On local 1-connectedness of Whitney continua

by

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Dedicated to Professor Yukihiro Kodama on his 60th birthday

Abstract. In [23], S. B. Nadler proved that the property of being locally connected is a Whitney property. In [25], A. Petrus proved that there is a Whitney map ω on a 2-cell D such that $\omega^{-1}(t)$ is not locally 2-connected and not 2-connected for some $t > 0$, which implies that the property of being locally 2-connected (or 2-connected) is not a Whitney property. Naturally, the following problem will be raised: Is it true that the property of being locally 1-connected is a Whitney property? In this paper, we prove the following. (1) If X is a locally 1-connected continuum contained in a 2-dimensional manifold, then each Whitney continuum of X is also locally 1-connected. Moreover, if X is simply connected, then each Whitney continuum of X is simply connected. (2) There exist a 2-dimensional compact AR Z and a Whitney map ω for $C(Z)$ such that $\omega^{-1}(t)$ is not locally 1-connected and not simply connected for some $t > 0$.

0. Introduction. By a *continuum* we mean a nonempty compact connected metric space. Let X be a continuum with metric d. By the *hyperspace* of X we mean $C(X) = \{A|\ A$ is a subcontinuum of $X\}$ with *Hausdorff metric* d_H. In [29], H. Whitney proved that for any continuum X there exists a map $\omega\colon C(X) \to [0, \infty)$ satisfying

(1) $\omega(\{x\}) = 0$ for every $x \in X$, and

(2) if $A, B \in C(X)$, $A \subset B$ and $A \neq B$, then $\omega(A) < \omega(B)$.

Any such a map ω is called a *Whitney map* and $\omega^{-1}(t)$ $(0 \leqslant t < \omega(X))$ is called a *Whitney continuum*. A topological property P is called a *Whitney property* if whenever X has property P, so does every Whitney continuum in $C(X)$. Many properties of Whitney continua have been studied by many authors (e.g., see references).

In [23], S. B. Nadler proved that the property of being locally connected is a Whitney property. In [25], A. Petrus proved the following result:

There exists a Whitney map ω for $C(D)$ such that $\omega^{-1}(t)$ is not locally 2-connected and not 2-connected for some $t > 0$, where D is a 2-cell. But, in [26] J. T. Rogers proved that if X is a continuum with $\check{H}^1(X) = 0$, then for any Whitney map ω for $C(X)$ $\check{H}^1(\omega^{-1}(t)) = 0$ for any t, where $\check{H}^1(X)$ denotes the Čech 1-dimensional cohomology of X. Also, in [10], we proved that the property of being shape 1-connected is a Whitney property. Naturally, the following problem will be raised: Is it true that the property of being locally 1-connected is a Whitney property? In this paper, we answer to this problem. In fact, we prove the following:

(1) If X is a locally 1-connected continuum contained in a 2-dimensional manifold, then each Whitney continuum of X is also locally 1-connected. Moreover, if X is simply connected, then each Whitney continuum of X is simply connected. Hence every Whitney continuum of D is locally 1-connected and simply connected.

(2) There exist a 2-dimensional compact AR Z and a Whitney map ω for $C(Z)$ such that $\omega^{-1}(t)$ is not locally 1-connected and not simply connected for some $t > 0$.

We refer readers to [15] and [24] for hyperspace theory.

1. Preliminaries. In this section, we list some notations and facts which will be needed in the sequel.

A metric space X is *locally n-connected* if for each $x \in X$ and neighborhood U of x in X there exists a neighborhood V of x in U such that each map f: $S^i \to V$ $(i \leqslant n)$ is null homotopic in U, where S^i denotes the i-sphere. A metric space X is *approximately locally n-connected* (see [1]) if for each $x \in X$ and neighborhood U of x in X there exists a neighborhood V of x in U such that for any $\varepsilon > 0$ and map f: $S^i \to V$ $(i \leqslant n)$ such that $x \in f(S^i)$ there is a map g: $S^i \to V$ such that g is ε-closed to f and g is null homotopic in U. We need the following (see [1, (2.1)]).

(1.1) *Let X be a locally compact metric space. If X is locally $(n-1)$-connected and approximately locally n-connected $(n \geqslant 1)$, then X is locally n-connected.*

A compactum X is *nearly* 1-*movable* [21] if for some (and hence for every) embedding X into the Hilbert cube Q, the following holds: For each neighborhood U of X in Q there exists a neighborhood V of X in U such that for each loop f: $S^1 = \partial D \to V$ and for each neighborhood W of X in Q there is a finite, disjoint collection of disks D_i in D and an extension f^*: $(\overline{D - \bigcup D_i}) \to U$ of f such that $f^*(\bigcup \partial D_i) \subset W$. Clearly, "1-movable" implies "nearly 1-movable". It is well-known that solenoids are not nearly 1-movable.

