présente que la méthode de Laurent Schwartz, employée dans son travail „*Généralisation de la notion de fonction...*" [1]), peut être appliquée à des espaces abstraits beaucoup plus généraux que celui des fonctions. Elle ouvre ainsi la voie à des nouvelles applications bien différentes.

Nous nous appuyons dans nos considérations ci-dessous sur la notion de convergence faible.

1. Soient donnés trois ensembles quelconques F,Φ,C. On définit une „composition" qui fait correspondre à chaque couple d'éléments f,φ ($f\in F$, $\varphi\in\Phi$) un élément c de C: $f\varphi=c$. On suppose que l'ensemble Φ est „total" par rapport à cette composition, c'est-à-dire que la relation „$f\varphi=g\varphi$ pour tout $\varphi\in\Phi$" entraîne $f=g$.

On définit ensuite dans C une convergence quelconque qui fait correspondre univoquement à certaines suites $c_n\in C$ des éléments c de C: $\lim c_n=c$. [On suppose toujours que si $c_n=c_0$ pour $n=1,2,...$, alors $\lim c_n=c_0$].

On dira qu'une suite $f_n\in F$ converge faiblement vers f: $\widetilde{\lim} f_n=f$, lorsque $\lim f_n\varphi=f\varphi$ pour tout $\varphi\in\Phi$.

Il se peut que la suite f_n étant donnée, les suites $f_n\varphi$ convergent dans C pour tout $\varphi\in\Phi$, mais qu'il n'existe pas d'élément $f\in F$, tel que $\lim f_n\varphi=f\varphi$. Désignons, dans ce cas, par $\bar{f}$ l'ensemble de toutes les suites $f'_n\in F$, telles que $\lim f'_n\varphi=\lim f_n\varphi$ (pour tout $\varphi\in\Phi$). Nous

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[1]) *Généralisation de la notion de fonction, de dérivation, de transformation de Fourier et applications mathématiques et physiques*, Annales de l'Université de Grenoble, **21** (1945). Un nouvel article sur le même sujet a paru tout récemment: L. Schwartz, *Généralisation de la notion de fonction et de dérivation, Théorie des distributions,* Annales des Télécommunications, T. 3, N° 4, 1948, p. 135-140.