On pairs of compacta with $\dim(X \times Y) < \dim X + \dim Y$

by

Stanisław Spież (Warszawa)

Abstract. In this note we give a necessary and sufficient condition for maps $f_1: X_1 \to Y \times (0) \subset \mathbb{D}$, $f_2: X_2 \to (0) \times Y \subset \mathbb{D}$ of 2-dimensional compacta into discs to admit mappings $g_1: X_1 \to \mathbb{D}$, $g_2: X_2 \to \mathbb{D}$ which have disjoint images and coincide with $f_i$ on $f_i^{-1}(\partial \mathbb{D})$, for $i = 1, 2$. As a corollary we obtain that $\dim(X_1 \times X_2) < 4$ if any two mappings $X_1 \to \mathbb{R}^n$, $X_2 \to \mathbb{R}^n$ can be approximated by mappings with disjoint images. We also characterize pairs $X_1$, $X_2$ of finite dimensional compacta with $\dim(X_1 \times X_2) < n = \dim X_1 + \dim X_2$ in terms of extensions of mappings from closed subsets of $X$ into certain CW-complexes. In the Appendix we also give an alternative proof of the latter result and of a result of [Sp2] by applying Eilenberg-MacLane spaces.

Introduction. Recall that a mapping $h: Z \to I^m$, where $I = [-1, 1]$, is said to be inessential (in the sense of Alexandrov-Hopf) if there exists a mapping $h': Z \to \partial I^m$ such that $h(x) = h'(x)$ for each $x \in h'^{-1}(\partial I^m)$. In this note we prove the "only if" part in the case $m = 2 = n$ of the following

Theorem 1. Let $f: X \to I^m$, $g: Y \to I^n$ be mappings of finite dimensional compacta, so that $m = \dim X$ and $n = \dim Y$. Then, the product mapping $f \times g: X \times Y \to I^{m+n}$ is inessential if and only if there exist two mappings $F: X \to I^{m+n}$ and $G: Y \to I^{m+n}$ with disjoint images and such that

$$F(x) = (f(x), 0) \quad \text{and} \quad G(y) = (0, g(y))$$

for each $x \in f^{-1}(\partial I^m)$ and each $y \in g^{-1}(\partial I^n)$ (i.e. the mappings $f$ and $g$ are transversely trivial in the sense of [Kr]).

The "if" part of the above theorem was proved in [K-L]. The case $m = 1$ of the "only if" part was proved also in [K-L] and the case $m \geq 3$, $n \geq 2$ of this part in [Sp1]. The proof of the missing case $(m = 2 = n)$ of the theorem is the same as that given in [Sp1] except that one has to apply a homotopy version of the Whitney trick for 4-dimensional manifolds (which is described in § 1) instead of the isotopy Whitney trick for higher dimensional manifolds.

A consequence of Theorem 1 and of (2.1) in [Kr] (cf. [M-R]) is the "only if" part of the following characterizations of compacta with the properties which occur in the famous constructions of L. S. Pontryagin [Po] and V. Boltyanskii [Bo] (the "if" part was proved in [Kr]).
**Corollary 1.** Any two mappings \( X \to R^k \) and \( Y \to R^k \) of finite dimensional compacta, where \( k = \text{dim} \, X + \text{dim} \, Y \), can be approximated arbitrarily closely by mappings with disjoint images if and only if \( \text{dim} \, (X \times X) < \text{dim} \, X + \text{dim} \, Y \).

**Corollary 2** ([Sp1], [Sp2]). Let \( X \) be an \( m \)-dimensional compactum. Then \( \text{dim} \, (X \times X) < 2m \) if and only if the set of all imbeddings \( X \to R^{2m} \) is dense in the space of all mappings \( X \to R^{2m} \).

We are also interested in characterizing pairs \( X, Y \) of compacta with the property \( \text{dim} \, (X \times Y) < \text{dim} \, X + \text{dim} \, Y \) in terms of extensions of mappings of closed subsets of \( X \) and \( Y \) into certain CW-complexes. In the statement of the following result, which in a weaker form was announced in [Sp2], we need the following notation. Let \( S_{m-1} \vee S_{l-1} \) be the one point union, with the base point \(* \), of \((m-1)\)-spheres \( S_{m-1} \) and \( S_{l-1} \). Let \( a_i \) be a generator of the group \( \pi_{m-1}(S_{m-1}, *, *) \) for \( j \in \{0, 1\} \). By \( P_i(a_j) \), where \( m \geq 2 \), we denote the CW-complex obtained by attaching to \( S_{m-1} \vee S_{l-1} \) two \( m \)-cells by mappings corresponding to \( a_i a_j \) and \( a_i \), and by \( P_i(b_j) \) the CW-complex obtained by attaching to \( S_{m-1} \vee S_{l-1} \) one \( m \)-cell by a mapping corresponding to \( (a_i a_j)^n \). (We use multiplicative notation for the higher homotopy groups also.)

