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The nonexistence of expansive homeomorphisms of dendroids

by

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Abstract. In this paper, we prove that no dendroid admits an expansive homeomorphism. Also, we show that no uniformly arcwise connected continuum admits an expansive homeomorphism.

0. Introduction. Let X be a compact metric space with metric d. A homeomorphism f of X is *expansive* if there exists $c > 0$ (called an expansive constant for f) such that $d(f^n(x), f^n(y)) \leqslant c$ for all integers n implies $x = y$. This property is important in the topological theory of dynamical systems.

It is well known that the Cantor set, the 2-adic solenoid and the 2-torus admit expansive homeomorphisms ([11]).

Also, Bryant, Jacobson and Utz proved that there exists no expansive homeomorphism on an arc and a circle (see [1] and [6]). By using those results, Kawamura showed that if X is a Peano continuum and X contains a free arc, then X admits no expansive homeomorphism ([7]). In [8], we showed that if X is a Peano continuum which contains a 1-dimensional open ANR then X does not admit an expansive homeomorphism. In particular, 1-dimensional compact ANRs admit no expansive homeomorphism. Also, Jacobson and Utz [6] asserted that the shift homeomorphism of the inverse limit of any continuous surjection of an arc is not an expansive homeomorphism (see [5] for a simple proof). The limit is a special type of arc-like continua and arc-like continua are tree-like. Naturally, the following problem arises: Is it true that no tree-like continuum admits an expansive homeomorphism?

The purpose of this paper is to prove that no dendroid (= arcwise connected tree-like continuum) admits an expansive homeomorphism, and no uniformly arcwise connected continuum admits an expansive homeomorphism.

1. Preliminaries. All spaces under consideration are assumed to be metric. A *continuum* is a compact connected nondegenerate space.

A continuum X is said to be *unicoherent* provided that if $X = A \cup B$ and A, B are subcontinua of X, the intersection $A \cap B$ is connected. A continuum X is *hereditarily*

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unicoherent provided that each subcontinuum of X is unicoherent. A continuum X is *tree-like* provided that for any $\varepsilon > 0$ there is a finite open cover $\mathcal{T}$ of X such that the nerve $N(\mathcal{T})$ is a polyhedral tree and mesh $\mathcal{T} < \varepsilon$. A continuum X is said to be a *dendroid* provided that X is an arcwise connected hereditarily unicoherent space. It is well known that X is a dendroid if and only if X is an arcwise connected and tree-like continuum ([4]). Also, every nondegenerate subcontinuum of a dendroid is a dendroid. A continuum X is said to be *uniformly arcwise connected* (cf. [3]) provided that it contains an arc and if for every number $\eta > 0$ there is a natural number n such that every arc A in X contains points $a_0, a_1, \ldots, a_n$ such that $A = \bigcup_{i=0}^{n-1} \langle a_i a_{i+1} \rangle$ and diam $\langle a_i a_{i+1} \rangle < \eta$ for each $i = 0, 1, \ldots, n-1$, where $\langle a_i a_{i+1} \rangle$ denotes the subarc from a_i to a_{i+1} in A. Note that every 1-dimensional compact connected ANR is uniformly arcwise connected and the 2-adic solenoid is not uniformly arcwise connected.

(1.1) EXAMPLE. In the Euclidean plane E^2, let $[a, b]$ denote the segment from a to b, where $a, b \in E^2$. Consider the following points in E^2:

$$p = (0, 0), \quad a_n = (2, 2/2^{n-1}) \quad (n = 1, 2, \ldots,),$$

$$a_n^k = (2, (n-k)/2^n n + 2/2^{n+1}), \quad b_n^k = (1, (n-k)/2^{n+1} n + 1/2^{n+1})$$

$$(k = 1, 2, \ldots, n-1), \quad q = (2, 0).$$

Let $X = \bigcup_{n=1}^{\infty} ([p, a_n] \cup [a_n, b_n^1] \cup [b_n^1, a_n^1] \cup \ldots \cup [b_n^{n-1}, a_n^{n-1}]) \cup [p, q]$ (see Figure 1). Clearly, X is a dendroid, but it is not uniformly arcwise connected.

