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Non-trivial homeomorphisms of $\beta N \setminus N$ without the continuum hypothesis

by

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Abstract. The problem of constructing non-trivial homeomorphisms of $\beta N \setminus N$ without assuming the continuum hypothesis is examined.

In [3] Shelah showed that it is consistent that all automorphisms of $\mathscr{P}(\omega)/Finite$, or, equivalently, all autohomeomorphisms of $\beta N \setminus N$, are trivial in the sense that they are induced by almost-permutations of the integers (an almost-permutation of ω is an injective function from ω to ω whose domain and range are both cofinite). In [2] W. Rudin showed that the continuum hypothesis implies thet there is a non-trivial autohomeomorphism by showing that there are in fact $2^{2^{\aleph_0}}$ such homeomorphisms. It is the purpose of this paper to examine the question of how to construct non-trivial autohomeomorphisms in the absence of the continuum hypothesis. The reader should be warned that $\beta N \setminus N$ and $\mathscr{P}(\omega)/Finite$ will be used almost interchangeably. As well, subsets of the integers will routinely be confused with clopen sets in $\beta N \setminus N$.

At this point the reader may be wondering why the argument assuming $2^{\aleph_0} = \aleph_1$ does not generalize to $MA_{\aleph_1}$ and make the rest of this paper pointless. The reason, of course, is that an induction of length greater than ω_1 may run into a Hausdorff gap and stop. In fact it will be shown in [4] that PFA implies that all autohomeomorphisms of $\beta N \setminus N$ are trivial and so this is consistent with $MA_{\aleph_1}$. This raises the following unanswered question:

QUESTION. Is it consistent with $MA_{\aleph_1}$ that there is a non-trivial autohomeomorphism of $\beta N \setminus N$?

The first result towards obtaining non-trivial autohomeomorphisms of $\beta N \setminus N$ without the continuum hypothesis is due to Frolik [1]. He showed that the set of fixed points of any 1-1 continuous function from an extremally disconnected space to itself form a clopen set. To see how this can be used to construct non-trivial autohomeomorphisms of $\beta N \setminus N$ consider the following lemma.

LEMMA 1. *Suppose that $\mathscr{I}$ is an ideal on ω generated by an $\subseteq^*$-ascending sequence $\{A_\alpha: \alpha \in \varkappa\}$. Suppose further that f_α is an almost-permutation of A_α for* *each $\alpha \in \varkappa$ and that if $\alpha \in \beta$ then $f_\alpha \subseteq^* f_\beta$. Then these functions $\{f_\alpha:\ \alpha \in \varkappa\}$ induce an isomorphism, Φ, of the subalgebra of $\mathcal{P}(\omega)/Finite$ generated by $\mathcal{I}$ defined by*

$$\Phi(X) = \begin{cases} f_\alpha'' X \text{ if there is some } \alpha \text{ such that } X \subseteq^* A_\alpha, \\ \omega \setminus f_\alpha''(\omega \setminus X) \text{ if there is some } \alpha \text{ such that } \omega \setminus X \subseteq^* A_\alpha. \end{cases}$$

Proof. Easy.

Now suppose that $\{A_\alpha:\ \alpha \in \omega_1\}$ is a neighbourhood base of clopen sets for a p-point, $\mathcal{F}$, in $\beta N \setminus N$. It is easy to construct permutations f_α of $\omega \setminus A_\alpha$ such that if $\alpha \in \beta$ then $f_\alpha \subseteq^* f_\beta$ and such that homeomorphism of the clopen set $A_\alpha \setminus A_{\alpha+1}$ induced by $f_{\alpha+1}$ is not the identity. By Lemma 1 these permutations induce an autohomeomorphism Φ of $\beta N \setminus N$ (since the subalgebra generated by $\mathcal{F}$ is all of $\mathcal{P}(\omega)/Finite$). Note that $\mathcal{F}$ is an isolated fixed point of Φ. If Φ was trivial then either it or its inverse would extend to a continuous 1-1 function from βN to $\beta(N \setminus a)$ where a is finite. But then $\mathcal{F}$ would still be an isolated fixed point contradicting the fact that, since βN is extremally disconnected, the set of fixed points must be clopen. Consequently, in any model where there is a p-point of character $\aleph_1$ (and there are many such models where $2^{\aleph_0} > \aleph_1$) there is a non-trivial autohomeomorphism of $\beta N \setminus N$. A similar proof due to Baumgartner appears in [5]. Recently van Douwen (unpublished) has shown that a point of character $\aleph_1$ is sufficient to imply the existence of a non-trivial autohomeomorphism of $\beta N \setminus N$.