(1.2) (D. R. McMillan, Jr. [21]). *Each continuum in a 2-dimensional manifold is movable, hence nearly* 1-*movable.*

(1.3) (J. Dydak [2]). *Let f: $X \to Y$ be an onto map between continua such that $f^{-1}(y)$ is a nearly 1-movable continuum for each $y \in Y$. If X is locally 1-connected, then Y is also locally 1-connected and $\pi_1(f)$: $\pi_1(X) \to \pi_1(Y)$ is an epimorphism.*

(1.4) (J. Dydak [2]). *If X is a subcontinuum of a locally 1-connected continuum Y and the decomposition space Y/X is locally 1-connected, then X is nearly 1-movable.*

(1.5) (J. Krasinkiewicz [16]). *Let X be a continuum and let ω be a Whitney map for $C(X)$. Then if $\omega(A) = \omega(B)$ and $\varepsilon > 0$, there is neighborhood U of A in X such that $B \subset U$ implies $d_H(A, B) < \varepsilon$.*

Let $(P, <)$ be a partially ordered space. Then a map ω: $P \to [0, \infty)$ is said to be a *Whitney map* if (i) $\omega(p) = 0$ for $p \in \operatorname{Min} P$, (ii) $\omega(p) < \omega(q)$ for $p < q$, and (iii) $\omega(p) = \omega(q)$ for $p, q \in \operatorname{Max} P$. Thus a Whitney map ω for $C(X)$ is a Whitney map in the above sense for $C(X)$ ordered by inclusion. Then the following fact is very useful.

(1.6) (L. E. Ward, Jr. [28]). *Let P be a compact metric partially ordered space such that $\operatorname{Min} P$ and $\operatorname{Max} P$ are disjoint closed sets and let Q be a closed subset of P such that $\operatorname{Min} Q \subset \operatorname{Min} P$ and $\operatorname{Max} Q \subset \operatorname{Max} P$. Then a Whitney map for Q can be extended to a Whitney map for P.*

2. Local 1-connectedness of Whitney continua. In this section, we prove the following theorem.

(2.1) THEOREM. *Let X be a continuum such that each subcontinuum of X is nearly 1-movable. If X is locally 1-connected, then for any Whitney map ω for $C(X)$, $\omega^{-1}(t)$ is also locally 1-connected for each t. Moreover, if X is simply connected, then $\omega^{-1}(t)$ is also simply connected for each t.*

Proof. Let ω be a Whitney map for $C(X)$ and let $0 < t < \omega(X)$. Consider the following subset Y in $X \times \omega^{-1}(t)$:

$$Y = \{(x, A) \in X \times \omega^{-1}(t) \mid x \in A\}.$$

Then Y is a continuum. In fact, the map $p = p_2|Y$: $Y \to \omega^{-1}(t)$ is an open and monotone map, where p_2: $X \times \omega^{-1}(t) \to \omega^{-1}(t)$ is the projection map. Note that $p^{-1}(A) \cong A$ for each $A \in \omega^{-1}(t)$ and A is nearly 1-movable. Hence, if Y is locally 1-connected, by (1.3) we see that $\omega^{-1}(t)$ is locally 1-connected. Now, we shall show that Y is locally 1-connected. First, we shall show that Y is locally connected. Let $(x, A) \in Y$ and $\mathscr{U}$ be any neighborhood of (x, A) in Y. Take $\varepsilon > 0$ and a neighborhood U of x in X such that if $y \in U$ and $d_H(A, B) < \varepsilon$, then $(y, B) \in \mathscr{U}$. By (1.5), there is a closed neighborhood A^* of A in X such that if $B \in \omega^{-1}(t) \cap C(A^*)$, then $d_H(A, B) < \varepsilon$. We may assume that A^* is a locally connected continuum. Take a path connected neighborhood V of x in X such that $V \subset U \cap A^*$. Set

