Rigid P-spaces

by

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Abstract. There is a rigid Lindelöf P-space of size and weight ω_1. There is also a compact space which is rigid in its G_δ topology. It is consistent that there is a P-space of size ω_1 in which all points have different characters.

§ 0. **Introduction.** All spaces considered are T_3. A *P-space* is a space in which all G_δ sets are open. A space is *rigid* iff the identity is the only homeomorphism of the space.

Van Mill (unpublished) showed that there is a rigid (infinite) P-space. His method (reminiscent of Watson's [Wat]) was to construct X so that all points in X have different characters. Note that if X is such a space and $\varkappa = |X|$, then $2^{\varkappa} > \omega_{\varkappa}$. In particular, under GCH, $\varkappa = \omega_{\varkappa}$, and this was true of van Mill's space. In § 1, we use independent sets to show that under suitable cardinal arithmetic, there is a P-space of size ω_1 in which all points have different characters. Obviously, this arithmetic must include $2^{\omega_1} > \omega_{\omega_1}$, but our construction requires some further (but consistent) assumptions as well.

If $\varkappa = \omega_{\varkappa}$, it is easy to modify van Mill's construction to make our P-space a LOTS (a totally ordered space with the order topology) of size $\varkappa$ in which all points have different uncountable characters; here the assumption $\varkappa = \omega_{\varkappa}$ is necessary under any cardinal arithmetic. However, a much smaller rigid P-space LOTS will be constructed in § 2. Note that when we are talking about a LOTS, rigidity still refers to the topological notion, not the (weaker) statement that the order is rigid.

In § 2, we build P-spaces using trees, in the manner of Juhász and Weiss [JW], and make the space rigid by making the tree rigid. We show that there is a rigid P-space, X, of size and weight ω_1. Furthermore, X is Lindelöf and X is a LOTS.

A similar tree argument will give us a compact LOTS whose G_δ topology is rigid. This space cannot be quite so small; since points of countable character in X become isolated in the G_δ topology, X can have at most one such point, whence $|X| \geq 2^{\omega_1}$ by the Čech–Pospíšil Theorem. Our space has size $\max((2^{\omega})^+, 2^{\omega_1})$.

* Research supported by NSF grant DMS-8501521.

A slightly larger such space is due first to Watson [Wat], who constructs a compact LOTS in which all points have different characters. The cardinality of such a space must be at least as large as the first weakly Mahlo cardinal (see [G] and [M]), and some form of $\diamondsuit$ seems to be required for its construction. This space is rigid in its G_δ topology because in a LOTS, points of uncountable character keep their same character in the G_δ topology.

§ 1. An independent set construction. In this section, we prove,

1.1. THEOREM. *Suppose that $2^{\omega_1} > \omega_{\omega_1}$, and that for each $\xi < \omega_1$, $\omega_{\xi+1}^{\omega} = \omega_{\xi+1}$. Then there is a P-space topology on the set ω_1 such that each point, ξ, has character $\omega_{\xi+1}$.*

Proof. Since our hypotheses imply CH, there is a family, $\mathscr{F} \subset \mathscr{P}(\omega_1)$ of size 2^{ω_1} which is σ-independent; that is, every countable intersection of sets in $\mathscr{F}$ and complements of such sets is uncountable. Since $2^{\omega_1} > \omega_{\omega_1}$, we may fix disjoint subsets $\mathscr{F}_\xi \subset \mathscr{F}$, for $\xi < \omega_1$, where each $|\mathscr{F}_\xi| = \omega_{\xi+1}$. Since modifying each member of $\mathscr{F}$ on a countable set preserves independence, we may assume also that for each $\xi < \omega_1$,

$$(*) \qquad \forall X \in \mathscr{F}_\xi (\xi+1 \subset X) \ \wedge \ \forall \eta > \xi \, \exists Y \in \mathscr{F}_\xi (\eta \notin Y).$$