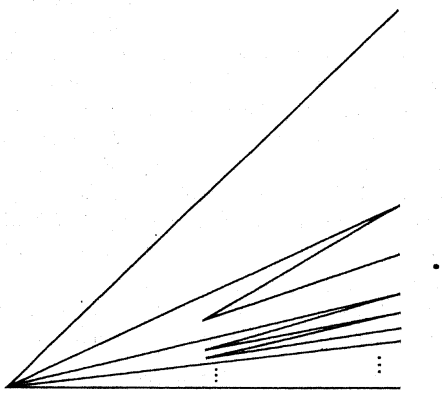

Fig. 1

2. Expansive homeomorphisms and uniformly arcwise connected continua. In this section, we show that if X is a uniformly arcwise connected continuum, then X does not admit an expansive homeomorphism. We need the following.

(2.1) (Mañé [10]). *Let f: $X \to X$ be an expansive homeomorphism of a compactum X and let $c > 0$ be an expansive constant for f and $0 < \varepsilon < c/2$. Then there exists $\delta > 0$ such that if $x, y \in X$, $d(x, y) < \delta$, and $\varepsilon \leqslant \sup\{d(f^j(x), f^j(y)) \mid 0 \leqslant j \leqslant n\} \leqslant 2\varepsilon$ for some $n \geqslant 0$, then $d(f^n(x), f^n(y)) \geqslant \delta$.*

(2.2) LEMMA. *Let f: $X \to X$ be an expansive homeomorphism of a compactum X. Then there exists $\delta > 0$ such that if A is a nondegenerate subcontinuum of X, there exists a natural number n_0 $(n_0 \geqslant 0)$ such that one of the following conditions holds:*

(*) $\qquad \operatorname{diam} f^n(A) \geqslant \delta \quad$ *for* $n \geqslant n_0$.

(**) $\qquad \operatorname{diam} f^{-n}(A) \geqslant \delta \quad$ *for* $-n \leqslant -n_0$.

Proof. By (2.1), there is $\delta > 0$ satisfying the conditions as in (2.1). Choose a subcontinuum A' of A such that diam $A' < \delta$ and choose two points a and b in A' $(a \neq b)$. Since f is expansive, there is an integer $m \in Z$ such that $d(f^m(a), f^m(b)) > \varepsilon$. Suppose that $m \geqslant 0$. Set $n_0 = m$. Let $n \geqslant n_0$. We must show that diam $f^n(A) \geqslant \delta$. Choose $\eta > 0$ such that $d(x, y) < \eta$ implies $d(f^j(x), f^j(y)) < \varepsilon$ for each $0 \leqslant j \leqslant n$. Since A' is connected, there is a finite sequence $a = a_0, a_1, \ldots, a_p = b$ of points in A' such that $d(a_i, a_{i+1}) < \eta$ for each i. For each $0 \leqslant r \leqslant p$, define $S_r = \sup\{d(f^j(a_0), f^j(a_r)) \mid 0 \leqslant j \leqslant n\}$. Note that $S_0 = 0$ and $S_p > \varepsilon$. Also, note that $|S_{r+1} - S_r| \leqslant \varepsilon$ for all r. Hence we can choose r such that $S_{r-1} \leqslant \varepsilon$ and $S_r > \varepsilon$. Then $S_r \leqslant 2\varepsilon$. By (2.1), $d(f^n(a_0), f^n(a_r)) \geqslant \delta$. Hence diam $f^n(A) \geqslant \delta$. The case for $m < 0$ is the same as above, because f^{-1} is expansive as well.

(2.3) THEOREM. *No uniformly arcwise connected continuum admits an expansive homeomorphism.*

Proof. Let X be a uniformly arcwise connected continuum. Suppose, on the contrary, that there exists an expansive homeomorphism f on X. Choose an arc A in X. Let $\delta > 0$ be a positive number as in (2.2). Since X is uniformly arcwise connected, there is a natural number m such that (#) if B is an arc, there are points $b_0, b_1, \ldots, b_m$ of B such that $B = \bigcup_{i=0}^{m-1} \langle b_i, b_{i+1} \rangle$ and diam $\langle b_i, b_{i+1} \rangle < \delta$ for each i. Choose a sufficiently large natural number p $(p > 2(m+1))$ and subarcs A_j $(j = 1, 2, \ldots, p)$ of A such that

$$A \supset \bigcup_{j=1}^{p} A_j, \quad A_j \cap A_k = \emptyset \quad (j \neq k).$$

Clearly, we may assume that $|J| \geqslant m+1$, where $J = \{j \in \{1, 2, \ldots, p\} \mid$ there is a natural number n_j such that diam $f^n(A_j) \geqslant \delta$ for $n \geqslant n_j\}$ and $|S|$ denotes the cardinal number of a set S. Put $n_0 = \max\{n_j \mid j \in J\}$. Then the arc $f^n(A)$ does not satisfy the condition (#) if $n \geqslant n_0$. This is a contradiction.