$$\mathscr{W} = \{(y, B) \in Y \mid y \in V,\ B \in C(A^*)\}.$$

Then $\mathscr{W}$ is a neighborhood of (x, A) in Y and $\mathscr{W} \subset \mathscr{U}$. We show that $\mathscr{W}$ is path connected. Let $(y, B) \in \mathscr{W}$. Since $y \in V$, there is an arc α: $I \to V$ such that $\alpha(0) = x$, $\alpha(1) = y$. By [15], there are two segments β_1, β_2: $I \to \omega^{-1}([0, t])$ such that $\beta_1(0) = x$, $\beta_1(1) = A$, $\beta_2(0) = y$ and $\beta_2(1) = B$. Define an arc γ from A to B in $\omega^{-1}([0, t])$ by

$$\gamma(s) = \begin{cases} \beta_1(1-3s), & \text{for } 0 \leqslant s \leqslant 1/3, \\ \alpha(3s-1), & \text{for } 1/3 \leqslant s \leqslant 2/3, \\ \beta_2(3s-2), & \text{for } 2/3 \leqslant s \leqslant 1. \end{cases}$$

Since A^* is locally connected, there is a retraction

$$R\colon \omega^{-1}([0,t])\cap C(A^*)\to \omega^{-1}(t)\cap C(A^*)$$

such that $R(C)\supset C$ for each $C\in\omega^{-1}([0,t])\cap C(A^*)$ (see [4, (1.2)]). Then $\sigma = R\gamma\colon I\to\omega^{-1}(t)\cap C(A^*)$ is an arc from A to B such that $x\in\sigma(s)$ for $0\leqslant s\leqslant 1/3$, $\gamma(s)\in\sigma(s)$ for $1/3\leqslant s\leqslant 2/3$ and $y\in\sigma(s)$ for $2/3\leqslant s\leqslant 1$. Define an arc γ' from x to y in V by

$$\gamma'(s)=\begin{cases} x, & \text{for } 0\leqslant s\leqslant 1/3\,,\\ \alpha(3s-1), & \text{for } 1/3\leqslant s\leqslant 2/3\,,\\ y, & \text{for } 2/3\leqslant s\leqslant 1\,.\end{cases}$$

Define an arc $\theta\colon I\to X\times\omega^{-1}(t)$ by $\theta(s)=(\gamma'(s),\sigma(s))$ for $s\in I$. Then $\theta(s)\in Y$ and $\theta(s)\in\mathscr{W}$ for each $s\in I$. This implies that Y is locally connected. Next, we shall show that Y is approximately locally 1-connected. Let $(x,A)\in Y$ and $\mathscr{U}$ be any neighborhood of (x,A) in Y. Take a neighborhood A^* of A in X and a neighborhood U of x in X such that $(U\times C(A^*))\cap Y\subset\mathscr{U}$. We may assume that A^* is a locally connected continuum and $U\subset A^*$. Since X is locally 1-connected, there is a neighborhood V of x in U such that any loop $f\colon S^1\to V$ is null homotopic in U. Set $\mathscr{V}=(V\times C(A^*))\cap Y$. Let $\varepsilon>0$ and let $f\colon S^1\to\mathscr{V}$ be any map with $(x,A)\in f(S^1)$. We assume that $f(0)=(x,A)$. Consider the following maps $f_1=p_1f\colon S^1\to V$ and $f_2=p_2f\colon S^1\to\omega^{-1}(t)\cap C(A^*)$, where $p_1\colon X\times\omega^{-1}(t)\to X$ and $p_2\colon X\times\omega^{-1}(t)\to\omega^{-1}(t)$ are the projection maps. Since f_1 is null homotopic in U, there is an extension $f_1'\colon D\to U$ of f_1. Choose points $0=a_0,a_1,\dots,a_n,a_{n+1}=0$ of S^1 such that $\operatorname{diam}(f_2([a_i,a_{i+1}]))<\varepsilon/2$, $\operatorname{diam}\big[C(\bigcup\{B\in f_2([a_i,a_{i+1}])\})\cap\omega^{-1}(t)\big]<\varepsilon/2$ (see (1.5)), and $\operatorname{diam}(f_1([a_i,a_{i+1}]))<\varepsilon/2$ for each $i=0,1,\dots,n$ (see Figure 1).