Our topology will be generated by certain Boolean combinations of our independent sets. For each ξ, let $\mathscr{A}_\xi \subseteq \mathscr{P}(\omega_1)$ be the set of all countable Boolean combinations of the sets in $\bigcup \{\mathscr{F}_\mu : \mu \leqslant \xi\}$. By recursion on ξ, define $\mathscr{B}_\xi$ to be the set of all $B \in \mathscr{A}_\xi$ such that for all $\mu < \xi$,

$$(\dagger) \qquad (\mu \in B \to \exists X \in \mathscr{B}_\mu (\mu \in X \subseteq B)) \wedge \big(\mu \notin B \to \exists X \in \mathscr{B}_\mu (\mu \in X \subseteq (\omega_1 \setminus B))\big).$$

Then, $\mathscr{B}_0 = \mathscr{A}_0$, since for $\xi = 0$, condition ($\dagger$) is vacuous. By induction on ξ, $\mathscr{B}_\xi$ is a σ-algebra. If $\xi < \eta$, then $\mathscr{B}_\xi \subseteq \mathscr{B}_\eta$; this is proved by induction on η; note that for $B \in \mathscr{B}_\xi$, condition ($\dagger$) is trivial for $\xi \leqslant \mu < \eta$, since $\mathscr{B}_\xi \subseteq \mathscr{B}_\mu$ by the inductive hypothesis. By induction on η and condition ($*$), any set of the form $\bigcup \{A_\mu : \mu \leqslant \eta\}$, where each $A_\mu \in \mathscr{F}_\mu$, is in $\mathscr{B}_\eta$; by independence, this set is not in $\mathscr{B}_\xi$ (or even $\mathscr{A}_\xi$) when $\xi < \eta$. Let $\mathscr{B} = \bigcup \{\mathscr{B}_\xi : \xi < \omega_1\}$. $\mathscr{B}$ is a σ-algebra, since each $\mathscr{B}_\xi$ is. Thus, $\mathscr{B}$ is a base for a P-space topology, $\mathscr{T}$, on ω_1, and $\mathscr{T}$ is 0-dimensional (i.e., set in $\mathscr{B}$ are clopen in $\mathscr{T}$). Furthermore, $\mathscr{T}$ separates points (and is thus Hausdorff); to see this, note that if $\xi < \eta$, then, by ($*$), ξ has a basic neighborhood of the form $\bigcup \{A_\mu : \mu \leqslant \xi\}$ (where each $A_\mu \in \mathscr{F}_\mu$) which excludes η.

Finally, we must compute the character of each point, ξ. By ($\dagger$), $\mathscr{B}_\xi$ is a local base at ξ, so $\chi(\xi) \leqslant \omega_{\xi+1}^{\omega} = \omega_{\xi+1}$. If $\chi(\xi) \leqslant \omega_\xi$, we derive a contradiction as follows. For $\mu < \xi$, fix $A_\mu \in \mathscr{F}_\mu$ such that $\xi \notin A_\mu$. For $X \in \mathscr{F}_\xi$, let $N_X = X \cup \bigcup \{A_\mu : \mu < \xi\}$. By $\chi(\xi) \leqslant \omega_\xi$, we may fix a $B \in \mathscr{B}$ such that $\xi \in B$ and $\mathscr{E} = \{X : B \subseteq N_X\}$ has size $\omega_{\xi+1}$. By independence of $\mathscr{F}$, the fact that $\mathscr{E}$ is uncountable implies that $B \subseteq \bigcup \{A_\mu : \mu < \xi\}$. But this is impossible, since $\xi \in B$ and $\xi \notin \bigcup \{A_\mu : \mu < \xi\}$. ■

§ 2 Tree constructions. We plan to build a rigid P-space as the space of paths through a tree. It will be sufficient to consider only trees of binary sequences, so we make all our definitions only for this case to simplify notation. A *binary sequence* is a function, s, from an ordinal, $\alpha = lh(s)$ into $2 = \{0, 1\}$. A *tree* is a set, T, of binary sequences such that whenever $s \in T$ and $\alpha < lh(s)$, $s|\alpha \in T$. The tree order on T is just $\subseteq$, and the αth level of T is $T \cap 2^\alpha$. If $s \in T$, let $T_s = \{t \in T : s \subseteq t\}$. The *height* of T is $\sup\{lh(s)+1 : s \in T\}$. We will be considering trees whose height is a regular uncountable cardinal.