This raises the question of whether it is necessary to have points of small character in order to have non-trivial autohomeomorphisms. It will be shown that in the model obtained by adding $\aleph_2$ Cohen reals to a model of the continuum hypothesis there is a non-trivial autohomeomorphism whereas it is known that in this model every point in $\beta N \setminus N$ has character $\aleph_2$. The result for p-points of character $\aleph_1$ will also be extended. It will be shown that if X is a closed p-set of character $\aleph_1$ in $\beta N \setminus N$ then the quotient space obtained by shrinking X to a point has a non-trivial autohomeomorphism. This is equivalent to saying that if $\mathcal{I}$ is a p-ideal of character $\aleph_1$ then there is an automorphism of the Boolean algebra generated by $\mathcal{I}$ which is not induced by any function from ω to ω. Moreover, it will be shown that this automorphism is absolutely non-trivial in the sense that even in any ω_1-preserving extension of the set-theoretic universe it is not induced by a function from ω to ω. The automorphisms constructed by using the method of Baumgartner and Frolik need not have this property.

The significance of this absolute non-triviality becomes apparent upon considering the method used in [3] to construct a model where all automorphisms of $\mathcal{P}(\omega)/Finite$ are trivial. The construction consists of trapping non-trivial automorphisms and adding subsets of ω to which it is impossible to extend the automorphism. One might wonder whether it is possible to obtain such a model by adding generic permutations which turn a non-trivial automorphism into a trivial one. The absolute non-triviality of the automorphisms to be constructed shows that this is not possible. The final point worth noting in this regard is the connection with uniformization properties. In [3] page 58 it is shown that $MA_{\aleph_1}$ implies that if $\{A_\alpha:\ \alpha \in \omega_1\}$ is a certain type of almost disjoint family and $f_\alpha:\ A_\alpha \to \omega$ are functions then there is $F:\ \omega \to \omega$ such that for all $\alpha \in \omega_1$ $F \restriction A_\alpha \equiv^* f_\alpha$. One might reasonably conjecture that a similar uniformization property is true for towers $\{A_\alpha:\ \alpha \in \omega_1\}$ and functions $f_\alpha:\ A_\alpha \to \omega$ so long as $\alpha \in \beta$ implies $f_\alpha \subseteq^* f_\beta$. Again the example to be constructed shows that this is not possible.

THEOREM 1. *If $\aleph_2$ Cohen reals are added to a model where $2^{\aleph_0} = \aleph_1$ then there is a non-trivial automorphism of $\mathcal{P}(\omega)/Finite$ in the resulting model.*

Proof. Let C_γ represent the partial order for adding $\omega_1\gamma$ Cohen reals. If G is C_{ω_2} generic over a model of the continuum hypothesis then let $V_\gamma = V[G \cap C_\gamma]$. It will be shown by induction on $\gamma \in \omega_2$ that there is an automorphism of $(\mathcal{P}(\omega)/Finite) \cap V_{\gamma+1}$, which will be referred to as F_γ. Moreover, the automorphism F_γ will be constructed so that if $\delta \in \gamma$ then $F_\delta \subseteq F_\gamma$. The desired automorphism will be $\bigcup\{F_\gamma;\ \gamma \in \omega_2\}$. The fact that is non-trivial will follow from the fact that each automorphism F_γ will be constructed so that there is no permutation of ω in $V_{\gamma+1}$ which induces F_γ.

