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On composants of solenoids

by

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Abstract. It is proved that any two composants of any two solenoids are homeomorphic.

1. Introduction. Solenoids were introduced by van Dantzig [5]. His original description was the following. Let $P = (p_1, p_2, \ldots)$ be a sequence of primes. The solenoid S_P is the intersection of a descending sequence of solid tori $T_1 \supset T_2 \supset T_3 \supset \ldots$ such that T_{i+1} is wrapped around inside T_i longitudinally p_i times without folding back. Van Heemert proved that solenoids are indecomposable continua [8].

There exists a classification theorem for solenoids, conjectured by Bing [3] and proved by McCord [9], giving necessary and sufficient conditions for two solenoids S_P and S_Q to be homeomorphic (see also [2]).

The composants of solenoids coincide with the arc components. Since solenoids are topological groups, any two composants of the same solenoid are homeomorphic. The main theorem of this paper is

THEOREM 1. *Any two composants of any two solenoids are homeomorphic.*

The composants of solenoids are examples of orbits in dynamical systems that are not locally compact. A locally compact orbit is either a singleton, a simple closed curve, or a topological copy of the real line. For orbits which are not locally compact the situation is much more complicated. Fokkink has proved the existence of uncountably many of them [6, 7].

In [4] Bandt shows that any two composants of the bucket handle are homeomorphic. Following a suggestion of Fokkink, we shall adapt the ideas from that article to prove the result of this paper.

For the proof, we need a different description of solenoids. We define the cascade (C_P, σ) as follows. C_P is the Cantor set represented as the

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topological product $C_P = \prod_{i=1}^{\infty} \overline{p}_i$ of discrete spaces $\overline{p}_i = \{0, 1, \ldots, p_i - 1\}$. The homeomorphism $\sigma : C_P \to C_P$ has the form

$$\sigma(x_1, x_2, \ldots) = (x_1 + 1, x_2, x_3, \ldots) \quad \text{if } x_1 < p_1 - 1,$$
$$\sigma(p_1 - 1, \ldots, p_{k-1} - 1, x_k, \ldots) = (0, 0, \ldots, 0, x_k + 1, \ldots) \quad \text{if } x_k < p_k - 1,$$
$$\sigma(p_1 - 1, p_2 - 1, \ldots) = (0, 0, \ldots).$$

Now we define the solenoid S_P to be the suspension $\Sigma(C_P, \sigma)$, obtained from the product $C_P \times [0,1]$ by identifying each $(x, 1)$ with $(\sigma(x), 0)$.

Since C_P is the topological group of P-adic integers and the map σ corresponds to the addition of $(1, 0, 0, \ldots)$, it is easy to see that the composant of S_P containing the point zero can be represented as a suspension $\Sigma(\mathbb{Z}, \tau)$, where $\mathbb{Z}$ is the set of integers with the P-adic topology and $\tau : \mathbb{Z} \to \mathbb{Z}$ is defined by $\tau(x) = x + 1$.

In proving the theorem, we may clearly confine ourselves to the case of the composants of the zero point of the solenoids S_P and S_Q, where $P = (2, 2, \ldots)$ and $Q = (q_1, q_2, \ldots)$ is an arbitrary sequence of primes. We now describe these composants.

Let I denote the set of integers with the 2-adic topology. Then a local basis for $x \in I$ is given by $\{x + U_n\}_{n \geq 0}$, where $U_n = 2^n \mathbb{Z}$. Now $\Sigma(I, \tau)$ (with $\tau(x) = x + 1$) represents the composant of the zero point of the 2-solenoid $S_{(2,2,\ldots)}$. Similarly, we let $\Sigma(J, \tau)$ represent the composant of the zero point of the solenoid S_Q by setting $J = \mathbb{Z}$ and taking $\{x + V_m\}_{m \geq 0}$ with $V_m = q_1 \ldots q_m \mathbb{Z}$ as a local basis for $x \in J$.

For a clopen subset A of I, we define the return map $\tau_A : A \to A$ by $\tau_A(x) = \min\{y \in A : y > x\}$. If there exist clopen subsets A and B of I and J, respectively, and a homeomorphism $f : A \to B$ such that $f \circ \tau_A = \tau_B \circ f$, we say that (I, τ) and (J, τ) are *first return equivalent*. From Theorem 5.2 in [1] it follows that (I, τ) and (J, τ) are first return equivalent if and only if $\Sigma(I, \tau)$ and $\Sigma(J, \tau)$ are homeomorphic.