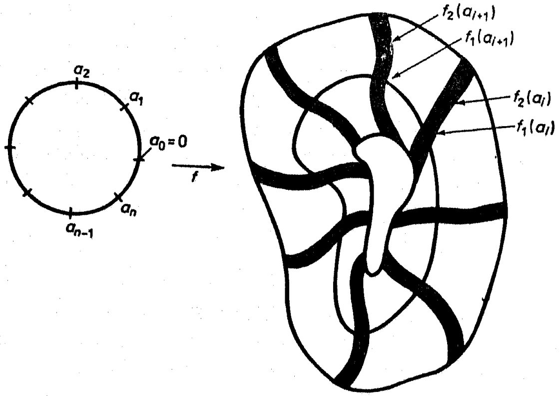

Fig. 1

For each i, take a neighborhood A_i of $\bigcup\{B\in f_2([a_i,a_{i+1}])\}$ in A^* such that $\operatorname{diam}(C(A_i)\cap\omega^{-1}(t))<\varepsilon/2$. We may assume that each A_i is a locally connected continuum. As before, take arcs $\alpha_i\colon I\to\omega^{-1}([0,t])$ $(i=0,1,\dots,n)$ such that

$\alpha_i(0)=f_1(a_i)$, $\alpha_i(1)=f_2(a_i)$ and $\alpha_i(s)\subset\alpha_i(s')$ if $s\leqslant s'$. Since each A_i is locally connected, there is a strong deformation retract $R_i\colon(C(A_i)\cap\omega^{-1}([0,t]))\to C(A_i)\cap\omega^{-1}(t)$ such that $R_i(B,s)\subset R_i(B,s')$ if $s\leqslant s'$. We identify $[a_i,a_{i+1}]$ with I, where $[a_i,a_{i+1}]$ is the arc from a_i to a_{i+1} in S^1. Define maps $g_i\colon I\times I\to C(A_i)\cap\omega^{-1}([0,t])$ by

$$g_i((1-s)z+sh(z))=\begin{cases} R_i(\alpha_i(z),s), & \text{for } z\in\{0\}\times I,\\ R_i(f_1(z),s), & \text{for } z\in I\times\{0\},\\ R_i(\alpha_{i+1}(z),s), & \text{for } z\in\{1\}\times I,\end{cases}$$

where $0\leqslant s\leqslant 1$, and for each $z\in\{0\}\times I\cup I\times\{0\}\cap\{1\}\times I$, $h(z)$ is the unique point of $I\times\{1\}$ such that the segment in $I\times[0,2]$ from the point $(1/2,3/2)$ to z contains $h(z)$ (see Figure 2). Define a map $g\colon D\times\{0\}\cup S^1\times I\to C(A^*)\cap\omega^{-1}([0,t])$ by

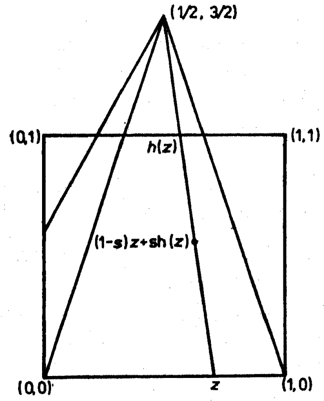

Fig. 2

$g|D\times\{0\}=f_1'$, $g|[a_i,a_{i+1}]\times I=g_i$. Since A^* is locally connected, there is a strong deformation retract $R\colon(C(A^*)\cap\omega^{-1}([0,t]))\times I\to C(A^*)\cap\omega^{-1}(t)$ such that $R(B,s)\subset R(B,s')$ if $s\leqslant s'$. Define a map $F_2\colon D\times\{0\}\cup S^1\times I\to C(A^*)\cap\omega^{-1}(t)$ by $F_2(y)=R(g(y),1)$. Then $d_H(F_2|S^1\times\{1\},f_2)<\varepsilon$. By identifying $[a_i,a_{i+1}]$ with I, define maps $k_i\colon I\times I\to A_i$ by

$$k_i((1-s)z+sh(z))=\begin{cases} f_1(a_i), & \text{for } z\in\{0\}\times I\,,\\ f_1(z), & \text{for } z\in I\times\{0\}\,,\\ f_1(a_{i+1}), & \text{for } z\in\{1\}\times I\,.\end{cases}$$

By using k_i, we have a map $F_1\colon D\times\{0\}\cup S^1\times I\to U$ such that $F_1|D\times\{0\}=f_1'$ and $F_1|[a_i,a_{i+1}]\times I=k_i$. Then we have $d(F_1|S^1\times\{1\},f_1)<\varepsilon$. By the construction, $F_1(y)\in F_2(y)$ for each $y\in D\times\{0\}\cup S^1\times I$. Define a map

$$F\colon D\times\{0\}\cup S^1\times I\to(U\times C(A^*))\cap Y \text{ by } F(y)=(F_1(y),F_2(y))$$

for $y \in D \times \{0\} \cup S^1 \times I$. Then $F|S^1 \times \{1\}$ is sufficiently near to f. Hence Y is approximately locally 1-connected. By (1.1), we see that Y is locally 1-connected. Hence we conclude that $\omega^{-1}(t)$ is locally 1-connected. Similarly, moreover, if X is simply connected, we see that $\omega^{-1}(t)$ is also simply connected. This completes the proof.

