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Addendum to "A note on the MacDowell–Specker theorem"

by

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In my paper, "A note on the MacDowell–Specker theorem", Fundamenta Mathematicae 127 (1987), 163–170, there is a serious gap in one of the arguments. This was pointed out by K. Skandalis to C. Dimitracopoulos, during a series of talks given by Dimitracopoulos on end extensions at the University of Crete. In what follows, all notation and definitions can be found in the FM127 paper.

This paper attempts to remove a cardinality assumption of a well-known theorem of L. Kirby and J. Paris. Their theorem states that for $n \geq 2$, a countable model M satisfies $B\Sigma_n$ if and only if M has a proper n-elementary end extension $K \models I\Delta_0$. Clearly, if M is of any cardinality and has a proper n-elementary end extension $K \models I\Delta_0$, then $M \models B\Sigma_n$. For the base case $n = 2$ of the converse, I attempted to construct a Σ_1-complete Σ_1-ultrafilter under the assumption of $B\Sigma_2$. Claim 1 on page 168 states that

$$M \models \forall m > n\ \exists k > m\ (\sigma_m|n \neq \sigma_k|n)$$

where σ_m is the leftmost node of level m (i.e. the length $\mathrm{lh}(\sigma_m) = m$) in the Σ_1 definable tree

$$T = \{\sigma \in 2^{<M} : \text{there exist } \mathrm{lh}(\sigma) \text{ many } x \text{ in } \bigcap\{A_i : i < \mathrm{lh}(\sigma) \wedge \sigma(i) = 0\}\}.$$

Here A_i is the ith Σ_1 definable subset of M (with parameters). Assertion (1) and (2) of Claim 1 are correct, as well as the statement that

$$h : m \mapsto \text{least } k > m[\sigma_m|n \neq \sigma_k|n]$$

is total in M. It was then claimed that using $I\Delta_2$ (which is equivalent to $B\Sigma_2$), there is a sequence $\langle m_0, \ldots, m_{2^n+1}\rangle$ such that $m_i = h^{(i)}(m)$, the i-fold iteration of h on m. This last claim is doubtful, since it appears that $I\Sigma_2$, rather than $I\Delta_2$, is required to establish the existence of this sequence.

Since I cannot bridge the gap of the argument, I formally retract the claimed theorem. What does follow from the techniques of the paper is that for $n \geq 2$, if $M \models I\Sigma_n$ is of any cardinality, then M admits an n-elementary end extension $K \models I\Delta_0$. After becoming aware of the gap in my original argument, as outlined above, C. Dimitracopoulos and C. Cornaros proved the latter statement in a different manner, using a modification of the arithmetized completeness theorem. This and other results are presented in their paper [1].

References

[1] C. Cornaros and C. Dimitracopoulos, *A note on end extensions*, typescript under revision.

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