A *path* through a tree T of height $\varkappa$ is an f: $\varkappa \to 2$ such that $f|\alpha \in T$ for all $\alpha < \varkappa$. The *path space*, $P(T)$ is the set of all paths through T. We will consider two topologies on $P(T)$. One, is the *order topology*, induced by the lexical order on $P(T)$. The other is the *tree topology*. This has as basic open (actually, clopen) sets all sets of the form

$$N_s = \{f \in P(T) : s \subset f\}$$

for each $s \in \mathscr{T}$. Equivalently, the tree topology is the topology that $P(T)$ inherits as a subspace of $2^\varkappa$ in the $<\varkappa$ box topology. Every set open in the order topology is open in the tree topology, but not conversely in general. Since the tree topology does not depend on the ordering, it is a little simpler to deal with, but since we wish to construct a LOTS, we obviously want to use the order topology. Fortunately, we need only consider trees on which the two topologies agree. Call a tree, T, of height $\varkappa$, *agreeable* iff for each $f \in P(T)$, $f^{-1}(\{0\})$ and $f^{-1}(\{1\})$ are both cofinal in $\varkappa$, and for each $s \in T$, $N_s \neq \emptyset$.

LEMMA 1. *If T is agreeable, then the order topology and the tree topology agree on $P(T)$.* ■

Call a space *$\varkappa$-compact* iff every open cover has a subcover of size less than $\varkappa$; thus, Lindelöf is the same as ω_1-compact. For any regular uncountable $\varkappa$, $P(T)$ is clearly a P-space in the tree topology. We shall show that for a suitable T, $P(T)$ will also be rigid and $\varkappa$-compact.

In the tree topology, $P(T)$ and all its subspaces are *$\varkappa$-metrizable* (see [JW], [S] and [Wan]), and we recall some facts about these spaces in our notation. $P(T)$ is paracompact; in fact, every open cover, $\mathscr{U}$, has a disjoint refinement of basic sets, namely

$$\{N_s : N_s \lhd \mathscr{U} \wedge \neg \exists \alpha < lh(s)(s|\alpha \lhd \mathscr{U})\}$$

($\lhd$ means "refines"). Thus, if $P(T)$ fails to be $\varkappa$-compact, T has an antichain, A, of size at least $\varkappa$ such that $\{N_s : s \in A\}$ covers $P(T)$. The subtree consisting of nodes below a member of A would then have no paths, and hence be Aronszajn if each level of T has size less than $\varkappa$. This proves the following result of Juhász and Weiss:

LEMMA 2 ([JW]). *Let T be a tree of height $\varkappa$, where $\varkappa$ is regular. Assume that $\forall_s \in T$ $(N_s \neq \emptyset)$. Assume also that:*

(i) *All levels of T have size less than $\varkappa$.*

(ii) *T has no $\varkappa$-Aronszajn subtrees.*

Then $P(T)$ is $\varkappa$-compact in the tree topology. ■

[JW] also shows that (i) and (ii) are necessary for $\varkappa$-compactness, but we do not need that fact here.

Carrying this idea further, we can represent open sets and continuous functions on $P(T)$ (in the tree topology) directly in terms of T. If U is any open set, let

$$R(U) = \{N_s\colon N_s \subseteq U \wedge \neg \exists \alpha < lh(s)(s|\alpha \subseteq U)\}.$$

Then $R(U)$ is an antichain in T and $U = \bigcup \{N_s\colon s \in R(U)\}$. If U is clopen and $P(T)$ is $\varkappa$-compact, then $R(U)$ has size less than $\varkappa$. Suppose $F\colon N_u \to N_v$ is continuous. Define its *representing function*, $\mathrm{Rep}(F)$ to be the function mapping T_v into subsets of T_u defined by:

$$\mathrm{Rep}(F)(s) = R(F^{-1}(N_s)).$$

Assuming the hypotheses of Lemma 2, $\mathrm{Rep}(F)$ is non-increasing in levels on a club (closed unbounded subset) of $\varkappa$:

LEMMA 3. *If T satisfies the hypotheses of Lemma 2, and $F\colon N_u \to N_v$ is continuous, then there is a club $C \subset \varkappa$ such that for each $\gamma \in C$ and $s \in T_v \cap 2^\gamma$, all members of* $\mathrm{Rep}(F)(s)$ *have length* $\leqslant \gamma$.