To see how to construct the automorphisms notice that since V_1 is a model of the continuum hypothesis it is easy to construct F_0 inductively to satisfy the induction hypothesis. Now suppose that F_γ has been constructed on $(\mathcal{P}(\omega)/Finite) \cap V_{\gamma+1}$. To construct $F_{\gamma+1}$ on $(\mathcal{P}(\omega)/Finite) \cap V_{\gamma+2}$ proceed by induction on $(\mathcal{P}(\omega)/Finite) \cap V_{\gamma+2} \setminus V_{\gamma+1}$. Let $\{\pi_\xi:\ \xi \in \omega_1\}$ enumerate all almost-permutations of ω and $\{X_\xi:\ \xi \in \omega_1\}$ enumerate $\mathcal{P}(\omega)/Finite$ in $V_{\gamma+2}$. Suppose that B is a countable subalgebra of $\mathcal{P}(\omega)/Finite$ and that the automorphism $F_{\gamma+1}$ has been defined on the algebra generated by $\big((\mathcal{P}(\omega)/Finite) \cap V_{\gamma+1}\big) \cup B$ (call this algebra B'). Let $X_\xi \in (\mathcal{P}(\omega)/Finite) \setminus B'$ and let X be an equivalence class representative of X_ξ. Define

$$\mathcal{F}(X) = \{b \in B';\ X \subseteq^* b\}, \quad \mathcal{I}(X) = \{b \in B';\ X \supseteq^* b\}.$$

It will first be shown that $\mathcal{F}(X)$ and $\mathcal{I}(X)$ are countably generated and then this fact will be used to extend the automorphism.

Since B is countable, it suffices to show that $\{b \in B' \cap V_{\gamma+1};\ X \subseteq^* b\}$ and $\{b \in B' \cap V_{\gamma+1};\ X \supseteq^* b\}$ are countably generated. Let C be a countable completely embedded subalgebra of $C_{\gamma+1}$ such that X has a C name. For each $q \in C$ let $X(q) = \{n \in \omega;\ q \text{ forces } \text{“}n \in N\text{”}\}$. It will be shown that $\{X(q);\ q \in C\}$ generate $\{b \in B' \cap V_{\gamma+1};\ X \subseteq^* b\}$ (a similar proof will work for $\{b \in B' \cap V_{\gamma+1};\ X \supseteq^* b\}$). Now suppose that $p \in C_{\gamma+1}$ is a condition forcing that $X \subseteq b \cup k$ for some integer k. Then let p^* be the projection of p on C. To see that $b \subseteq X(p^*) \cup k$ suppose that $m \in b \setminus k$. If $m \notin X(p^*)$ then there is some $q \supseteq p^*$ such that q forces “$m \notin X$”. Moreover, it may be assumed that $q \in C$ since C is completely embedded in $C_{\gamma+1}$. Hence $q \cup p$ is a condition which forces contradictory statements.

Now to extend $F_{\gamma+1}$ it suffices to define $F_{\gamma+1}(X_\xi) = Y_\xi$ where Y is an equivalence class representative of Y_ξ and the following conditions are satisfied:

1. $Y_\xi \notin B'$ and $Y \neq \pi_\xi(X)$;
2. $F_{\gamma+1}(b) \supseteq^* Y_\xi \supseteq^* F_{\gamma+1}(c)$ for every $b \in \mathcal{F}(X)$ and $c \in \mathcal{I}(X)$;

3. if neither b nor $\omega\setminus b$ is in $\mathscr{F}(X)\cup\mathscr{I}(X)$ then $Y\cap b$ and $b\setminus Y$ are both infinite.

It is left to the reader to verify that extending $F_{\gamma+1}$ to the algebra generated by $B'\cup\{Y_\xi\}$ in the natural way is an isomorphism onto the algebra generated by $(F''_{\gamma+1}B')\cup\{Y_\xi\}$.

So it suffices to see why Y can be found. Since $V_{\gamma+2}$ is obtained from $V_{\gamma+1}$ by adding ω_1 Cohen reals and B is countable it is possible to find a Cohen generic real of B'. Notice that the partial order for splitting the gap formed by $(F_{\gamma+1}''\mathscr{F}(X)$ and $F_{\gamma+1}''\mathscr{I}(X)$ is a countable partial order since the gap is countably generated. Hence the Cohen real can be thought of as filling this gap. It is easy to check that such a Cohen real satisfies properties (1), (2) and (3). So if the induction is carried out and $F_{\gamma+1}$ and $F_{\gamma+1}^{-1}$ are dealt with alternately so as to make the limit function surjective then $F_{\gamma+1}$ will have been defined as wanted.

