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Correction to the paper "The Bohr compactification, modulo a metrizable subgroup"

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by

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Given a LCA group G and a closed subgroup N of its Bohr compactification $\mathrm{b}G$, let $r : G \twoheadrightarrow G^+ \subseteq \mathrm{b}G$ be the identity function (so G^+ denotes G with the topology inherited from $\mathrm{b}G$), and with $\pi : \mathrm{b}G \twoheadrightarrow \mathrm{b}G/N$ let $\phi = \pi \circ r$. The principal positive contribution of our paper [1] is Theorem 2.10, which asserts that every LCA group G *strongly respects compactness* in the sense that for every subset A of G and for every closed metrizable subgroup N of $\mathrm{b}G$, the set $\phi[A]$ is compact (in $\mathrm{b}G$) if and only if $A\cdot(N\cap G)$ is compact (in G). [This result generalizes a celebrated theorem of Glicksberg [3], which is in fact the case $N = \{1_G\}$.]

Crucial to our proof of Theorem 2.10 is Lemma 2.9, which is this special case: G is discrete.

We are indebted to Jorge Galindo and Salvador Hernández of Universitat Jaume I (Castellón, Spain) for informing us that the proof given in [1] of Lemma 2.9 is misleading and perhaps incorrect. Accordingly, we herewith propose the following modification of the proof of that Lemma.

Let $A \subseteq G$ with $\phi[A]$ compact. Since $G \cap N$ is compact (cf. [1](2.5)) it is enough to show that A itself is compact, i.e., is finite. Suppose instead that $|A| \geq \omega$, let G' be the subgroup of G generated by some countably infinite subset A' of A, and define $G_0 = G \cap (\mathrm{b}G' \cdot N)$ and $A_0 = A \cap G_0$; evidently from $A_0 \supseteq A'$ we have $|A_0| \geq \omega$. From $\mathrm{b}G_0 = \overline{G_0}^{\mathrm{b}G} \subseteq \mathrm{b}G' \cdot N$ and

$$\mathrm{b}(G_0/G') = \mathrm{b}G_0/\mathrm{b}G' \subseteq (\mathrm{b}G' \cdot N)/\mathrm{b}G' = N/(N \cap \mathrm{b}G') \quad \text{(cf. [4](5.33))}$$

it follows that $\mathrm{b}(G_0/G')$ is metrizable, so that G_0/G' is finite; hence G_0 is countably infinite. Now let $N_0 = N \cap \mathrm{b}G_0$ and define $r_0 : G_0 \twoheadrightarrow G_0^+ \subseteq \mathrm{b}G_0$, $\pi_0 : \mathrm{b}G_0 \twoheadrightarrow \mathrm{b}G_0/N_0$, and $\phi_0 = \pi_0 \circ r_0$ as usual. We claim that the homomorphism $f : \mathrm{b}G_0/N_0 \twoheadrightarrow \pi[\mathrm{b}G_0]$ given by $f(gN_0) = gN$, which (by [4](5.31 & 5.33)) is an isomorphism and a homeomorphism, carries $\phi_0[A_0]$ onto $\phi[A] \cap \pi[\mathrm{b}G_0]$. Indeed, given $a \in A$ and $g \in \mathrm{b}G_0$ such that $\phi(a) = aN = gN = \pi(g)$, choose $n \in N$ so that $a = gn$ and then choose $g' \in \mathrm{b}G'$ and $n' \in N$ so that $g \in \mathrm{b}G_0 \subseteq \mathrm{b}G' \cdot N$ satisfies $g = g'n'$; then $a = gn = g'n'n \in \mathrm{b}G' \cdot N$ and hence

$$a \in A \cap (\mathrm{b}G' \cdot N) = (A \cap G) \cap (\mathrm{b}G' \cdot N) = A \cap (G \cap (\mathrm{b}G' \cdot N)) = A \cap G_0 = A_0,$$

as claimed. The proof then concludes as in [1]: $\phi_0[A_0]$ is compact in $\mathrm{b}G_0/N_0$, and since G_0 strongly respects compactness ([1](2.6)) the set $A_0 \cdot (N_0 \cap G_0)$ is compact in G_0 (hence finite) and we have the contradiction $\omega \le |A_0| \le |A_0 \cdot (N_0 \cap G_0)| < \omega$.

Remark. Substantially generalizing our result [1](2.10), the authors of [2] have shown that many other maximally almost periodic (not necessarily locally compact) Abelian groups also strongly respect compactness.

References

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