2. Composants of solenoids. We now define some subsets of I and J. These will be the building blocks in the construction of the sets A and B. First, choose sequences of integers $0 = n_0 < n_1 < n_2 < \ldots$ and $0 = m_0 < m_1 < m_2 < \ldots$ We define blocks $\{I^k\}_{k \geq 0}$ inductively. We set $I^0 = \{0\}$ and

$$I^{k+1} = \bigcup \{I^k + v 2^{n_k} : -2^{n_{k+1} - n_k - 1} \leq v \leq 2^{n_{k+1} - n_k - 1} - 1\}.$$

Note that $I^0 \subset I^1 \subset I^2 \subset \ldots$, that $\bigcup I^k = I$ and that I^k consists of $2^{n_k - n_l}$ consecutive copies of I^l $(0 \leq l \leq k)$. Next, for $v \in \mathbb{Z}$ we set $I^k_v = I^k + v 2^{n_k}$. Analogously, we define $J^0 = \{0\}$, let J^{k+1} denote

$$\bigcup \Big\{ J^k + wq_1 \ldots q_{m_k} : \\ \Big[-\frac{q_{m_k+1} \ldots q_{m_{k+1}} - 1}{2} \Big] \le w \le \Big[\frac{q_{m_k+1} \ldots q_{m_{k+1}} - 1}{2} \Big] \Big\}$$

and set $J^k_w = J^k + wq_1 \ldots q_{m_{k+1}}$. We also define translations $\varphi^k_v : I^k \to I^k_v$ and $\psi^k_w : J^k \to J^k_w$ by $\varphi^k_v(x) = x + v2^{n_k}$ and $\psi^k_w(x) = x + wq_1 \ldots q_{m_k}$.

The blocks I^k and J^k will be called *central* blocks. Also, blocks I^k_v and J^k_w will be referred to as k-blocks. The special k-blocks of the form $I^k + v'2^{n_{k+1}}$ and $J^k + w'q_1 \ldots q_{m_k}$ will be called *k-return* blocks. At places where we want to make clear that we are dealing with return blocks, we shall use the indices r and s (so that e.g. r is assumed to be a multiple of $2^{n_{k+1}-n_k}$). Note that each $(k+1)$-block contains exactly one k-return block, which is positioned in the middle of the $(k+1)$-block. By an *interval* of k-blocks we shall mean a union of consecutive k-blocks.

The following lemma shows how we should piece the blocks together to form the sets A and B.

Lemma 1. *Suppose we have non-empty subsets $A \subset I$, $B \subset J$ and a bijection $f : A \to B$ such that*

(i) *f is strictly increasing*,

(ii) *for $k \ge 0$, f maps $I^k \cap A$ onto $J^k \cap B$*,

(iii) *for even $k \ge 0$ and each k-return block I^k_r, $I^k_r \cap A = \varphi^k_r(I^k \cap A)$ and there exists w such that $f(\varphi^k_r(x)) = \psi^k_w(f(x))$ for $x \in I^k \cap A$*,

(iv) *for odd $k \ge 0$ and each k-return block J^k_s, $J^k_s \cap B = \psi^k_s(J^k \cap B)$ and there exists v such that $f(\varphi^k_v(x)) = \psi^k_s(f(x))$ for $x \in I^k \cap A$.*

Then the sets A, B are clopen, f is a homeomorphism and $f \circ \tau_A = \tau_B \circ f$.

Proof. First we show that A is clopen. Let $x \in I$ and choose an even integer k with $x \in I^k$. Let $v \in \mathbb{Z}$. Since $I^k + v2^{n_{k+1}}$ is a k-return block, $I^k_v \cap A = \varphi^k_v(I^k \cap A)$. Therefore $x + v2^{n_{k+1}} \in A$ iff $x \in A$. Hence either $x + U_{n_{k+1}} \subset A$ or $x + U_{n_{k+1}} \subset I \setminus A$, so that A is clopen. In the same way it follows that B is clopen.