The only thing left to consider is the limit stages. Limits of cofinality ω_1 take care of themselves. The limits of cofinality ω are handeled almost the same as the successor case; the only difference is that if γ has cofinality ω then F_γ has been defined on

$$(\mathscr{P}(\omega)/\mathit{Finite})\cap(\bigcup\{V_\delta;\ \delta\in\gamma\}).$$

Instead of defining $F_{\gamma+1}$ on $V_\gamma\setminus\bigcup\{V_\delta;\ \delta\in\gamma\}$ we define it on $V_{\gamma+1}\setminus\bigcup\{V_\delta;\ \delta\in\gamma\}$ so that there will be enough Cohen reals to make the argument work.

THEOREM 2. *If $\mathscr{I}$ is the dual of a p-filter of character $\aleph_1$ then there is an automorphism of the Boolean algebra generated by $\mathscr{I}$ which is not induced by any function from ω to ω. Moreover, this is upward absolute with respect to models preserving ω_1. (The automorphism itself does extend canonically to the Boolean algebra generated by $\mathscr{I}$ in any extensions of the universe.)*

Proof. Let $\{A_\gamma;\ \gamma\in\omega_1\}$ be $\subseteq^*$-ascending. It may be assumed that $A_\gamma\subseteq A_{\gamma+1}$ for each γ. Furthermore, if $\gamma=\lambda+k$ where λ is a limit then it may be assumed that $k\in A_\gamma$. Hence if ν is a limit then $A_\nu\subseteq\bigcup\{A_{\sigma(n)};\ n\in\omega\}$ for some increasing sequence σ approaching ν. Transfinite induction on ω_1 will be used to define bijections $f_\alpha\colon A_\alpha\to A_\alpha$ such that the following conditions are satisfied:

4. $f_\alpha\circ f_\alpha=\mathrm{id}$;
5. if $\alpha\in\beta$ then $f_\alpha\subseteq^* f_\beta$;
6. if $P_X(f,g)$ represents the statement

$$\text{“}\forall m\in\omega\setminus X)\ (\text{if } m\in\mathrm{dom}(f)\cap\mathrm{dom}(g) \text{ then } f(m)=g(m))\text{”}$$

then $\{\beta\in\alpha;\ P_n(f_\beta,f_\alpha)\}$ is finite for each n.

Condition (6) is reminiscent of Hausdorff's construction of an (ω_1,ω_1^*) gap. Finally, if f is a function and $X\in[\omega]^{<\omega}$ then let $f\dagger X$ represent the restriction of f to $\mathrm{dom}(f)\setminus X$. Lemma 1 and condition (5) assure that the functions $\{f_\alpha\colon \alpha\in\omega_1\}$ will generate an automorphism Φ of the Boolean algebra generated by $\mathscr{I}$.

Note that (6) implies that there is no $g\colon\omega\to\omega$ such that $g\supseteq^* f_\beta$ for all β because otherwise there is some k such that

$$\{\beta\in\omega_1;\ g\supseteq f_\beta\dagger k\}$$

is uncountable. Let γ belong to $\{\beta\in\omega_1;\ g\supseteq f_\beta\dagger k\}$ with infinitely many predecessors in this set. Now if

$$\alpha\in\{\beta\in\gamma;\ g\supseteq f_\beta\dagger k\}$$

then $(f_\alpha\dagger k)\cup(f_\gamma\dagger k)\subseteq g$ and hence it follows that $P_k(f_\alpha,f_\gamma)$ holds contradicting (6). It follows that Φ is non-trivial.