Proof. By the usual Löwenheim–Skolem argument, we may choose C to be a club of limit ordinals such that for each $\gamma \in C$ and each $s \in T_v \cap 2^{<\gamma}$, $\mathrm{Rep}(F)(s) \subseteq 2^{<\gamma}$. Now fix $\gamma \in C$, $s \in T_v \cap 2^\gamma$, and $t \in \mathrm{Rep}(F)(s)$. For each $\alpha < \gamma$, $N_t \subseteq F^{-1}(N_{s|\alpha})$, so $N_{t|\gamma} \subseteq F^{-1}(N_{s|\alpha})$. Thus, $N_{t|\gamma} \subseteq F^{-1}(N_s)$, so $t|\gamma = t$, so $\mathrm{lh}(t) \leqslant \gamma$. ∎

Actually, we can get all members of $\mathrm{Rep}(F)(s)$ to have length equal to γ unless F is constant on some open set.

LEMMA 4. *If T, F are as in Lemma 3 and F is not constant on any non-empty open set, then there is a club $D \subset \varkappa$ such that for each $\gamma \in D$ and $s \in T_v \cap 2^\gamma$, all members of* $\mathrm{Rep}(F)(s)$ *have length* γ.

Proof. Fix C as in the proof of Lemma 3. If no club $D \subseteq C$ satisfies 2.3, then there is a stationary $A \subset C$ such that for each $\alpha \in A$, there is an $s_\alpha \in T \cap 2^\alpha$ and a $t_\alpha \in \mathrm{Rep}(F)(s_\alpha)$ with $\mathrm{lh}(t_\alpha) < \alpha$. Then, by the pressing-down lemma, there is a $t \in T$ and a stationary $B \subseteq A$ such that $t_\alpha = t$ for all $\alpha \in B$. Thus, for all $\alpha \in B$, $N_t \subseteq F^{-1}(N_{s_\alpha})$, so $F''(N_t) \subseteq N_{s_\alpha}$. Thus, the s_α for $\alpha \in B$ cohere; that is, $g = \bigcup \{s_\alpha\colon \alpha \in B\}$ is a function, and $F''(N_t) = \{g\}$. ∎

If we forget about making the space rigid, it is quite easy to find a T satisfying the hypotheses of Lemma 2. Fix any function $h\colon \varkappa \to 2$. Following Kurepa, let

$$T_0 = \{s \in 2^{<\varkappa}\colon |\{\alpha < \mathrm{lh}(s)\colon s(\alpha) \neq h(\alpha)\}| < \omega\}.$$

It is easy to verify that the hypotheses to Lemma 2 hold. Also, assuming $\varkappa$ is uncountable,

$$P(T_0) = \{f \in 2^\varkappa\colon |\{\alpha < \varkappa\colon f(\alpha) \neq h(\alpha)\}| < \omega\}.$$

Thus, if $\varkappa = \omega_1$, we have a Lindelöf P-space of size and weight ω_1. It is also easy to see that neither the tree topology nor the order topology are rigid. We will get a rigid example by using a slightly larger tree. See § 4 of Todorčević [T] for some other ways to make T_0 larger and still have no Aronszajn subtrees.

Obviously, the choice of h does not affect the tree topology on $P(T_0)$. So, from now on, consider h fixed for all time, and to be neither eventually 0 nor eventually 1. This will guarantee that T_0 (and the larger tree that we build) will be agreeable, so that the tree and order topologies coincide.

If $\alpha < \beta <_! \varkappa$ and $s \in 2^\alpha$, we define the *h-extension* of s, $\mathrm{ext}(s, \beta)$ to be the $t \in 2^\beta$ such that $t|\alpha = s$ and $t(\xi) = h(\xi)$ whenever $\alpha \leqslant \xi < \beta$. Sequences of the form $\mathrm{ext}(s, \beta)$ where β is a limit ordinal and $\mathrm{lh}(s) < \beta$ are called *h-tails*.