Next we show that f is continuous. Let $x \in A$ and choose k with $x \in I^k$. Let V be a neighborhood of $f(x)$. There is $l > k$ with l even and $f(x) + V_{m_l} \subset V$. Since $I^l + v2^{n_{l+1}}$ is an l-return block, for some l-block J^l_w we have $f(x+v2^{n_{l+1}}) = f(x)+wq_1 \ldots q_{m_l} \in f(x)+V_{m_l}$. So $f(x+U_{n_{l+1}}) \subset V$ and hence f is continuous. The continuity of f^{-1} is proved in the same way. Finally, we claim that $f \circ \tau_A = \tau_B \circ f$. Let $x \in A$. Since $\tau_A(x) > x$, it follows from (i) that $f(\tau_A(x)) > f(x)$. Hence $f(\tau_A(x)) \ge \tau_B(f(x))$. Since f^{-1} is also increasing, the same reasoning gives $f^{-1}(\tau_B(x)) \ge \tau_A(f^{-1}(x))$ for $x \in B$. Therefore $\tau_B(f(x)) \ge f(\tau_A(x))$ for $x \in A$. ∎

In the next section, sets A and B and a bijection $f : A \to B$ will be constructed satisfying the conditions of this lemma.

3. Construction of f. We shall now describe our construction of f. In what follows we tacitly assume that all choices are made in such a way that f remains order-preserving. In order to facilitate the construction, we introduce some terminology.

To indicate which points of I are to be included in A, we introduce the special symbol $\perp$. For all $x \in I$ that are not to be included in A, we shall set $f(x) = \perp$. So $A = \{x \in I : f(x) \neq \perp\}$ and $B = f(A)$. We let $\psi_w^k(\perp) = \perp$. In this notation, the condition $I_v^k \cap A = \varphi_v^k(I^k \cap A)$ and $f(\varphi_v^k(x)) = \psi_w^k(f(x))$ for $x \in I^k \cap A$ is equivalent to $f(\varphi_v^k(x)) = \psi_w^k(f(x))$ for all $x \in I^k$.

If f has been defined on k-blocks I_v^k and $I_{v'}^k$ and there exist k-blocks J_w^k and $J_{w'}^k$ such that $(\psi_w^k)^{-1}(f(\varphi_v^k(x))) = (\psi_{w'}^k)^{-1}(f(\varphi_{v'}^k(x)))$ for $x \in I^k$, then we say that I_v^k *is a copy of* $I_{v'}^k$. Note that condition (iii) now says that for even $k \geq 0$ each k-return block in I is a copy of I^k. We use the same terminology for blocks in J. For the construction it is important to note that a copy of a copy is again a copy.

Note that there is a symmetry in the conditions of Lemma 1. To emphasize this symmetry, we view f as a relation between $I \cup \{\perp\}$ and $J \cup \{\perp\}$. In that way we can use the symmetry in defining f. So instead of $f(x) = y$ we write xfy. We also set $\hat{I}^l = I^l \cup \{\perp\}$ and $\widehat{J}^l = J^l \cup \{\perp\}$.

In order to satisfy condition (ii), we define f on $\widehat{I}^l \times \widehat{J}^l$ by induction on l. To define f on I^0, we set $0f0$. In making the inductive step, we have to take care that the conditions (iii) and (iv) are satisfied. To make clear how we proceed and to indicate what difficulties arise, we shall now describe the extension of f from $\widehat{I}^3 \times \widehat{J}^3$ to $\widehat{I}^4 \times \widehat{J}^4$.

First we consider the return blocks inside I^4 and J^4. Since the only 3-return block in J^4 is the central block J^3, and f has already been defined on this block, we do not have to worry about condition (iv) for $k = 3$.

We take care of condition (iii) for $k = 2$. The block I^4 is built up out of 3-blocks, each containing a 2-return block. We have already defined f on the 2-return block inside I^3. For the 2-return blocks in $I^4 \setminus I^3$ we choose 2-blocks in $J^4 \setminus J^3$ in an almost linear fashion (this will be explained later). We copy the definition of f on $\widehat{I}^2 \times \widehat{J}^2$ to each of these blocks.

Next, we deal with (iv) for $k = 1$. The complement of J^3 and the chosen 2-blocks in J^4 is a union of intervals of 2-blocks. To each of these intervals corresponds an interval in the remaining part of I^4. For each of the 1-return blocks in such an interval we choose a 1-block in the corresponding interval, again per interval in an almost linear fashion.