It will now be shown how to carry out the induction. Suppose that $\{f_\alpha;\ \alpha\in\gamma\}$ have been constructed. If $\gamma=\beta+1$ then let

$$f_\gamma(m)=\begin{cases} m & \text{if } m\in A_\gamma\setminus A_\beta,\\ f_\beta(m) & \text{if } m\in A_\beta.\end{cases}$$

Notice that since $A_{\beta+1}\supseteq A_\beta$ it follows that $f_\gamma\supseteq f_\beta$ and hence (5) holds. To see that (6) holds note that for each $k\in\omega$

$$\{\mu\in\beta;\ P_k(f_\mu,f_\gamma)\}\subseteq\{\mu\in\beta;\ P_k(f_\mu,f_\beta)\}$$

because $f_\gamma\supseteq f_\beta$.

If γ is a limit then let $\{\sigma(n);\ n\in\omega\}$ be an increasing sequence cofinal in γ such that $\bigcup\{A_{\sigma(n)};\ n\in\omega\}\supseteq A_\gamma$. Define by induction on ω finite sets $K_i\subseteq\omega$ and finite functions h_i from $A_{\sigma(i)}\cap K_i$ to A_γ such that:

7. K_i is closed under $f_{\sigma(i)}$;
8. $i\subseteq K_i$;
9. if $F_i=\bigcup\{h_j\cup(f_{\sigma(j)}\dagger K_j);\ j\leqslant i\}$ then F_i is injective
10. $\mathrm{domain}(F_i)=\mathrm{range}(F_i)\supseteq\bigcup\{A_{\sigma(j)}\cap A_\gamma;\ j\leqslant i\}$;
11. $h_i\circ h_i=\mathrm{id}$;
12. if $\beta\in\sigma(i+1)\setminus\sigma(i)$ and both $P_{K_i}(f_\beta,F_i)$ and $P_{K_{i+1}}(f_\beta,f_{\sigma(i+1)})$ hold then $((K_{i+1}\setminus K_i)\cap A_{\sigma(i+1)}\cap A_\beta)\setminus\mathrm{range}(F_i)\neq 0$;
13. if $\beta\in\sigma(i+1)\setminus\sigma(i)$, $P_{K_{i+1}}(f_\beta,f_{\sigma(i+1)})$ holds and j belongs to

$$((K_{i+1}\setminus K_1)\cap A_{\sigma(i+1)}\cap A_\beta)\setminus\mathrm{range}(F_i)$$

then $h_{i+1}(j)\neq f_\beta(j)$;

14. there is some $j\in K_{i+1}\setminus K_i$ such that $h_{i+1}(j)\neq f_{\sigma(i+1)}(j)$.

If the induction can be carried out then let $f_\gamma=\bigcup\{F_i;\ i\in\omega\}$. The fact that f_γ is a bijection follows from (9) and (10) and the choice of the sequence $\{\sigma(n);\ n\in\omega\}$. The fact that (4) is satisfied follows from (11) and (5) follows from the construction. To see that (6) holds let $n\in\omega$. Then $n\subseteq K_n$ and it suffices to show that

$$\{\beta\in\gamma;\ P_{K_n}(f_\beta,f_\gamma)\}$$

is finite. But note that

$$\{\beta \in \sigma(n);\ P_{K_n}(f_\beta, f_\gamma)\} \subseteq \{\beta \in \sigma(n);\ P_{K_n}(f_\beta, f_{\sigma(n)} \dagger K_n)\}$$
$$= \{\beta \in \sigma(n);\ P_{K_n}(f_\beta, f_{\sigma(n)})\}$$

and the last set is finite by the induction hypothesis.

Hence, it suffices to show that if $\beta \geqslant \sigma(n)+1$ then $P_{K_n}(f_\beta, f_\gamma)$ fails. First note that (14) ensures that $P_{K_n}(f_\gamma, f_{\sigma(m)})$ fails for every $m \geqslant n+1$ and so it may be assumed that $\sigma(m) < \beta < \sigma(m+1)$, where $m \geqslant n$. But now it follows that $P_{K_{m+1}}(f_\beta, f_{\sigma(m+1)})$ holds if $P_{K_n}(f_\beta, f_\gamma)$ does since $K_n \subseteq K_{m+1}$. Hence (12) ensures that either $P_{K_m}(f_\beta, F_m)$ fails or $((K_{m+1} \setminus K_m) \cap A_{\sigma(m+1)}) \cap A_\beta \setminus \mathrm{range}(F_m) \neq 0$. But the first possibility implies that $P_{K_m}(f_\beta, f_\gamma)$ fails and so the second must hold. Now an application of (13) yields that $P_{K_m}(f_\beta, h_{m+1})$ fails, and hence so does $P_{K_n}(f_\beta, f_\gamma)$.