**Theorem 2.** Let \( X \) and \( Y \) be compacta such that \( \text{dim} \, X = m \) and \( \text{dim} \, Y = n \). Then, \( \text{dim} \, (X \times Y) < m + n \) if and only if there exists a permutation \((U, V)\) of the letters \( P \) and \( R \) such that for arbitrary closed subsets \( A \) of \( X \) and \( Y \), respectively, any mappings

\[
A \to S_{m-1} \quad \text{and} \quad B \to S_{l-1}
\]

admit extensions

\[
X \to U_{i,j} \quad \text{and} \quad Y \to V_{i,j}
\]

for some integers \( k \) and \( l \).

In this note we prove the "only if" case in the theorem \( m = n = 2 \), see § 3 (the proof in the higher dimensions is the same). In the proof, Lemma (3.9) in [Sp2] plays an essential role. The inverse implication can be proved similarly to (1.5) in [Kr] (compare also the proof of the "if" part of Theorem 3 in [Sp2], § 4).

In the Appendix, we give an alternative proof of Theorem 3 in [Sp2] that is independent of the results of [Sp2] and we sketch an alternative proof of Theorem 2.

I would like to remember that recently I have learned by letters from D. Repovš that he, A. N. Dranishnikov, and B. V. Shepkin have obtained results along the lines of Corollary 1, case \( m = n = 2 \) (see [D–R–S]) and that some results of [Kr] and [Sp1] have been very recently recovered in [D–S] in an alternative way. (Added in proof: After this paper was accepted the author received a preprint [D–R–S2] containing an alternative proof of Corollary 1 in the case \( m = n = 2 \).)

1. **A version of the Whitney lemma.** The proof of the case \( m = n = 2 \) of the "only if" part of Theorem 1 is the same as that of Theorem (2.1) in [Sp1] except that one has to apply Lemma (1.1) below instead of Lemma (2.1) in [Sp1]. The proof of Lemma (1.1) is a modification of Lemma (3.1) in [Fr] (compare also Lemma (1.1) in [Fr]).
This can be achieved by a PL-homotopy of \( f(X_2) \) which removes points of intersection of \( h(\text{Int} D) \) and \( f(X_2) \) by pushing these points along disjointly imbedded arcs in \( h(D) \) to the edge \( e_2 \). These are "Casson moves" (or finger moves), see the proof of (3.1) in [F] and section 1 in [F-Q]. Moreover, we may assume that this procedure does not introduce new points of intersection of \( f(X_2) \) and \( f(X_1) \).

Observe that there are open neighborhoods \( P_1 \) and \( P_2 \) of \( s_1 \) and \( s_2 \) in \( f(\text{Int} e_1) \) and \( f(\text{Int} e_2) \), respectively, which satisfy the assumptions of Lemma (1.2). Applying (1.2), we can modify the map \( f \) on the inverse image of a small neighborhood of \( e_2 \) in \( P_2 \) in order to get a map \( f' \colon X \to M_3 \), which satisfies the conditions (a), (b), (c) and (d) of (1.1) and such that

\[
f'(X_1) \cap f'(X_2) = f(X_1) \cap f(X_2) \setminus \{a, b\}
\]

Thus the lemma follows by applying successively the above argument.

2. Some lemmas concerning presentations of groups. First we introduce some conventions. By \( \text{cdim}_Q X \) we denote the cohomological dimension of \( X \) with respect to \( G; Q \) the additive group of all rationals, \( Q_p \) the additive group of all rationals whose denominators are coprime with a prime \( p \) and \( Z(k) = Z/kZ \). By \( (k, l) \) we denote the greatest common divisor of integers \( k \) and \( l \). If \( (k, l) = 1 \) for each \( i \) belonging to a set \( \mathcal{F} \) of integers then we write \( (k, \mathcal{F}) = 1 \). In the sequel of the paper, we fix a copy of the unit circle and denote it by \( S_0 \) or by \( S \). Let \( \sqrt{\mathcal{F}} \), where \( 0 \in \mathcal{J} \), denote the one point union, with the basepoint \( \ast \), of circles \( \{S_j : j \in \mathcal{J}\} \) and let \( g \) be a generator of \( \pi_1^i(S, \ast) \). If \( \{g_i : i \in J\} \) is any collection of words in symbols \( \{s_1 : j \in J\} \), then one can form a 2-dimensional CW-complex whose 1-skeleton is \( \bigvee \{S_j : j \in J\} \) and whose 2-cells \( \{g_i : i \in I\} \) are such that the attaching map of \( s_1 \) is given by \( s_1 \in \pi_1 \bigvee \{S_j : j \in J\} \). We call

