FUNDAMENTA
MATHEMATICAE
141 (1992)

Corrections to "On the computation of the Nielsen numbers and the converse of the Lefschetz coincidence theorem"

(Fund. Math. 140 (1992), 191–196)

by

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I thank Professor Boju Jiang for pointing out that Theorem 2.3 of [3] which is quoted as Lemma 1.2 of [1], is false in general. Consequently, without additional hypotheses, the main results in [1, §2] do not hold in the generality as stated. Let $f, g : M_1 \to M_2$ be as in [1, 2.1]. In addition, we assume that M_1, M_2 are compact, M_1 is triangulable and $\pi_1(M_1)$ is finite so that the universal cover $\widetilde{M}_1$ is also compact. By a result of Schirmer, we may assume without loss of generality that the coincidence set of f and g is given by $C_{f,g} = \{x_1, \ldots, x_k\}$ such that each x_i is a distinct coincidence class. It follows from [1, §1] that each root class of $\eta : \widetilde{M}_1 \to M_2$ must lie entirely inside the fiber $p_1^{-1}(x_i)$ over x_i for some i. Following [2, Cor. 5], the root classes of η have the same root index. Furthermore, η has exactly $|K| = |\pi_1(M_2)|$ root classes if $\deg \eta \neq 0$. It is shown in the proof of [1, 2.1] that every point of $p_1^{-1}(x_i)$ has the same root index which coincides with the coincidence index at x_i. By summing all the indices, we obtain $\deg \eta = L(f, g) \cdot |\pi_1(M_1)|$.

Case (I): If K is infinite, then $\deg \eta = 0$ and hence every x_i is inessential. Thus $N(f, g) = 0$ and f and g are deformable to be coincidence free.

Case (II): Suppose that K is finite. It follows that $\deg \eta = |K| \cdot \omega = L(f, g) \cdot |\pi_1(M_1)|$ where ω is the root index of a root class of η. If $L(f, g) = 0$ then $\deg \eta = 0$ and hence $N(f, g) = 0$. Again, f and g are deformable to be coincidence free. Let $r = |\pi_1(M_2)|/|\pi_1(M_1)|$. Now suppose that $L(f, g) \neq 0$.

1991 *Mathematics Subject Classification*: Primary 55M20.
Key words and phrases: fixed points, coincidences, roots, Lefschetz number, Nielsen number.

If $\pi_1(M_1) = 1$ then $M_1 = \widetilde{M}_1$ and the x_i's have the same coincidence (root) index. Thus, $N(f,g)$ divides $L(f,g)$. If $\omega = \pm 1$, then every point in $p_1^{-1}(x_i)$ is a root class of index ± 1. Therefore, $N(f,g) = |L(f,g)| = r$. Moreover, if $\gcd(\omega, |\pi_1(M_1)|) = 1$, it follows that every point in $p_1^{-1}(x_i)$ is a root class, which implies that the x_i's have the same coincidence index. Therefore, $N(f,g)$ divides $L(f,g)$.

We now summarize the above in the following

THEOREM A. *Let $f, g : M_1 \to M_2$ be maps between closed, connected, triangulable and orientable n-manifolds $(n \ge 3)$ such that $|\pi_1(M_1)| < \infty$ and $M_2 = \widetilde{M}_2/K$ where $\widetilde{M}_2$ is a connected simply connected topological group and K is a discrete subgroup. If K is infinite or $L(f,g) = 0$ then $N(f,g) = 0$. Hence f and g are deformable to be coincidence free. If K is finite and we let $r = |K|/|\pi_1(M_1)|$, $\omega = L(f,g)/r$ then*

(1) $\pi_1(M_1) = 1 \Rightarrow N(f,g)$ *divides* $L(f,g)$;
(2) $\omega = \pm 1 \Rightarrow N(f,g) = |L(f,g)| = r$;
(3) $\gcd(\omega, |\pi_1(M_1)|) = 1 \Rightarrow N(f,g)$ *divides* $L(f,g)$ *and* $N(f,g) = r$.

Cor. 2.2 of [1] will hold true if $|\pi_1(M_1)| < \infty$. When $M_1 = M_2$, $r = 1$. Thus, for the fixed point case, we replace 2.3 and 2.4 of [1] by the following

COROLLARY B. *Let M_1, M_2 be as in Theorem* A *and $M_1 = M_2 = M$. Let $f : M \to M$ be a map. If $L(f) = 0$ then $N(f) = 0$ and f is deformable to be fixed point free. If* (i) $L(f) = \pm 1$ *or* (ii) $\gcd(L(f), |\pi_1(M_1)|) = 1$, *then* $N(f) = 1$.

It is worthwhile to note that if M_1 is compact and M_2 is a compact Lie group, then it can be shown easily, along the lines of [1], that the coincidence classes of f and g are the root classes of φ, where $f, g : M_1 \to M_2$ and $\varphi : M_1 \to M_2$ is given by $\varphi(x) = f(x)^{-1}g(x)$. Furthermore, the coincidence index of f and g is the same as the root index of φ. Hence we have the following

THEOREM C. *Let $f, g : M_1 \to M_2$ be maps from a closed connected oriented n-manifold M_1 $(n \ge 1)$ to a compact connected Lie group M_2 of the same dimension. If the Lefschetz coincidence number $L(f,g) = 0$ then the Nielsen coincidence number $N(f,g) = 0$. Otherwise, $N(f,g) > 0$ and $N(f,g)$ divides $L(f,g)$.*

I thank the referee for some helpful suggestions and comments.

References

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Received 24 April 1992;
in revised form 22 May 1992