(2.5) COROLLARY [8, (2.3) and (3.2)]. *If X is a 1-dimensional compact* ANR, *then X admits no expansive homeomorphism.*

3. The nonexistence of expansive homeomorphisms of dendroids. In this section, we prove the following main theorem in this paper.

(3.1) THEOREM. *No dendroid admits an expansive homeomorphism.*

To prove (3.1), we need the following (cf. [9]).

Let $\delta > 0$ be a positive number and let n be a natural number ($n \geqslant 1$). Let C be an arc containing points a, b. Then (C, a, b) is said to be *(δ, n)-folding* provided that there exist points $a = x_0 < x_1 < \ldots < x_n = b$ in C such that $\operatorname{diam}\langle x_i, x_{i+1}\rangle \geqslant \delta$ for each $i = 0, 1, \ldots, n-1$. Let A, B, C be subsets of X. A triple (A, B, C) is contained in a triple (A', B', C') provided that $A \subset A'$, $B \subset B'$ and $C \subset C'$. Moreover, if A', B' and C' are neighborhoods of A, B, and C, respectively, then we call (A', B', C') a neighborhood of (A, B, C).

Consider the following set:

$$D(X) = \{(A, x, y) \in C(X) \times X \times X \mid x, y \in A\},$$

where $C(X)$ denotes the hyperspace of subcontinua of X with the Hausdorff metric.

For any subset $M \subset D(X)$ and any $\delta > 0$, define

$M_\delta^f = \{(A, x, y) \in D(X) \mid$ for each neighborhood (G, U, V) of (A, x, y) and for any natural number n, there exists $(A', x', y') \in M$ such that (G, U, V) is a neighborhood of (A', x', y') and any arc from x' to y' in A' is (δ, n)-folding$\}$.

In this case, we say that (A', x', y') is (δ, n)-folding. For simplicity, set $M_\delta^f = M^f$.

(3.2) PROPOSITION. *M^f is closed in $D(X)$.*

(3.3) PROPOSITION. $(M^f)^f \subset M^f$.

For a subset M of $D(X)$ and for ordinal numbers, define

$$M_1 = M^f, \quad M_{\alpha+1} = (M_\alpha)^f, \quad M_\lambda = \bigcap_{\alpha<\lambda} M_\alpha, \quad \text{where } \lambda \text{ is a limit ordinal.}$$

(3.4) THEOREM. *If X is a dendroid, then $M_\alpha = \emptyset$ for some countable ordinal α.*

Proof. Note that $D(X)$ is separable. Since M_α is closed in $D(X)$ and $M_\alpha \supset M_\beta$ for $\beta \geqslant \alpha$, there is a countable ordinal α such that $M_\alpha = M_\beta$ for all $\beta \geqslant \alpha$. In particular, $(M_\alpha)^f = M_\alpha$. We shall show that $M_\alpha = \emptyset$. Suppose, on the contrary, that $M_\alpha \neq \emptyset$. Choose $(A_1, x_1, y_1) \in M_\alpha$. Since A_1 is a tree-like continuum, there is a finite open cover $\mathscr{I}_1$ of A_1 such that

a(1) $G_1 = \bigcup \mathscr{I}_1 (= \bigcup\{W \in \mathscr{I}_1\})$ is a neighborhood of A_1,
b(1) $\operatorname{mesh} \mathscr{I}_1 < 1/2$,
c(1) the nerve $N(\mathscr{I}_1)$ is a tree, and
d(1) $x_1 \in U_1 \in \mathscr{I}_1$ and $y \in V_1 \in \mathscr{I}_1$.