\[
P = \{(a_j : j \in J) ; (a_i : i \in I)\}
\]

the presentation of the arising complex \( K(P) \). The elements \( e_i \) are called relators of \( P \).

We say (see [Sp2]) that a mapping \( f \colon (L, L_0) \to (K, S) \) is admissible if for any homomorphism \( h \) of abelian groups the following condition is satisfied

\[
H^2 F(\phi \otimes h) = 0 \quad \text{iff} \quad H^2 F(\phi) \otimes h = 0,
\]

where \( \phi \colon (L, L_0) \to (D^2, S) \) and \( \psi \colon (K, S) \to (D^2, S) \) are extensions of \( f \mid L_0 \) and the identity map on \( S \), respectively. (By \( D^2 \) we denote the unit 2-ball.)

The following lemma is essential in the proof of Theorem 2.

(2.2) LEMMA (see [Sp2], (3.9) and (2.5)). Let \( (L, L_0) \) be a pair of compact polyhedra such that \( \text{dim} L = 2 \). Then any mapping \( L_0 \to S \) can be extended to an admissible mapping \( (L, L_0) \to (K(P), S) \) where \( P \) has the following form

\[
P = \{a_0, \ldots, a_k ; a_0^{q_0}, a_k^{q_k} \mid 1 \leq i \leq k, \{a_i, a_j \mid 0 < i < j \leq k\},
\]

such that \( n(j) \neq 0 \) and \( n(j) \mid m(i) \) for each \( j \in \{1, \ldots, k\} \).

We will also need the following lemmas:

(2.3) LEMMA. Let \( P \) be given by (2.1) and let \( P' \) be defined as \( P \) except that, for some \( i \in I \) and some positive integer \( n \), the relator \( e_i \) is replaced by \( (a_i)^n \). If \( (X, A) \) is a pair of compacta such that \( \text{dim} X = 2 \) and \( \text{cdim}_{a_0} A, X \leq 1 \), then a map \( f \colon A \to S \) admits an extension \( X \to K(P') \) provided \( f \) admits an extension \( X \to K(P) \).

The proof is essentially the same as that of Lemma (4.4) in [Sp2], since the assumption \( \text{dim}(X \times X) < 4 \) can be relaxed to \( \text{cdim}_{a_0} A, X \leq 1 \) (see Remark (4.3) in [Sp2]).

(2.4) LEMMA. Let \( P \) and \( P' \) be as assumed in (2.3). Then there exists a mapping \( K(P) \to K(P) \) which is the identity on \( S \).

(2.5) LEMMA. Let us consider two presentations

\[
P = \{a_0, a_1, a_2; a_0^{q_0}, a_1^{q_1}, a_2^{q_2}, [a_0, a_1]\}\]

and

\[
P' = \{a_0, a_1, a_2; a_0^{q_0}, a_1^{q_1}, a_2^{q_2}, [a_0, a_1] \mid 0 \leq i < j \leq 2\},
\]

where \( n, k, l \) and \( I \) are positive integers. If \( (k, l) = 1 \), then there exists a mapping \( K(P) \to K(P') \) which is the identity on \( S \).

Proof. Let \( i \) and \( s \) be integers such that \( s \cdot k \cdot l \cdot i = 1 \). Then the required map is induced by the substitutions \( a_0 \to a_0^{-1} \) and \( a_1 \to a_1^{-1} \).