Let X be a continuum in a 2-dimensional manifold. Then X is locally 1-connected if and only if X is an ANR. Hence we have

(2.2) COROLLARY. *Let X be a continuum in a 2-dimensional manifold. If X is an* ANR, *then for each Whitney map ω for $C(X)$, $\omega^{-1}(t)$ is locally* 1-*connected for each* t. *In particular, every Whitney continuum of a* 2-*cell D is locally* 1-*connected and simply connected.*

Proof. (1.2) and (2.1) imply (2.2).

3. The property of being locally 1-connected is not a Whitney property. In this section, we prove, by showing a counterexample, that the property of being locally 1-connected (or simply connected) is not a Whitney property.

In fact, we show that there exist a 2-dimensional AR Z and a Whitney map ω for $C(Z)$ such that $\omega^{-1}(t)$ is not locally 1-connected and not simply connected for some $t > 0$.

It is known that if Z is a 1-dimensional ANR (resp. AR), then for any Whitney map ω for $C(Z)$, $\omega^{-1}(t)$ is an ANR (resp. AR) for each t (see [20]).

(3.1) EXAMPLE. Let $X = \{X_n, p_{n,n+1}, N\}$ be the inverse sequence as follows:

(1) $X_1 = \{*\}$ and $X_n (n \geqslant 2)$ is the unit circle S^1, and

(2) the bonding maps $p_{n,n+1}\colon X_{n+1} \to X_n$ are covering projections with degree 2 $(n \geqslant 2)$, and $p_{1,2}\colon X_2 \to X_1$ is the constant map.

Set $X = \operatorname{invlim} X$. Then X is the dyadic solenoid, and hence it is not nearly 1-movable. Let $p_n\colon X \to X_n$ be the projection. Now, consider the infinite telescope $T(X) = \bigcup_{n=1}^{\infty} M(p_{n,n+1})$, where $M(p_{n,n+1})$ denotes the mapping cylinder obtained by $p_{n,n+1}\colon X_{n+1} \to X_n$, i.e., $M(p_{n,n+1})$ is obtained by identifying points

$$(x, 1/n) \in X_{n+1} \times \{1/n\}$$

and $p_{n,n+1}(x) \in X_n$ for $x \in X_{n+1}$ in a topological sum $X_n \cup (X_{n+1} \times [1/(n+1), 1/n])$, and $T(X)$ is obtained by identifying each point of $X_n \times \{1/n\}$ in $M(p_{n-1,n})$ and the corresponding point of X_n in $M(p_{n,n+1})$. Let $Z = S(X) = T(X) \cup X$ be an AR having the same topology as in [17, (4.1)] (see Figure 3). Define a map $\mu\colon Z \to I$ by $\mu([x, t]) = t$ if $[x, t] \in T(X)$ and $\mu(x) = 0$ if $x \in X$. Also, define a natural retraction $\psi_t\colon Z \to \mu^{-1}([t, 1])\ (t \in I)$ by $\psi_t(z) = p_{q(t)}(x)$ for $x \in X$, $\psi_t(z) = [p_{q(t),n}(x), t]$ for $z = [x, s] \in \mu^{-1}((0, t])$ and $x \in X_n$, $\psi_t(z) = z$ for $z \in \mu^{-1}([t, 1])$, where $q(t)$ is the natural number such that $1/q(t) \leqslant t < 1/(q(t)-1)$ (for more detail construction of $S(X)$, see [17]). Note that $\mu^{-1}(t)$ is homeomorphic to S^1 for $0 < t < 1$. Then Z is a 2-dimensional AR. Next, we construct a Whitney

