We work in $L[a]$. Suppose $Y = \bigcup_n \bigcap_m B_{nm}$ where each B_{nm} is $\underset{\sim}{\Sigma}^0_{\alpha+2}$. For $n \in \omega$ let

$$D_n = \{p \in \boldsymbol{P}_\alpha \colon \exists m p \Vdash_{\boldsymbol{P}_\alpha} \mathring{T} \notin B_{nm}\}.$$

CLAIM 2. *D_n is dense.*

Let $p \in \boldsymbol{P}_\alpha$. Fix $\delta > \alpha \delta < \aleph_1^{L[\alpha]}$. Since $\boldsymbol{P}_\alpha \subseteq \boldsymbol{P}_\delta$, $p \in \boldsymbol{P}_\delta$. Suppose $\langle T^*, H\rangle \subseteq \boldsymbol{P}_\delta$ is generic over $L[a]$ and $p \in \langle T^*, H\rangle$. Clearly $\omega_1^{T^*} \geqslant \delta$, thus $T^* \notin Y$. Thus $p \nVdash_{P_\delta} \bigvee_n \bigwedge_m \mathring{T} \in B_{nm}$. Thus $p \nVdash_{P_\delta} \bigvee_m \mathring{T} \in B_{nm}$. Thus there is $m \in \omega$ and $r \leqslant p$ such that $r \Vdash_{P_\delta} \mathring{T} \notin B_{nm}$. Let $\bar{r} \in \boldsymbol{P}_\alpha$ be the retagging of r. By Lemma 2.2 $\bar{r} \Vdash_{P_\alpha} \mathring{T} \notin B_{nm}$, since $\neg B_{nm}$ is $\underset{\sim}{\Pi}^0_\alpha$. Clearly $r \leqslant \bar{p}$ so D_n is dense.

Let $\langle T, H\rangle$ be $\boldsymbol{P}_\alpha$-generic over $L[a]$. Since then D_n are dense $\forall n \in \exists m \in \omega \exists p \in \langle T, H\rangle\, p \Vdash_{P_\alpha} \mathring{T} \notin B_{nm}$. Thus $T \in \bigcap_n \bigcup_m \neg B_{nm} = \neg Y$. So $\omega_1^T \neq \alpha$, contradicting Lemma 2.2.

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Representability of $V[h]$ as intersection of Λ-bounded variation classes

by

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Abstract. It is proved that the generalized bounded variation class $V[h]$ of Čanturija is the intersection of all classes of Λ-bounded variation with $\Lambda = \{\lambda_i\}$ satisfying $\Sigma h(i)(\lambda_i^{-1} - \lambda_{i+1}^{-1}) < \infty$, but it is not the intersection of any countable subcollection of them. As a consequence of this result, a version of Helly's theorem for the classes $V[h]$ is proved.

1. Two important generalizations of the concept of bounded variation have been given by D. Waterman [4] and Z. A. Čanturija [2] by introducing, respectively, the functions of Λ-bounded variation (ΛBV) and the classes $V[h]$. These spaces have been studied mainly because of their applicability to the theory of Fourier series. An interesting connection between the class of functions of bounded variation (BV) and the classes ΛBV has been pointed out by Perlman [3], who has proved that the space BV is the intersection of all ΛBV classes but not of any countable collection of them. We shall prove an extension of Perlman's result to study the representability of the classes $V[h]$ as intersections of ΛBV classes. This theorem will allow us to prove a version for the classes $V[h]$ of the well-known Helly's theorem.

Let f be a function defined on an interval $[a, b]$. If $I = [x, y]$, we write $f(I) = f(y) - f(x)$. Let $\{I_i\}$ be a collection of nonoverlapping intervals $I_i \subseteq [a, b]$.

If $\Lambda = \{\lambda_i\}$ is a nondecreasing sequence of positive real numbers such that $\sum 1/\lambda_i = \infty$, we say that f is of Λ-bounded variation (ΛBV) on $[a, b]$ if $\sum |f(I_i)|/\lambda_i < \infty$ for every $\{I_i\}$. This is known to imply that the supremum $V_\Lambda(f)$ of the collection of the above sums is finite [4]. Also, if $f \in \Lambda BV$, then f is regulated, i.e., has only simple discontinuities.