3. Proof of Theorem 2. As discussed in the Introduction, we will restrict ourselves to the case \( m = 2 = n \) and the "only if" part. Let \( X_1, X_2 \) be 2-dimensional compacta such that \( \text{dim}(X_1 \times X_2) < 4 \) and let

\[
g_i : (X_s, A_s) \to (D^2, S), \quad s = 1, 2,
\]

be mappings, where \( A_s \) is a closed subset of \( X_s \), \( s \in \{1, 2\} \). We need the following

(3.1) LEMMA. There exist mappings

\[
h_i : (X_s, A_s) \to (K(P_s), S), \quad s = 1, 2,
\]

such that \( h_i(x) = \hat{g}_i(x) \) for \( x \in A_s \), and \( P_s \) is a presentation of the following form

\[
P_s = \{a_0, \ldots, a_k ; a_0^{q_0}, a_k^{q_k} \mid 1 \leq j \leq k, \{a_0, a_j \mid 0 \leq i < j \leq k\},
\]

where \( n(s, j) \) divides \( m(s, j) \), \( n(s, j) \neq 0 \), \( m(s, j) \) is either 0 or a power of a prime, and the greatest common divisor of the integers \( m(1), m(2) \) divides the product \( n(1), n(2) \) for each pair \( (i, j) \in \{1, \ldots, k\} \times \{1, \ldots, k\} \).

Proof. By (2.5), we may additionally require that in (2.2) each \( m(j) \) is either 0 or a power of a prime. The proof follows from (2.2) by the argument used to prove (4.2) in [Sp2].

Observe that under assertions of (3.1) either all \( m(1)/j \) s are nonzero or all \( m(2)/j \) s are nonzero. Without loss of generality we will assume that the latter
condition is satisfied. Let \( k \) denote the lowest common multiple of the integers \( m(2, i) = m(2, j)/n(2, j) \) for \( 1 \leq j \leq k(2) \). Let us observe that

(3.2) Remark. If, in (3.1), we have \( m(1, j) = 0 \) then \( k \) divides \( n(1, j) \).

For \( s \in \{1, 2\} \), let \( S \) denote the set of all primes \( p \) such that \( \gcd(p, X_s) = X_s \). Let us note (see [KoJ]) that \( \dim(X_1 \times X_2) < 4 \) implies that \( S \cap X_1 = \emptyset \). The proof of Theorem 2 is divided into the following two lemmas:

(3.3) Lemma. There exist an integer \( l \) such that \( (l, S_2) = 1 \) and a mapping

\[ f_1: X_1 \to K_1 = K((a_0, a_1; a_0^0, a_1^0)) \]

such that \( f_2(x) = g_2(x) \) for each \( x \in A_2 \).

Proof. By (2.5) in [Sp2], and (3.1), (3.2) and (2.3), there exists a mapping \( h_1: (X_1, A_1) \to (K(P_1), S) \) as asserted in (3.1) and (3.2) except that, for each \( j \) such that \( (m(1, j), S) = 1 \), the condition \( n(1, j) \) divides \( m(1, j) \) is replaced by the condition \( k \) divides \( n(1, j) \). By Lemma (2.4), we may additionally require that \( n(1, j) = 1 \) for each \( j \) such that \( (m(1, j), S) = 1 \) and \( m(1, j) \neq 0 \). Let \( l \) be the lowest common multiple of all integers \( m(1, j)'s \) such that \( (m(1, j), S) = 1 \) and \( m(1, j) \neq 0 \). Observe that then \( (l, S_2) = 1 \).

Now, there exists a mapping

\[ K(P_1) \to K_1, \]

which is the identity on \( S_1 \); it is defined by the following substitutions

\[ a_j \to \text{trivial element} \quad \text{if} \quad (m(1, j), S_1) = 1, \]

\[ a_j \to (a_j)^{m(1, j)} \quad \text{if} \quad (m(1, j), S_1) 
eq 1 \quad \text{and} \quad m(1, j) \neq 0, \]

and by applying (2.4).

(3.4) Lemma. Let \( l \) be an integer such that \( (l, S_2) = 1 \). Then there exists a mapping

\[ f: X_2 \to K_2 = K((a_0, a_1; a_0 a_1^0)) \]

such that \( f(x) = g_2(x) \) for each \( x \in A_2 \).

Proof. Let \( A_2 \) be as asserted in (3.1). By (2.4), there exists a mapping \( K(P_2) \to K(P_1) \), which is the identity on \( S_2 \), where \( P_2 \) is defined as \( P_3 \) except that \( n(2, j) \) is replaced by \( 1 \) and \( m(2, j) \) is replaced by \( m(2, j)/n(2, j) \) for each \( 1 \leq j \leq k(2) \). Then there exists a mapping

\[ K(P_2) \to K((a_0, a_1; a_0 a_1^0)), \]

which is the identity on \( S_1 \); it is induced by the substitutions \( a_j \to (a_0)^{m(2, j)} \). The lemma follows by applying (2.3) successively.