Finally, we take care of (iii) for $k = 0$. Now we have intervals of 1-blocks and each 1-block has its 0-return block. In the corresponding intervals in J^4

we now choose 0-blocks again in an almost linear fashion. The points $x \in I^4$ ($x \in J^4$ respectively) on which f has not yet been defined are taken care of by setting $xf\perp$ ($\perp fx$ respectively).

At three stages in this construction, blocks have to be selected. It is conceivable that at some stage there will not be enough room to select the destination blocks. We shall later show how to avoid this.

We now give a general procedure to define f on subsets of $I \times J$. We assume that we have already defined f on $\widehat{I}^l \times \widehat{J}^l$. If $\widetilde{I}$ is an interval of $(k+1)$-blocks inside I^{l+1} not containing a $(k+1)$-return block and $\widetilde{J}$ is a similar subset of J^{l+1}, we can use the following recursive procedure to define f on $\widetilde{I}$ and $\widetilde{J}$:

- If k is even (odd), then for each k-return block in $\widetilde{I}$ (in $\widetilde{J}$), we select a k-block in $\widetilde{J}$ (in $\widetilde{I}$).
- We define f on these blocks by copying the central blocks I^k and J^k.
- Let $\widetilde{I}_0, \ldots, \widetilde{I}_c$ and $\widetilde{J}_0, \ldots, \widetilde{J}_c$ denote the intervals of k-blocks that remain. If $k \geq 1$, we use the same procedure to define f on each of these blocks. Otherwise, we set $xf\perp$ for $x \in \bigcup \widetilde{I}_i$ and $\perp fx$ for $x \in \bigcup \widetilde{J}_i$.

Of course, we still have to show that this procedure does indeed work. To define f on $\widehat{I}^{l+1} \times \widehat{J}^{l+1}$, we apply this procedure with $k = l-1$ to each of the two components of $I^{l+1} \setminus I^l$ and the corresponding components of $J^{l+1} \setminus J^l$.

We shall now show how we can make the procedure above work. We have to make sure that there will always be enough room to select the destination blocks. It will turn out that we need to choose the sequences $\{n_k\}_k$ and $\{m_k\}_k$ in such a way that for all k, $n_{k+1} - n_k \geq k+22$, $m_{k+1} - m_k \geq k+22$ and $1/2 < 2^{n_k}/(q_1 \ldots q_{m_k}) < 2$. It is easily seen that this is possible by using induction on k and first choosing m_k and then n_k. Let $P_k = 2^{n_{k+1} - n_k}$ and $Q_k = q_{m_k+1} \cdots q_{m_{k+1}}$. Then a $(k+1)$-block in I (in J respectively) contains P_k (Q_k respectively) k-blocks. Note that $P_k, Q_k \geq 2^{k+22}$ and that for $j \leq k$, $1/4 < (P_j \ldots P_k)/(Q_j \ldots Q_k) < 4$. The following lemma shows that, given intervals $\widetilde{I}$ and $\widetilde{J}$ of $(k+1)$-blocks satisfying certain conditions, we can define f on all k-return blocks inside $\widetilde{I}$ (in case k is even). It also provides information on the blocks we are left with. For blocks in J mapped to blocks in I the same assertion is true and has a similar proof.

LEMMA 2. *Suppose $\widetilde{I} \subset I$ and $\widetilde{J} \subset J$ are intervals of c, respectively d, $(k+1)$-blocks, where $1/\alpha \leq c/d \leq \beta$ and $\alpha, \beta \leq 16$. If $I^k_{p_1}, \ldots, I^k_{p_c}$ are the k-return blocks in $\widetilde{I}$, we can find c many k-blocks $J^k_{w_1}, \ldots, J^k_{w_c}$ in $\widetilde{J}$, such that after removal of these blocks we end up with non-empty intervals of*

k-blocks $\widetilde{I}_0, \ldots, \widetilde{I}_c$ and $\widetilde{J}_0, \ldots, \widetilde{J}_c$ with the following property: if $\widetilde{I}_i$ contains c_i and $\widetilde{J}_i$ contains d_i k-blocks, then

$$\frac{1}{\alpha + \frac{1}{4} \cdot 2^{-(k+1)}} \frac{P_k}{Q_k} \le \frac{c_i}{d_i} \le \left(\beta + \frac{1}{4} \cdot 2^{-(k+1)}\right) \frac{P_k}{Q_k}.$$

Proof. The set $\widetilde{I}$ contains cP_k many k-blocks that we number from left to right with the integers $1, \ldots, cP_k$. We do the same with the dQ_k many k-blocks in $\widetilde{J}$.