Let

$$v(n, f, [a, b]) = v(n, f) = \sup \sum_{i=1}^{n} |f(I_i)|,$$

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the supremum being taken over all finite collections $\{I_i\}_{i=1}^n$. For a nondecreasing, concave function h on the positive integers, satisfying $h(0) = 0$, let

$$V[h, [a, b]] = V[h] = \{f \mid V_h(f) = \sup_n v(n, f, [a, b])/h(n) < \infty\}.$$

For h bounded, $V[h]$ is BV, and thus $h(n) \to \infty$ for all other classes $V[h]$. It may be shown that $V[h]$ consists only of regulated functions if and only if $h(n) = o(n)$, and therefore we will make this assumption on h, since our interest is to represent $V[h]$ as intersection of ΛBV classes, which contain only regulated functions We will also assume that $h(n) \to \infty$ as $n \to \infty$.

2. The following theorems establish some properties of the classes $V[h]$.

THEOREM 1. *If $f \in V[h, [a, b]]$ and $f \in V[h, [b, c]]$, then $f \in V[h, [a, c]]$.*

The proof of Theorem 1 is trivial.

THEOREM 2. *If $f \notin V[h, [a, b]]$, then there exists $x \in [a, b]$ such that $f \notin V[h, J]$ for any closed interval $J \subseteq [a, b]$ containing a neighborhood of x.*

Proof. We split $[a, b]$ into two closed intervals of equal length L_1 and L_2, and observe that, by Theorem 1, for one of L_1 or L_2, say $J_1, f \notin V[h, J_1]$. Dividing J_1 as we did $[a, b]$, thus by an inductive procedure, we obtain a nested sequence $J_1 \supseteq J_2 \supseteq \ldots$ of closed intervals of length approaching zero, and such that $f \notin V[h, J_i]$ for $i = 1, 2, \ldots$ The intersection of the J_i's is a single point x which satisfies the requirements of the conclusion of the theorem.

Theorem 2 implies the existence of a point $x \in [a, b]$ such that either $f \notin V[h, [x, x+\delta]]$ for all $\delta > 0$, or $f \notin V[h, [x-\delta, x]]$ for all $\delta > 0$.

It is observed from the definitions that $f \in V[h]$ if and only if there is a constant C such that

$$\Big(\sum_{i=1}^{n} |f(I_i)|\Big)/h(n) \leqslant C$$

for all collections $\{I_i\}_{i=1}^\infty$ and all n. The next theorem shows, however, that only requiring the above expression (as a sequence of n) to be bounded for each particular collection $\{I_i\}$ is sufficient to assure its uniform boundedness.

THEOREM 3. (i) *$f \in V[h, [a, b]]$ if and only if*

$$\sum_{i=1}^{n} |f(I_i)| = O(h(n)) \tag{1}$$

for each collection $\{I_i\}_{i=1}^\infty$.

(ii) *If f is regulated, then $f \in V[h, [a, b]]$ if and only if (1) is true for each collection $\{I_i\}_{i=1}^\infty$ satisfying $|f(I_n)| \downarrow 0$ as $n \to \infty$.*

Proof. We prove (ii) ((i) being similar). The "only if" part follows immediately from the definitions. For the "if" part of the theorem, suppose that $f \notin V[h, [a, b]]$.