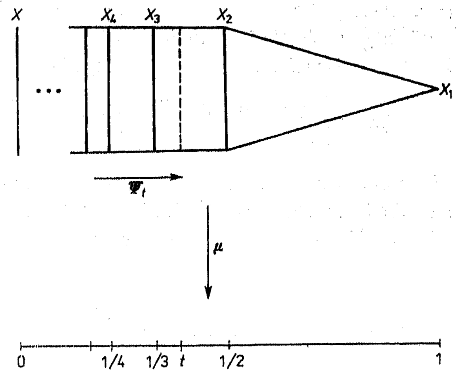

Fig. 3

map ω for $C(Z)$ as follows: Consider the arc $\mathscr{A} = \{\mu^{-1}(t) = \psi_t(X)|\ 0 \leqslant t \leqslant 1\}$ in $C(Z)$. Define a map $\omega_1\colon \mathscr{A} \to [0, 3/2]$ by $\omega_1(\mu^{-1}(t)) = t+1$ for $0 \leqslant t \leqslant 1/2$, and $\omega_1(\mu^{-1}(t)) = -3t+3$ for $1/2 \leqslant t \leqslant 1$. By (1.6), there exists a Whitney map ω_2 for $C(Z)$ which is an extension of ω_1. Define a map $\omega\colon C(Z) \to [0, \infty)$ by

$$\omega(A) = \sup\{\omega_2(\psi_t(A))|\ t \in \mu(A)\} + (\operatorname{diam} \mu(A))\, \omega_2(A)\,.$$

Clearly, $\omega(\{z\}) = 0$ for $z \in Z$. Suppose that $A, B \in C(Z)$, $A \subset B$ and $A \neq B$. If $B \subset \mu^{-1}(t)$ for some t, then $\omega(A) = \omega_2(A) < \omega_2(B) = \omega(B)$. If $\operatorname{diam} \mu(B) > 0$, then $\sup\{\omega_2(\psi_t(A))|\ t \in \mu(A)\} \leqslant \sup\{\omega_2(\psi_t(B))|\ t \in \mu(B)\}$ and

$$(\operatorname{diam} \mu(A))\, \omega_2(A) < (\operatorname{diam} \mu(B))\, \omega_2(B),$$

hence $\omega(A) < \omega(B)$. This implies that ω is a Whitney map for $C(Z)$. Note that $X \in \omega^{-1}(1)$. Now, we shall show that $\omega^{-1}(1)$ is not locally 1-connected. Consider the following decomposition space

$$(Z', *) = (\mu^{-1}([0, 1/2])/X, *)\,.$$

Let $q\colon \mu^{-1}([0, 1/2]) \to Z'$ be the quotient map. Since $\mu^{-1}([0, 1/2])$ is an ANR and X is not nearly 1-movable, (1.4) implies that Z' is not locally 1-connected at $*$. Let $\mathscr{U} = C(\mu^{-1}([0, 1/2])) \cap \omega^{-1}(1)$. Then $\mathscr{U}$ is a neighborhood of X in $\omega^{-1}(1)$. Let $A \in \mathscr{U}$. Suppose that $A \neq X$. Then we see that $\psi_{f(A)}(A)$ is an arc in $\mu^{-1}(f(A))$, where $f(A) = \sup\{s|\ s \in \mu(A)\}$. Then there is the unique point $\alpha(A)$ of $\psi_{f(A)}(A)$ such that there are two arcs A_1 and A_2 satisfying that $A_1 \cap A_2 = \{\alpha(A)\}$, $A_1 \cup A_2 = \psi_{f(A)}(A)$ and $\omega(A_1) = \omega(A_2)$. Define a map $F\colon \mathscr{U} \to Z'$ by

$$F(A) = \begin{cases} q\alpha(A) & \text{for } A \in \mathscr{U} \text{ and } A \neq X, \\ * & \text{for } A = X. \end{cases}$$

Let $z \in Z'$ and $z \neq *$. Then $q^{-1}(z)$ is one point set. Choose unique two arcs A_1 and A_2 in $\mu^{-1}(\mu(q^{-1}(z)))$ such that $A_1 \cap A_2 = \{q^{-1}(z)\}$, $\omega(A_1) = \omega(A_2)$ and $\omega(A_1 \cup A_2) = 1$. Set $\beta(z) = A_1 \cup A_2$. Define a map $G\colon Z' \to \mathscr{U}$ by

$$G(z) = \begin{cases} \beta(z) & \text{for } z \in Z' \text{ and } z \neq *, \\ X & \text{for } z = * \in Z'. \end{cases}$$

Then $FG = 1_{Z'}$. This implies that $\omega^{-1}(1)$ is not locally 1-connected at X. Similarly, we can conclude that Z/X is a retract of $\omega^{-1}(1)$. Since Z/X is not simply connected (see [22, Theorem 1]), $\omega^{-1}(1)$ is not simply connected.

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