Let $a_1, \ldots, a_c$ denote the ordinals of the return blocks in $\widetilde{I}$. We set $a_0 = 0$ and $a_{c+1} = cP_k + 1$. Since any k-return block is in the middle of a $(k+1)$-block, we know that $a_{i+1} - a_i = P_k$ for $1 \le i \le c-1$, that $a_1 - a_0 \ge (P_k - 1)/2$ and that $a_{c+1} - a_c \ge (P_k - 1)/2$. Therefore, $a_{i+1} - a_i \ge P_k/4$ for $0 \le i \le c$. For $1 \le i \le c$ we choose the block $J^k_{w_i}$ to be the block with ordinal $b_i = [(dQ_k/cP_k)a_i + 1/2]$. We can write $b_i = (dQ_k/cP_k)a_i + \varepsilon_i$ with $|\varepsilon_i| \le 1/2$. Let $b_0 = 0$ and $b_{c+1} = dQ_k + 1$. We have to show that the blocks we have chosen are distinct and satisfy the requirements. To do this, we fix an arbitrary i and let $c_i = a_{i+1} - a_i - 1$ and $d_i = b_{i+1} - b_i - 1$. Clearly, c_i equals the number of k-blocks in $\widetilde{I}_i$ and d_i the number of k-blocks in $\widetilde{J}_i$. It suffices to show that $\left(\alpha + \frac{1}{4} \cdot 2^{-(k+1)}\right)^{-1} P_k/Q_k \le c_i/d_i \le \left(\beta + \frac{1}{4} \cdot 2^{-(k+1)}\right) P_k/Q_k$.

Since

$$d_i = \frac{dQ_k}{cP_k}(a_{i+1} - a_i - 1) + \frac{dQ_k}{cP_k} + \varepsilon_{i+1} - \varepsilon_i - 1$$

we have

$$\frac{d_i}{c_i} \le \frac{dQ_k}{cP_k} + \frac{dQ_k/cP_k}{P_k/4 - 1}.$$

Also,

$$c_i = \frac{cP_k}{dQ_k}(b_{i+1} - b_i - 1) + \frac{cP_k}{dQ_k}(\varepsilon_i - \varepsilon_{i+1} + 1) - 1$$

and hence

$$\frac{c_i}{d_i} \le \frac{cP_k}{dQ_k} + \frac{2(cP_k/dQ_k) - 1}{(dQ_k/cP_k)P_k/4 - 2}.$$

Using the inequalities $1/64 < cP_k/dQ_k < 64$ and $P_k \ge 2^{k+22}$, it is now easy to check that

$$\frac{1}{\alpha + \frac{1}{4} \cdot 2^{-(k+1)}} \frac{P_k}{Q_k} \le \frac{c_i}{d_i} \le \left(\beta + \frac{1}{4} \cdot 2^{-(k+1)}\right) \frac{P_k}{Q_k}. \ \blacksquare$$

If we use Lemma 2 to select the destination blocks for the return blocks, we can define f on the whole of $\widehat{I}^{l+1} \times \widehat{J}^{l+1}$. This will be demonstrated in the next lemma, which completes the inductive step in the construction of f.