Applying Theorem 2, we may assume that $f \notin V[h, [x, x+\delta]]$ for some x and all $\delta > 0$. Let $M = \sup_t |f(t)|$. Let $\delta_1 > 0$. We choose n_1 such that $h(n_1+1) > 2M$ and $v(n_1+1, [x, x+\delta_1]) > 2h(n_1+1)$. There are subintervals of $[x, x+\delta_1]$, $I_1, \ldots, I_{n_1}, I_{n_1+1}$ (ordered from right to left, i.e., I_{j+1} lies to the left of I_j) such that

$$\sum_{i=1}^{n_1+1} |f(I_i)| > 2h(n_1+1).$$

We can assume that I_{n_1+1} has nonempty interior. Thus $x \notin I_{n_1}$. Now

$$\sum_{i=1}^{n_1} |f(I_i)| = \sum_{1}^{n_1+1} |f(I_i)| - |f(I_{n_1+1})| \geqslant \sum_{i=1}^{n_1+1} -2M$$
$$\geqslant 2h(n_1+1) - h(n_1+1) = h(n_1+1) > h(n_1).$$

Having chosen $n_1, \ldots, n_{k-1}$, and $I_1, \ldots, I_{n_{k-1}} \subseteq [x, b]$, ordered from right to left and $x \notin I_{n_{k-1}}$, let $I_{n_{k-1}} = [a_{k-1}, b_{k-1}]$ and $\delta_k = \min\{(a_{k-1}-x), 1/k\}$, and choose n_k such that

$$h(n_k+1) > 2M(n_{k-1}+1),$$

and

$$v(n_k+1, [x, x+\delta_k]) > (1+k)h(n_k+1).$$

Therefore, there are intervals $J_1, \ldots, J_{n_k+1} \subseteq [x, x+\delta_k]$, having nonempty interior and ordered from right to left, such that

$$\sum_{i=1}^{n_k+1} |f(J_i)| > (1+k)h(n_k+1).$$

Let $I_i = J_i$ for $n_{k-1} < i \leqslant n_k$, then $x \notin I_{n_k}$ and

$$\sum_{i=1}^{n_k} |f(I_i)| \geqslant \sum_{n_{k-1}+1}^{n_k} |f(I_i)| = \sum_{n_{k-1}+1}^{n_k} |f(J_i)|$$
$$\geqslant \sum_{1}^{n_k+1} |f(J_i)| - |f(J_{n_k+1})| - \sum_{1}^{n_{k-1}} |f(J_i)|$$
$$\geqslant (1+k)h(n_k+1) - 2M - 2Mn_{k-1} \geqslant (1+k)h(n_k+1) - h(n_k+1) \geqslant kh(n_k).$$

Since $\delta_k \downarrow 0$ as $k \to \infty$, we have that $f(I_i) \to 0$ as $i \to \infty$ and therefore $\{I_i\}$ can be rearranged into $\{I_i^*\}$ such that $|f(I_i^*)| \downarrow 0$ and

$$\sum_{i=1}^{n} |f(I_i^*)| \geqslant \sum_{i=1}^{n} |f(I_i)| \neq O(h(n)). \ \blacksquare$$

Throughout the rest of the paper, sequences $\Lambda = \{\lambda_i\}$ are assumed to satisfy the condition $\lambda_n \uparrow \infty$. For a sequence $\{a_i\}$ we will write $\Delta a_i = a_i - a_{i+1}$. The following relation between classes $V[h]$ and ΛBV is due to Avdispahić [1].

THEOREM 4. *If $\sum h(i)\Delta(1/\lambda_i) < \infty$, then $V[h] \subseteq \Lambda BV$.*

Proof. We first observe that

$$h(n)/\lambda_n = h(n)\sum_{n}^{\infty} \Delta(1/\lambda_i) \leqslant \sum_{n}^{\infty} h(i)\Delta(1/\lambda_i),$$

and therefore $h(n)/\lambda_n \to 0$ as $n \to \infty$. Now, for $f \in V[h]$ and a collection $\{I_i\}$ we have

$$\sum_{1}^{n} |f(I_i)|/\lambda_i \leqslant \sum_{i=1}^{n-1}\sum_{k=1}^{i} |f(I_k)|\Delta(1/\lambda_i) + \sum_{k=1}^{n} |f(I_k)|/\lambda_n$$
$$\leqslant V_h(f)\{\sum_{1}^{\infty} h(i)\Delta(1/\lambda_i) + h(n)/\lambda_n\} \leqslant CV_h(f)$$

for some constant C and all n. ∎

3. We now state the generalization of Perlman's result.

THEOREM 5. *$V[h]$ is the intersection of all ΛBV classes satisfying*

(2) $$\sum h(i)\Delta(1/\lambda_i) < \infty.$$

To prove this theorem we need the following results:

LEMMA 1. *Suppose $h, g \geqslant 0$ are nondecreasing concave functions of the positive integers. Then the function p defined by $p(n) = \{h(n)g(n)\}^{1/2}$ is also nondecreasing and concave.*

Proof. p is obviously increasing. If $0 \leqslant t \leqslant 1$ and $r = tn+(1-t)m$ is an integer, then by Hölder's inequality we have that

$$p(r) = \{h(r)g(r)\}^{1/2} \geqslant \{th(n)+(1-t)h(m)\}^{1/2}\{tg(n)+(1-t)g(m)\}^{1/2}$$
$$\geqslant t\{h(n)g(n)\}^{1/2}+(1-t)\{h(m)g(m)\}^{1/2} = tp(n)+(1-t)p(m). \ ∎$$

A consequence of this lemma is

THEOREM 6. *No $V[h]$ contains the class of regulated functions.*

Proof. Let $p(n) = \{nh(n)\}^{1/2}$. p is nondecreasing and concave by Lemma 1, and also $p(n) = o(n)$. Then the sequence defined by $b_n = p(n)-p(n-1)$ is decreasing and converges to zero. Consider the function f defined on $[a, b]$ by $f(b) = 0$,

$$f(x) = \sum_{i=1}^{k} (-1)^{i+1} b_i \quad \text{for } a+(b-a)/(k+1) \leqslant x < a+(b-a)/k,\ k = 1, 2, \ldots$$

and $f(a) = \sum_{1}^{\infty} (-1)^{i+1} b_i$. Then f has only simple discontinuities and $\nu(n, f, [a, b]) = p(n)$. But $p(n) \neq O(h(n))$ and therefore $f \notin V[h]$.

THEOREM 7. *Suppose f has only simple discontinuities. If $f \notin V[h]$, then $f \notin \Lambda BV$ for some $\Lambda = \{\lambda_i\}$ satisfying (2).*

Proof. By Theorem 3 there is a collection $\{I_i\}_{i=1}^{\infty}$ of nonoverlapping intervals such that $a_i = |f(I_i)| \downarrow 0$ as $i \to \infty$ and $\sum_{1}^{n} a_i \neq O(h(n))$. Thus $\sum_{1}^{\infty} a_i = \infty$. Let $n_0 = 0$. Choose n_1 such that

$$\sum_{i=1}^{n_1} a_i > h(n_1).$$

Having chosen $n_1, \ldots, n_{k-1}$, we can find n_k such that

$$\sum_{1}^{n_k} a_i > 2\sum_{1}^{n_{k-1}} a_i, \quad \text{and} \quad \sum_{1}^{n_k} a_i > 2k^2 h(n_k).$$

Therefore

$$\sum_{n_{k-1}+1}^{n_k} a_i > \tfrac{1}{2}\sum_{1}^{n_k} a_i > k^2 h(n_k).$$

Let $\lambda_i = k^2 h(n_k)$ for $n_{k-1} < i \leqslant n_k$. Then $\lambda_i \uparrow \infty$ and $\Delta(1/\lambda_i)$ is nonzero only when $i = n_k$ for some k. In this way,

$$\sum_{1}^{\infty} h(i)\Delta(1/\lambda_i) = \sum_{k=1}^{\infty} h(n_k)\left\{\frac{1}{k^2 h(n_k)} - \frac{1}{(k+1)^2 h(n_{k+1})}\right\} \leqslant \sum_{k=1}^{\infty} 1/k^2 < \infty.$$

Also

$$\sum_{i=1}^{\infty} |f(I_i)|/\lambda_i = \sum a_i/\lambda_i = \sum_{k=1}^{\infty}\{\sum_{i=n_{k-1}+1}^{n_k} a_i\}/\{k^2 h(n_k)\} \geqslant \sum_{1}^{\infty} 1 = \infty.$$

It is left only to show that $\sum 1/\lambda_i = \infty$. Since f is bounded, we have that $2\sup|f(x)|\sum 1/\lambda_i \geqslant \sum a_i/\lambda_i = \infty$, and thus $\sum 1/\lambda_i = \infty$.