Since $(A_1, x_1, y_1) \in (M_\alpha)^f$, we can choose $(A_2, x_2, y_2) \in M_\alpha$ such that (G_1, U_1, V_1) is a neighborhood of (A_2, x_2, y_2) and (A_2, x_2, y_2) is $(\delta, 1)$-folding. Take a finite open cover $\mathscr{I}_2$ of A_2 such that

a(2) $G_2 = \bigcup \mathscr{I}_2$ is a neighborhood of A_2 and $\operatorname{Cl} G_2 \subset G_1$,
b(2) $\operatorname{mesh} \mathscr{I}_2 < 1/2^2$,
c(2) $N(\mathscr{I}_2)$ is a tree,
d(2) $x_2 \in U_2 \in \mathscr{I}_2$, $y_2 \in V_2 \in \mathscr{I}_2$ and $\operatorname{Cl} U_2 \subset U_1$, $\operatorname{Cl} V_2 \subset V_1$ and
e(2) $(N(\mathscr{I}_2), U_2, V_2)$ is $(\delta, 1)$-folding,

where, in general, $((N)\mathscr{I}, U, V)$ is said to be *(δ, n)-folding* for a cover $\mathscr{I}$ with $U, V \in \mathscr{I}$ provided that for any chain $\mathscr{A}$ from U to V, there are $U = W_0 < W_1 < \ldots < W_n = V$ in $\mathscr{A}$ such that $\operatorname{diam}(\bigcup \mathscr{A}_i) \geqslant \delta$ $(i = 0, 1, \ldots, n-1)$, where $\mathscr{A}_1$ is the subchain from W_i to W_{i+1} in $\mathscr{A}$ (see Figure 2).

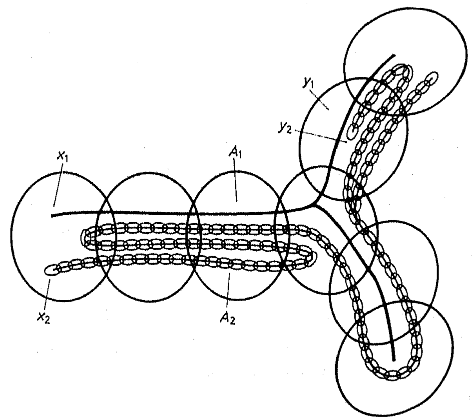

Fig. 2

By induction, we obtain $(A_i, x_i, y_i) \in M_\alpha$ and a finite open cover $\mathscr{I}_i$ of A_i such that $(G_{i-1}, U_{i-1}, V_{i-1})$ is a neighborhood of (A_i, x_i, y_i), (A_i, x_i, y_i) is (δ, i)-folding and $\mathscr{I}_i$ satisfies the following;

a(i) $G_i = \bigcup \mathscr{I}_i$ is a neighborhood of A_i and $\operatorname{Cl} G_i \subset G_{i-1}$,
b(i) $\operatorname{mesh} \mathscr{I}_i < 1/2^i$,
c(i) $N(\mathscr{I}_i)$ is a tree,
d(i) $x_i \in U_i \in \mathscr{I}_i$, $y_i \in V_i \in \mathscr{I}_i$ and $\operatorname{Cl} U_i \subset U_{i-1}$, $\operatorname{Cl} V_i \subset V_{i-1}$,
e(i) $(N(\mathscr{I}_i), U_i, V_i)$ is (δ, i)-folding.

Put $(A, x, y) = \left(\bigcap_{i=1}^{\infty} \operatorname{Cl} G_i, \bigcap_{i=1}^{\infty} \operatorname{Cl} U_i, \bigcap_{i=1}^{\infty} \operatorname{Cl} V_i\right)$. Then A is a continuum. Since X is a dendroid, A is a dendroid. Then there is an arc C from x to y in A. By the construction, (C, x, y) is (δ, i)-folding for all i. This is impossible.

Proof of (3.1). Let X be a dendroid. Suppose, on the contrary, that X admits an expansive homeomorphism $f\colon X \to X$. By (3.4), there is a countable ordinal α such that $M_\alpha = \emptyset$, where $M = D(X)$. Let $c > 0$ be an expansive constant and $0 < \varepsilon < c/2$. Choose $\delta > 0$ satisfying the condition as in (2.2). Let $0 < \eta < \delta/2$. Take an arc $A = B$ from x to $y = y'$. We may assume that $d(x, y) \geqslant \eta$. Let $m_1 < m_2 < \ldots,$ be a sequence of natural numbers. By the proof of (2.3), there exists a sequence $n(1), n(2), \ldots,$ of integers Z with $|n(1)| < |n(2)| < \ldots,$ satisfying one of the following conditions:

$\Delta(1)$ if $n(i) \geqslant 0$, for all $n \geqslant n(i)$, $(f^n(B), f^n(x), f^n(y'))$ is (δ, m_i+1)-folding.
$\nabla(1)$ if $n(i) < 0$, for all $n \leqslant n(i)$, $(f^n(B), f^n(x), f^n(y'))$ is (δ, m_i+1)-folding.