4. Appendix. An alternative approach to a result of [Sp2] and Theorem 2. We will give an alternative proof of Theorems 3 and 3' of [Sp2] and of the "only if" part of Theorem 2, which does not depend on Lemma (2.2) and uses Eilenberg–MacLane spaces. We shall consider the case of 2-dimensional compacta only, as the proof in higher dimensions is the same.

First, we introduce some notations. For a nonempty set \( S \) of primes, by \( Q_S \) we denote the intersection \( \bigcap \{Q_p: p \in S\} \). If \( S \) is empty then we define \( Q_S = Q \).

By \( Q^{<p} \) we denote the additive group of all rationals whose denominators are powers of a prime \( p \), and by \( N \) the set of nonnegative integers. The following informations, stated in (4.1) and (4.5), on the Eilenberg–MacLane spaces \( K(Q_S, 1) \) and \( K(Q^{<p}, 1) \) are necessary.

(4.1) Lemma. Let \( S \) be a set of primes and let \( S' \) denote the set of primes not belonging to \( S \). Suppose \( P \) is a presentation whose symbols form the set

\[ \mathcal{A} = \{a_0\} \cup \{a_{n, m}: p \in S', m \in N\}, \]

and whose relators form the set

\[ \{(a_{n, m})^p = a_{n-1, m}: p \in S', m \in N \} \cup \{(c, d): c, d \in \mathcal{A}\}, \]

where each \( a_{n, m} = a_{n, m} \) for each \( p \in S' \). Then, the 2-dimensional complex \( K(P) \) satisfies \( \pi_1(K(P, 1)) \cong Q_S \). Therefore, \( K(P) \) can be considered as a 2-skeleton of an Eilenberg–MacLane space \( K(Q_S, 1) \).

Proof. Let \( h: \pi_1(K(P, 1)) \to Q_S \) be a homomorphism such that \( h(a_0) = 1 \) and \( h(a_{n, m}) = p^m \). We will prove that \( h \) is an isomorphism, by using the following fact (see [Fu], Theorem 8.4, Chapter II):

\[ Q_S = \bigoplus \{Q^{<p}/Z: p \in S'\}. \]

Observe that any \( a \in \pi_1(K(P, 1)) \) can be presented in the following form

\[ a = a_0 \cdot \prod_{p \in S'} a_{n, m}^{k(p)} \]

where \( k(p) \) is 0 for almost all \( p \) and \( 0 \leq k(p) < p^{<p} \). Suppose that

\[ h(a) = n + \sum_{p \in S'} k(p) \cdot p^{<p} \]

is 0 (in \( Q_S \)). Then \( \sum_{p \in S'} k(p) \cdot p^{<p} \), considered as an element of \( Q_S/Z \), is equal to 0. Thus, by (4.2), we have that each \( k(p) \) is 0, and consequently \( n = 0 \). It follows that \( a = 0 \), which proves that \( h \) is a monomorphism.

Using (4.2), one can prove that any \( b \in Q_S \) can be presented in the form given by (4.4), where \( k(p) \) is 0 for almost all \( p \) and \( 0 \leq k(p) < p^{<p} \). Thus \( b = h(a) \), where \( a \) is given by (4.5). It follows that \( h \) is an epimorphism.

(4.5) Lemma. Suppose \( P \) is a presentation whose symbols form the set

\[ \{a_0\} \cup \{a_{n, m}: p \in S, m \in N\} \]

and whose relators form the set

\[ \{a_0\} \cup \{a_{n, m}^{-1}: n \in N\}. \]

Then, the 2-dimensional complex \( K(P) \) satisfies \( \pi_1(K(P, 1)) \cong Q^{<p}/Z \). Therefore, \( K(P) \) can be considered as a 2-skeleton of an Eilenberg–MacLane space \( K(Q^{<p}, 1) \).
Proof. The homomorphism \( \pi_1(\tilde{P}, \ast) \to \tilde{Q} \), given by \( h(a_0) = 0 \) and \( h(a_n) = p^n \) for \( n > 0 \), is an isomorphism.

As a consequence of (4.1), we obtain the following

\( (4.6) \text{ Lemma. Let } F \text{ be a set of primes and let } (X, A) \text{ be a pair of compacta such that } \dim X = 2 \text{ and that } \text{cdim}_{Q_{F}} X \leq 1. \text{ Then any mapping } A \to S \text{ can be extended to a mapping } X \to K(\langle a_0, a_1; a_0 a_1 \rangle) \)

for some integer \( n \) satisfying \( n(F) = 1 \).