We proceed now to prove Theorem 5.

Proof of Theorem 5. Theorems 6 and 7 guarantee that there is at least one space ΛBV with Λ satisfying (2). By Theorem 7 the intersection of such classes ΛBV is contained in $V[h]$. Finally, by Theorem 4, $V[h]$ is contained in such an intersection.

We will observe that Theorem 5 cannot be improved by considering only countable collections of classes ΛBV.

LEMMA 2. *If $\sum h(i)\Delta(1/\lambda_i) < \infty$, then*

$$\frac{h(n)}{n}\sum_{i=1}^{n} 1/\lambda_i \to 0 \quad \text{as } n \to \infty.$$

Proof. Let $\varepsilon > 0$. There is $N > 1$ such that $\sum_{N}^{\infty} h(i)\Delta(1/\lambda_i) < \varepsilon$. We can choose $M > N$ such that

$$\frac{h(n)}{n}\sum_{i=1}^{N-1} 1/\lambda_i < \varepsilon,$$

and $h(n)/\lambda_n < \varepsilon$ for $n > M$. Then, for $n > M$ we have

$$(3)\qquad \frac{h(n)}{n}\sum_{i=1}^{n} 1/\lambda_i \leqslant \frac{h(n)}{n}\sum_{1}^{N-1} 1/\lambda_i + \frac{h(n)}{n}\sum_{N}^{n} 1/\lambda_i = \mathrm{I}+\mathrm{II}\,.$$

$\mathrm{I} < \varepsilon$, and applying summation by parts

$$\mathrm{II} = \frac{h(n)}{n}\sum_{i=N}^{n-1}(i+1-N)\Delta(1/\lambda_i) + \frac{h(n)(n+1-N)}{n\lambda_n}$$

$$\leqslant \sum_{i=N}^{n-1}\frac{h(n)i}{n}\Delta(1/\lambda_i) + \frac{h(n)}{\lambda_n}$$

$$\leqslant \sum_{i=N}^{\infty} h(i)\Delta(1/\lambda_i) + \varepsilon\,,$$

since $h(n)/n \leqslant h(i)/i$ for $i \leqslant n$. Therefore the left side of (3) is less than 3ε if $n > M$, and the conclusion follows.

THEOREM 8. *Let $\Lambda^l = \{\lambda_i^l\}$, $l = 1, 2, \ldots$ be a collection of sequences such that $\sum_{i=1}^{\infty} h(i)(1/\lambda_i^l - 1/\lambda_{i+1}^l) < \infty$, $\lambda_n^l \uparrow \infty$ as $n \to \infty$ and $\sum_{i=1}^{\infty} 1/\lambda_i^l = \infty$ for all l. Then there exists a function f in $\bigcap_{l=1}^{\infty} \Lambda^l BV$ which does not belong to $V[h]$.*

Proof. Perlman [3] has shown that if $\Gamma^l = \{\gamma_i^l\}$, where $1/\lambda_i^l = \sum_{k=1}^{l} 1/\lambda_i^k$, $i = 1, 2, \ldots$, then the intersection of all $\Lambda^l BV$, $l = 1, 2, \ldots$ equals the intersection of all $\Gamma^l BV$, $l = 1, 2, \ldots$ We observe that $\gamma_i^l \geqslant \gamma_i^{l+1}$ for all l and all i. Also, $\gamma_n^l \uparrow \infty$ as $n \to \infty$, and

$$\sum_{i=1}^{\infty} h(i)(1/\gamma_i^l - 1/\gamma_{i+1}^l) = \sum_{k=1}^{l}\sum_{i=1}^{\infty} h(i)(1/\lambda_i^k - 1/\lambda_{i+1}^k) < \infty\,.$$