The following three conditions describe a tree $T \subset 2^{<\varkappa}$ slightly larger than T_0.

1. $\forall s \in T(s^\frown 0 \in T \wedge s^\frown 1 \in t)$.
2. $\forall s \in T \forall \beta > \mathrm{lh}(s)(\mathrm{ext}(s, \beta) \in T)$.
3. For all ω-limits $\beta < \varkappa$, there is exactly one node, $e_\beta \in T \cap 2^\beta$, which is *not* an h-tail.

Such a T is a superset of T_0. T_0 satisfies (1) and (2), but in T_0, *all* nodes at limit levels are h-tails.

LEMMA 5. *Suppose that $\varkappa > \omega$ is regular and T satisfies* (1–3). *Then*:

a. *For all limits $\beta < \varkappa$, there is at most one node in $T \cap 2^\beta$ which is not an h-tail.*
b. *Each level of T has size less than $\varkappa$.*
c. $|P(T)| = \varkappa$.
d. *$P(T)$ is $\varkappa$-compact in the tree topology.*

Proof. For (a), if $\mathrm{cf}(\beta) > \omega$ and $s \in T \cap 2^\beta$ is not an h-tail, then there must be a club $C \subset \beta$ such that for all ω-limits, $\gamma \in C$, $s|\gamma$ is not an h-tail, so that $s|\gamma = e_\gamma$. Hence, for each such β, there can be at most one such s. For (b), induct on the levels, using (a) for levels of uncountable cofinality. For (c), observe that by the same argument as in (a), $P(T)$ contains at most one element other than all the h-tails, $\mathrm{ext}(s, \varkappa)$, for $s \in T$.

That $P(T)$ is $\varkappa$-compact will follow immediately from Lemma 2 if we can show that T has no $\varkappa$-Aronszajn subtrees. Let S be any subtree of T of height $\varkappa$. Assume S is Aronszajn. Then for each $s \in S$, $\mathrm{ext}(s, \varkappa)$ is not a path through S, so there is some limit ordinal $\varphi(s)$ with $\mathrm{lh}(s) < \varphi(s) < \varkappa$ and $\mathrm{ext}(s, \varphi(s)) \notin S$. There is then a club of limit ordinals, $C \subset \varkappa$ such that for each $\beta \in C$ and each $s \in S$, if $\mathrm{lh}(s) < \beta$ then $\varphi(s) < \beta$ (and hence $\mathrm{ext}(s, \beta) \notin S$). But then, for β an ω-limit in C, the only member of $S \cap 2^\beta$ is e_β, which is impossible if S is Aronszajn. ∎

To guarantee rigidity, we introduce a fourth condition:

4. For each $s \in T$, the set $W_s = \{\beta\colon s \subset e_\beta\}$ is stationary in $\varkappa$.

Note that if s and t are incompatible in T, then W_s and W_t are disjoint stationary sets. Conversely, using the fact that the ω-limits in $\varkappa$ can be partitioned into $\varkappa$ disjoint stationary sets, it is easy to construct a tree satisfying conditions (1–4) Before we prove rigidity, we note, as a trivial application of (4), that the tree and order topologies agree.

LEMMA 6. *Suppose that $\varkappa > \omega$ is regular and T satisfies* (1–4). *Then*

a. *T is agreeable.*

b. *No collection of $\varkappa$ of the e_β can form a chain.*

Proof. Every element in $P(T)$ is an h-tail, since, as we saw in the proof of Lemma 5, the only other possible element would arise from an ω-club of the e_β which form a chain, which is ruled out by (4). By our assumption on h, every path through T takes values 0 and 1 cofinally often, proving (a). For (b), a chain consisting of *any* $\varkappa$ of the e_β would yield an element of $P(T)$ which is not an h-tail. ■

LEMMA 7. *Suppose that $\varkappa > \omega$ is regular and T satisfies (1–4). Then $P(T)$ is rigid. In fact, if F: $P(T) \to P(T)$ is continuous, then*

$$\bigcup \{N_u\colon F|N_u \text{ is constant or is the identity}\}$$

is dense in $P(T)$.