LEMMA 3. *Suppose f has been defined on $\widehat{I}^l \times \widehat{J}^l$. Then f can be defined on $\widehat{I}^{l+1} \times \widehat{J}^{l+1}$.*

P r o o f. As has already been said, to define f on $\widehat{I}^{l+1} \times \widehat{J}^{l+1}$, we apply the procedure described above to each of the two components of $I^{l+1} \setminus I^l$ and the corresponding components of $J^{l+1} \setminus J^l$. We shall show, using Lemma 2, that the procedure can define f on the $(l-1)$-return blocks in $\widetilde{I}$ (or $\widetilde{J}$) and can then be applied to the remaining $(l-1)$-blocks. We assert that after repeating the procedure n times (where $0 \le n \le l$), the length ratio c/d of corresponding intervals of $(l-n)$-blocks $\widetilde{I}$ and $\widetilde{J}$, measured in $(l-n)$-blocks, satisfies the inequalities

$$\frac{1}{\alpha_n} \le \frac{c}{d} \le \beta_n,$$

where

$$\alpha_n = \Big(3 + \sum_{i=0}^{n-1} 2^{-(l-i)}\Big) \frac{Q_l Q_{l-1} \ldots Q_{l-n}}{P_l P_{l-1} \ldots P_{l-n}}$$

and

$$\beta_n = \Big(3 + \sum_{i=0}^{n-1} 2^{-(l-i)}\Big) \frac{P_l P_{l-1} \ldots P_{l-n}}{Q_l Q_{l-1} \ldots Q_{l-n}}.$$

Note that $\alpha_n, \beta_n \le 16$ for all k. So if these inequalities hold, the requirements of Lemma 2 will always be satisfied. We prove the inequalities by induction on n. It is easy to see that our bound on c/d holds for $n = 0$. Now suppose it holds for some $n \ge 0$ with $n < l$. Then we can apply Lemma 2 with $k = l - n - 1$, $\alpha = \alpha_n$ and $\beta = \beta_n$ to get a length ratio c'/d' of intervals of $(l-n-1)$-blocks with

$$\frac{1}{\alpha_n + \frac14 \cdot 2^{-(l-n)}} \cdot \frac{P_{l-n-1}}{Q_{l-n-1}} \le \frac{c'}{d'} \le \Big(\beta_n + \frac14 \cdot 2^{-(l-n)}\Big) \frac{P_{l-n-1}}{Q_{l-n-1}}.$$

Since

$$\begin{aligned}
&\Big(\alpha_n + \frac14 \cdot 2^{-(l-n)}\Big) \frac{Q_{l-n-1}}{P_{l-n-1}} \\
&\quad = \Big(\Big(3 + \sum_{i=0}^{n-1} 2^{-(l-i)}\Big) \frac{Q_l \ldots Q_{l-n}}{P_l \ldots P_{l-n}} + + \frac14 \cdot 2^{-(l-n)}\Big) \frac{Q_{l-n-1}}{P_{l-n-1}} \\
&\quad \le \Big(3 + \sum_{i=0}^{n-1} 2^{-(l-i)} + 2^{-(l-n)}\Big) \frac{Q_l \ldots Q_{l-n-1}}{P_l \ldots P_{l-n-1}} = \alpha_{n+1}
\end{aligned}$$

and similarly $\big(\beta_n + \frac14 \cdot 2^{-(l-n)}\big) P_{l-n-1}/Q_{l-n-1} \le \beta_{n+1}$, we get

$$\frac{1}{\alpha_{n+1}} \le \frac{c'}{d'} \le \beta_{n+1}.$$

This proves the inequalities. We conclude that f can be constructed on $\widehat{I}^{l+1} \times \widehat{J}^{l+1}$. ∎

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References

[1] J. M. Aarts, *The structure of orbits in dynamical systems*, Fund. Math. 129 (1988), 39–58.

[2] J. M. Aarts and R. J. Fokkink, *The classification of solenoids*, Proc. Amer. Math. Soc. 111 (1991), 1161–1163.

[3] R. H. Bing, *A simple closed curve is the only homogeneous bounded plane continuum that contains an arc*, Canad. J. Math. 12 (1960), 209–230.

[4] C. Bandt, *Composants of the horseshoe*, Fund. Math. 144 (1994), 231–241.

[5] D. van Dantzig, *Ueber topologisch homogene Kontinua*, ibid. 15 (1930), 102–125.

[6] R. J. Fokkink, *The structure of trajectories*, PhD thesis, Delft, 1991.

[7] —, *There exist uncountably many orbits in flows*, Fund. Math. 136 (1990), 147–156.

[8] A. van Heemert, *Topologische Gruppen und unzerlegbare Kontinua*, Compositio Math. 5 (1938), 319-326.

[9] M. C. McCord, *Inverse limit sequences with covering maps*, Trans. Amer. Math. Soc. 114 (1965), 197–209.

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