For each i, take a point a_i of $\langle f^{2n(i)}(x), f^{2n(i)}(y')\rangle$ such that $\operatorname{diam}\langle f^{2n(i)}(x), a_i\rangle \geqslant \delta$, and $(\langle a_i, f^{2n(i)}(y')\rangle, a_i, f^{2n(i)}(y))$ is (δ, m_i)-folding. Since $C(X)$ is a compact metric space, we may assume that $\lim f^{2n(i)}(A) = A_1$, $\lim f^{2n(i)}(y) = y_1$ and $\lim \langle f^{2n(i)}(x), a_i\rangle = B'_1$. Note that $B'_1 \subset A_1$. Since $\operatorname{diam} B'_1 \geqslant \delta > 2\eta$, there is a point x_1 of B'_1 and an arc B_1 $(x_1 \in B_1)$ such that $B_1 \subset B'_1$ and $\langle x_1, y_1\rangle \cap B_1 = B_1$. Let $B_1 = \langle x_1, y'_1\rangle$. We may assume that $d(x_1, y'_1) \geqslant \eta$, i.e., $\operatorname{diam} B_1 \geqslant \eta$. For a countable ordinal λ, we assume that we have obtained $(A_\alpha, x_\alpha, y_\alpha) \in D(X)$, B'_α and B_α for all $\alpha < \lambda$ as before. Let y'_α be the point $\langle x_\alpha, y'_\alpha\rangle = B_\alpha$.

Consider two cases:

(I) $\lambda = \alpha + 1$. As before, we have a sequence $n(1), n(2), \ldots$, of integers Z such that $|n(1)| < |n(2)| <, \ldots,$ and one of the following conditions holds:

$\Delta(\lambda)$ If $n(i) \geqslant 0$, then $(f^n(B_\alpha), f^n(x_\alpha), f^n(y'_\alpha))$ is (δ, m_i+1)-folding for all $n \geqslant n(i)$.

$\nabla(\lambda)$ If $n(i) < 0$, then $(f^n(B_\alpha), f^n(x_\alpha), f^n(y'_\alpha))$ is (δ, m_i+1)-folding for all $n \leqslant n(i)$.

For each i, choose a point a_i of $f^{2n(i)}(B_\alpha)$ such that $\operatorname{diam}\langle f^{2n(i)}(x_\alpha), a_i\rangle \geqslant \delta$ and $(\langle a_i, f^{2n(i)}(y'_\alpha)\rangle, a_i, f^{2n(i)}(y'_\alpha))$ is (δ, m_i)-folding, hence $(\langle a_i, f^{2n(i)}(y_\alpha)\rangle, a_i, f^{2n(i)}(y_\alpha))$ is (δ, m_i)-folding. We may assume that the sequences $\{f^{2n(i)}(A_\alpha)\}_i$, $\{\langle f^{2n(i)}(x_\alpha), a_i\rangle\}_i$ and $\{f^{2n(i)}(y_\alpha)\}_i$ are convergent in $C(X)$. Set $A_{\alpha+1} = \lim f^{2n(i)}(A_\alpha)$, $y_{\alpha+1} = \lim f^{2n(i)}(y_\alpha)$ and $B'_{\alpha+1} = \lim \langle f^{2n(i)}(x_\alpha), a_i\rangle$. Then $\operatorname{diam} B'_{\alpha+1} \geqslant \delta$. Choose a point $x_{\alpha+1}$ of $B'_{\alpha+1}$ and an arc $B_{\alpha+1}$ in $B'_{\alpha+1}$ such that $x_{\alpha+1} \in B_{\alpha+1}$, $\langle x_{\alpha+1}, y_{\alpha+1}\rangle \cap B_{\alpha+1} = B_{\alpha+1}$ and $d(x_{\alpha+1}, y'_{\alpha+1}) \geqslant \eta$ i.e., $\operatorname{diam} B_{\alpha+1} \geqslant \eta$, where $B_{\alpha+1} = \langle x_{\alpha+1}, y'_{\alpha+1}\rangle$.