Proof. It is sufficient to fix incompatible u and v in T such that F maps N_u into N_v and show that F is constant on some non-empty open subset of N_u. If this is not the case, then by Lemma 4 applied to $G = F|N_u$, there is a club D such that for all $\gamma \in D$ and $s \in T_v \cap 2^\gamma$, $\mathrm{Rep}(G)(s) \subseteq 2^\gamma$. In particular, for $\gamma \in D \cap W_v$, choose an element of $\mathrm{Rep}(G)(e_\gamma)$; since $\gamma \notin W_u$, this element is of the form $\mathrm{ext}(t_\gamma, \gamma)$, where $\mathrm{lh}(t_\gamma) < \gamma$. Then, by the pressing-down lemma, there is a fixed t and a stationary $A \subseteq D \cap W_v$ such that $t_\gamma = t$ for all $\gamma \in A$. Then $G(\mathrm{ext}(t, \varkappa)) \in N_{e_\gamma}$ for each $\gamma \in A$, so $\{e_\gamma\colon \gamma \in A\}$ forms a chain, contradicting Lemma 6. ■

Taking $\varkappa = \omega_1$, we have

2.1. THEOREM. *There is a rigid P-space, X, of size and weight ω_1. Furthermore, X is a Lindelöf* LOTS. ■

A somewhat larger tree will give us:

2.2. THEOREM. *There is a compact* LOTS, *X, of size* $\max((2^\omega)^+, 2^{\omega_1})$, *which is rigid in its G_δ topology.*

Proof. For a suitable $\varkappa$ and subtree $T \subseteq 2^{<\varkappa}$, X will be the set of *all* maximal chains in T; i.e.,

$$X = P(T) \cup \{s \in 2^{<\varkappa}\colon s \notin T \wedge \forall \alpha < \mathrm{lh}(s)(s|\alpha \in T)\}.$$

Order X lexicographically and give it the order topology; this is always compact. Unfortunately, if we use our previous construction verbatim, then X will not be rigid in its G_δ topology, since X will contain many elements of character ω, which will become isolated in the G_δ topology. To avoid this, make T countably closed. Thus, we keep conditions (1–2) the same, but replace (3) by:

$\tilde{3}$. T is countably closed and for all limit $\beta < \varkappa$ of cofinality ω_1, there is exactly one node, $e_\beta \in \mathrm{lev}_\beta(T)$ which is *not* an h-tail.

Condition (4) is the same; now all ordinals in W_s have uncountable cofinality, so $\varkappa$ obviously can no longer be ω_1; in fact, since our argument needed that each level of the tree had size less than $\varkappa$, we need that $\lambda^\omega < \varkappa$ for all $\lambda < \varkappa$; the least $\varkappa$ with this property is $(2^\omega)^+$. With these modifications, the argument that $P(T)$ is rigid goes through as before. Furthermore, $P(T)$ is dense in X (with the G_δ topology) and is defined topologically in X (with the G_δ topology) as the set of points of character $\varkappa$. It follows that X is rigid in its G_δ topology.

We now must compute $|X|$. As before, $P(T)$ has size $\varkappa$, since its only elements are of the form $\mathrm{ext}(s, \varkappa)$. Now, for each $\gamma < \varkappa$ of uncountable cofinality, consider the possible $s \in 2^\gamma \cap X$. Note that no such s is an h-tail. If $\mathrm{cf}(\gamma) \geqslant \omega_2$, then s must be formed by the e_β cohering on a club subset of γ, so there is at most one such s. If $\mathrm{cf}(\gamma) = \omega_1$, there are $|\gamma|^{\omega_1}$ such s. Thus, $|X|$ is the larger of $\varkappa$ and $\sup\{\lambda^{\omega_1}\colon \lambda < \varkappa\}$, which is as advertised if $\varkappa = (2^\omega)^+$. ■

This leaves open the question of whether, in ZFC, one can produce an X as in Theorem 2.2 which has size 2^{ω_1}. Even better, can all points in X have character ω_1? Under $V = L$, this is easy. Just replace the T of 2.2 with a suitable countably closed ω_2-Suslin tree. Then $P(T) = \emptyset$, but one can use $\diamondsuit$ instead, when constructing T, to kill all potential homeomorphisms.

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Received 24 September 1987