Proof. Suppose a mapping \( A \to S \) is given. Since \( \text{cdim}_{Q_{F}} X \leq 1 \) we can extend this mapping to a mapping \( f: X \to K(\langle Q_F, 1 \rangle) \), see [Ko] (let us recall that \( a_0 \) corresponds to \( S \).) Since \( X \) is a 2-dimensional compactum thus we may assume that \( f(X) \) is contained in a compact subset of \( K(P) \). Thus there exist a finite subset \( F' \{p_1, \ldots, p_k\} \) of \( F \) and an integer \( r \) such that

\( f(X) = K(P) \subseteq K(P') \)

where \( P' \) is the presentation whose symbols form the set

\( A' = \{a_0 \cup \{a_{m}; p \in F', m \in N \text{ and } m \leq r \} \}

and whose relations form the set

\( \{ (a_{m})^{m} a_{m}^{-1}; p \in F', m \in N \text{ and } m \leq r \} \cup \{ (c, d); c, d \in A' \} \).

The substitutions \( a_{m} \to a_{m}^{m} \text{ and } 0 < m \leq r \) induce a mapping

\( K(P') \to K(P) \)

which is the identity on \( S \), where \( P' \) is the presentation whose symbols form the set

\( A' = \{a_0 \cup \{a_{m}; p \in F' \} \}

and whose relations form the set

\( \{ (a_{m}^{-1} a_{m}^{m}; p \in F') \cup \{ (c, d); c, d \in A' \} \}.

Finally, there exists a mapping

\( K(P') \to K(\langle a_0, a_1; a_0 a_1 \rangle) \)

which is the identity on \( S \), where \( n = (p_1, \ldots, p_k) \), and Lemma (4.6) follows.

Let us consider \( k \) as the product \( \tau \text{ of integers such that } \tau_{p} = 1 \text{ and } (\tau, P') = 1 \).

By (4.7), any mapping \( B \to S \), where \( B \) is a closed subset of \( X \), admits an extension

\( Y \to K(\langle a_0, a_1; a_0 a_1 \rangle) \)

for some integer \( k \) satisfying \( (k, F) = 1 \).

By \( F(k) \) we denote the set of all primes \( p \) which divide \( k \). Since \( 41 < \text{cdim}_{Q_{F}} Y \) \( \leq 2 \), thus by (4.4) in [Ko] it follows that \( \text{cdim}_{Q_{F}} Y \leq 2 \). Let \( \mathcal{P} \) be the set of all primes \( \mathcal{P} \in P \) such that \( \text{cdim}_{Q_{\mathcal{P}}} Y \leq 2 \). Let us consider \( k \) as the product \( \tau \text{ of integers such that } \tau_{p} = 1 \text{ and } (\tau, P') = 1 \).

By (4.7), any mapping \( B \to S \), where \( B \) is a closed subset of \( Y \), admits an extension

\( Y \to K(\langle a_0, a_1; a_0 a_1 \rangle) \)

for some integer \( l \) which has the following property: if a prime \( p \) divides \( l \) then \( p \in \mathcal{P}(k) \cap \mathcal{P} \).

Observe that if a prime \( p \) divides \( s \) then \( \text{cdim}_{Q_{F}} Y \leq 2 \), and if a prime \( q \) divides \( l \) then \( \text{cdim}_{Q_{F}} Y = 2 \) and consequently \( \text{cdim}_{Q_{F}} X \leq 2 \) by Theorem 4.1-4 in [Ko].

Thus by (2.3), there exist extensions

\( X \to K(\langle a_0, a_1; a_0 a_1 \rangle) \) \quad \text{and} \quad Y \to K(\langle a_0, a_1; a_0 a_1 \rangle) \)

of the mappings \( A \to S \) and \( B \to S \) respectively, and Theorem 2 follows.

I am grateful to Henryk Toruńczyk for helpful conversations and suggesting the use of Eilenberg-MacLane spaces to get alternative proofs in the Appendix. I would also like to thank Zbigniew Karno for helpful discussions.
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INSTITUTE OF MATHEMATICS
POLISH ACADEMY OF SCIENCES
Słowackiego 8
00-908 Warszawa, Poland

Received 3 April 1989;
in revised form 18 May 1989

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