By Lemma 2 and the fact that $h(n) = o(n)$, there exist integers $1 = n_0, n_1, \ldots$ such that $a_k = kh(n_k)/(n_k - n_{k-1})$ is a decreasing sequence of k which converges to zero, $n_k > 2n_{k-1}$, and

$$\frac{h(n_k)}{n_k}\sum_{i=1}^{n_k} 1/\gamma_i^k \leqslant \frac{1}{k2^{k+1}}\,,$$

for $k = 1, 2, \ldots$ Let $b_i = a_k$ for $n_{k-1} < i \leqslant n_k$. Clearly, $b_i \downarrow 0$ as $i \to \infty$. Also

$$\sum_{i=1}^{n_k} b_i \geqslant \sum_{i=n_{k-1}+1}^{n_k} a_k = kh(n_k)\,.$$

Hence $\sum_{i=1}^{n} b_i \neq O(h(n))$. But, since $a_k < 2kh(n_k)/n_k$, and $\gamma_i^l \geqslant \gamma_i^k$ for $k \geqslant l$, it follows that

$$\sum_{i=n_{l-1}+1}^{\infty} b_i/\gamma_i^l = \sum_{k=l}^{\infty} a_k\Big(\sum_{i=n_{k-1}+1}^{n_k} 1/\gamma_i^l\Big) < \sum_{k=l}^{\infty}\frac{2kh(n_k)}{n_k}\sum_{i=1}^{n_k} 1/\gamma_i^k \leqslant \sum_{k=l}^{\infty} 2^{-k} < \infty\,.$$

Thus $\sum b_i/\gamma_i^l < \infty$. Finally, by using the sequence $\{b_i\}$ we define f as we did in the proof of Theorem 6, and the procedure above shows that f is contained in the intersection of all $\Lambda^l BV$, $l = 1, 2, \ldots$ but $f \notin V[h]$. ∎

4. For $f \in \Lambda BV$ let $\|f\|_\Lambda = |f(a)| + V_\Lambda(f)$. If $f \in V[h]$, let

$$\|f\|_h = |f(a)| + V_h(f)\,;$$

V_Λ and V_h as defined in §1. It is easy to see that ΛBV and $V[h]$ are Banach spaces under the norms $\|\ \|_\Lambda$ and $\|\ \|_h$ respectively.

As an application of Theorem 5 we will prove an analogue of the well-known Helly's Theorem for the classes $V[h]$.

THEOREM 9. *Let $\{f_k\}$ be a sequence in $V[h]$ such that $\|f_k\|_h \leqslant M$ for some M, $k = 1, 2, \ldots$ Then there exists a subsequence $\{f_{k_j}\}$ converging pointwise to some f in $V[h]$ with $\|f\|_h \leqslant M$.*

Proof. Theorem 5 guarantees the existence of a class ΛBV satisfying (2). For each collection $\{I_i\}$ and each k, by an argument similar to that given in the proof of Theorem 4, we have that

$$\sum_{i=1}^{\infty} |f_k(I_i)|/\lambda_i \leqslant CV_h(f_k)$$

for some $C > 0$ independent of k. Thus

$$\|f_k\|_\Lambda = |f_k(a)| + V_\Lambda(f_k) \leqslant |f_k(a)| + CV_h(f_k) \leqslant (C+1)\|f_k\|_h \leqslant (C+1)M\,.$$

By the analogue of Helly's Theorem for the classes ΛBV ([4], Theorem 5), there is a subsequence $\{f_{k_j}\}$ converging pointwise to some f. For a finite collection $\{I_i\}$, consisting of n elements, we have

$$|f_{k_j}(a)| + \Big(\sum_{i=1}^{n} |f_{k_j}(I_i)|\Big)/h(n) \leqslant \|f_{k_j}\|_h \leqslant M\,.$$

By letting $j \to \infty$, we observe that

$$\Big(\sum_{i=1}^{n} |f(I_i)|\Big)/h(n) \leqslant M - |f(a)|\,,$$

and thus $V_h(f) \leqslant M - |f(a)|$, which is the same as $\|f\|_h \leqslant M$.

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