(II) λ is a limit ordinal. In this case, take a sequence $\alpha_1 < \alpha_2 < \ldots$ of countable ordinal numbers such that $\lim \alpha_i = \lambda$. Then we may assume that the sequences $\{A_{\alpha_i}\}$, $\{y_{\alpha_i}\}$ and $\{B'_{\alpha_i}\}$ are convergent. Set $A_\lambda = \lim A_{\alpha_i}$, $y_\lambda = \lim y_{\alpha_i}$ and $B'_\lambda = \lim B'_{\alpha_i}$. Also, choose a point x_λ of B'_λ and an arc B_λ such that $x_\lambda \in B_\lambda$, $\langle x_\lambda, y_\lambda\rangle \cap B_\lambda = B_\lambda$ and $\operatorname{diam} B_\lambda \geqslant \eta$. Let $B_\lambda = \langle x_\lambda, y'_\lambda\rangle$. Hence we obtain $(A_\alpha, x_\alpha, y_\alpha) \in D(X)$, B'_α and B_α for all countable ordinal numbers α.

Next, we shall show that for each countable ordinal number α,

$$(f^n(A_\alpha), f^n(b), f^n(y_\alpha)) \in M_\alpha,$$

where $M = D(X)$, $M^f = M^f_\delta$, $b \in B'_\alpha$ and n is any integer.

We shall prove this by transfinite induction. Let (G, U, V) be a neighborhood of $(f^n(A_1), f^n(b), f^n(y_1))$. Since $(f^{-n}(G), f^{-n}(U), f^{-n}(V))$ is a neighborhood of (A_1, b, y_1), where $b \in B'_1$, we can choose $n(i)$ such that if $n(i) \geqslant 0$, $2n(i)+n \geqslant n(i)$, or if $n(i) < 0$, $2n(i)+n \leqslant n(i)$, and $(f^{2n(i)}(A), b_i, f^{2n(i)}(y))$ is contained in $(f^{-n}(G), f^{-n}(U), f^{-n}(V))$, where $b_i \in \langle f^{2n(i)}(x), a_i\rangle$ such that $\lim b_i = b$. By $\Delta(1)$ or $\nabla(1)$, $(f^{2n(i)+n}(A), f^n(b_i), f^{2n(i)+n}(y))$ is (δ, m_i)-folding. Hence $(f^n(A_1), f^n(b), f^n(y_1)) \in M_1$.

Assume that for each integer n,

$$(f^n(A_\alpha), f^n(b), f^n(y_\alpha)) \in M_\alpha \quad \text{for all } \alpha < \lambda, \text{ where } b \in B'_\alpha.$$

Consider two cases:

(I) $\lambda = \alpha + 1$. Let (G, U, V) be any neighborhood of $(f^n(A_\lambda), f^n(b), f^n(y_\lambda))$, where $b \in B'_\lambda$. By the construction, we can choose a large number $k > 0$ such that $(f^{2n(k)}(A_\alpha), b_k, f^{2n(k)}(y_\alpha))$, where $b_k \in \langle f^{2n(k)}(x_\alpha), a_k\rangle \subset f^{2n(k)}(B_\alpha)$ and $\lim b_k = b$, is (δ, m_k)-folding and is contained in $(f^{-n}(G), f^{-n}(U), f^{-n}(V))$. By the construction, $(f^{2n(k)+n}(A_\alpha), f^n(b_k), f^{2n(k)+n}(y_\alpha))$ is (δ, m_k)-folding and is contained in M_α by the inductive hypothesis. Hence $(f^n(A_\lambda), f^n(b), f^n(y_\lambda)) \in M_\lambda$.

(II) λ is a limit ordinal. By an argument similar to the above one, we can prove that

$$(f^n(A_\lambda), f^n(b), f^n(y_\lambda)) \in M_\alpha \quad \text{for } \alpha < \lambda,\ b \in B'_\lambda.$$

Hence $(f^n(A_\lambda), f^n(b), f^n(y_\lambda)) \in \bigcap_{\alpha<\lambda} M_\alpha = M_\lambda$.

In particular, $M_\alpha \neq \emptyset$ for all countable ordinal numbers α. This is in contradiction to (3.4). This completes the proof.

The following problem remains open.

PROBLEM. If a continuum X is tree-like (moreover arc-like), is it true that X does not admit an expansive homeomorphism?

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