It now suffices to show that the induction can be carried out. To this end suppose that h_i and K_i have been constructed so that properties (8) to (14) hold. (To begin the induction simply choose K_0 so that $A_{\sigma(0)} \setminus K_0 \subseteq A_\gamma$ and $(f_{\sigma(0)} \dagger K_0)''(A_{\sigma(0)} \setminus K_0) = A_{\sigma(0)} \setminus K_0$ and h_0: $K_0 \cap A_{\sigma(0)} \to A_\gamma \setminus A_{\sigma(0)}$ consists of 2-cycles.) Choose T to be finite and such that $F_i \dagger T \subseteq f_{\sigma(i+1)}$, T is closed under both F_i and $f_{\sigma(i+1)}$, $K_i \cup i \subseteq T$ and $(f_{\sigma(i+1)} \dagger T) \cup F_i$ is injective. Let

$$B = \{\beta \in (\sigma(i+1) \setminus \sigma(i));\ P_T(f_\beta, f_{\sigma(i+1)})\}.$$

Then B is finite. Now choose $K_{i+1} \supseteq T$ such that for each $\beta \in B$ there is

$$j \in ((K_{i+1} \setminus T) \cap A_\beta \cap A_{\sigma(i+1)}) \setminus \mathrm{range}(F_i)$$

and K_{i+1} is closed under $f_{\sigma(i+1)}$. As well, it may be assumed that $(K_{i+1} \setminus K_i) \cap A_{\sigma(i+1)} \neq 0$. Now let $C = \{\beta \in (\sigma(i+1) \setminus \sigma(i));\ P_{K_{i+1}}(f_\beta, f_{\sigma(i+1)})\}$. Once more the induction hypothesis implies that C is finite. Now define

$$h_{i+1}\colon K_{i+1} \to A_\gamma \setminus (\mathrm{range}(F_i) \cup A_{\sigma(i+1)})$$

such that h_{i+1} consists of 2-cycles and $h_{i+1}(j) \neq f_\beta(j)$ and $h_{i+1}(j) \neq f_{\sigma(i+1)}$ for all $\beta \in C$ and $j \in K_{i+1}$.

Except for (12) it is easy to verify that properties (7) to (14) all hold. To see that (12) holds suppose that $\beta \in \sigma(i+1) \setminus \sigma(i)$, and both $P_{K_i}(f_\beta, F_i)$ and $P_{K_{i+1}}(f_\beta, f_{\sigma(i+1)})$ hold. There are two possibilities. The first one is that $P_T(f_\beta, f_{\sigma(i+1)})$ holds. In this case $\beta \in B$ and so the choice of K_{i+1} ensures that (12) is satisfied. If, on the other hand, $P_T(f_\beta, f_{\sigma(i+1)})$ fails then, since $P_{K_{i+1}}(f_\beta, f_{\sigma(i+1)})$ does hold, there is some $j \in K_{i+1} \setminus T$ such that $f_\beta(j) \neq f_{\sigma(i+1)}(j)$. Hence

$$j \in (K_{i+1} \setminus K_i) \cap A_{\sigma(i+1)} \cap A_\beta .$$

It suffices to show that $j \notin \mathrm{range}(F_i)$. But since range $(F_i \dagger T) = \mathrm{domain}(F_i \dagger T)$ and $F_i \dagger T \subseteq f_{\sigma(i+1)}$ it follows that, if $j \in \mathrm{domain}(F_i)$, then $F_i(j) = f_{\sigma(i+1)}(j) \neq f_\beta(j)$. This contradicts that $P_{K_i}(f_\beta, F_i)$ holds.

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