monotone subsequence of the points of E representing P_i, every monotone subsequence of the points P_i' represents $\mathscr{P}_0$.

By the definition of $f(\mathscr{P}_i)$, we may choose P_i' so near P_0 that

$$|F(P_i') - f(\mathscr{P}_i)| < \frac{1}{i};$$

and, by the definition of $f(\mathscr{P}_0)$,

$$\lim_{i \to \infty} F(P_i') = f(\mathscr{P}_0),$$

(because this is true for every monotone subsequence of the points P_i').

Hence also

$$\lim_{i \to \infty} f(\mathscr{P}_i) = f(\mathscr{P}_0).$$

Q. E. D.

Thus there is a complete correspondence between the continuous functions of a directed point and the functions of position with unique limits in every open quadrant at each point.

Each continuous function of a directed point represents the limit, in the corresponding open quadrant, of a function of position with unique limits in each open quadrant at every point. And each function of position of this type defines uniquely a continuous function of a directed point representing its limits in the open quadrants.

Concerning triodic continua in the plane.

By

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In a recent paper [1]) I defined the term *triod* and showed that there does not exist, in the plane, an uncountable set of mutually exclusive triods. In the present paper I will generalize this notion and establish a correspondingly more general theorem.

Lemma. *If the metric space S contains a countable collection of compact point sets $S_1, S_2, S_3, \ldots$ such that every compact subset of S is contained in the sum of a finite number of point sets of this collection, then every uncountable collection of closed and compact subsets of S contains an uncountable subcollection G such that if e is any positive number and g_0 is any point set of the collection G then there exist uncountably many point sets g of G such that every point of g is at a distance less than e from some point of g_0 and every point of g_0 is at a distance less than e from some point of g.*

Proof [2]). Let E_n denote the compact point set $S_1 + S_2 + \ldots + S_n$. For each pair of natural numbers m and n, E_n contains a finite point set S_{mn} such that every point of E_n is at a distance less than $1/m$ from some point of S_{mn}. Let T denote the collection of all point sets X such that, for some m and n, X is a subset of S_{mn}. Each

[1]) *Concerning triods in the plane and the junction points of plane continua*, Proceedings of the National Academy of Sciences, vol. 14 (1928), pp. 85—88.

[2]) For the case where the space S is Euclidean space of a finite number of dimensions this lemma may be proved (even though the requirement that the point sets of the collection G be compact is removed) by a modification of an argument given by Zarankiewicz to prove a related theorem. Cf. Casimir Zarankiewicz, *Sur les points de division dans les ensembles connexes*, Fundamenta Mathematicae, vol. IX (1927), Theorem 2, page 6.

closed and compact subset of S is the sequential limiting set [1] of some sequence of point sets belonging to T. But the collection T is clearly countable. Thus the space H whose elements are the closed and compact subsets of S is separable. If the distance between two closed and compact point sets x and y of the space S (and thus the distance between the elements x and y of the space H) is defined as the smallest number d such that no point of either of the sets x and y is at a distance greater than d from every point of the other one, then, with respect to this definition of distance, the separable space H is [2] metric. Hence, by a theorem established by Gross [3], every uncountable set of elements of H contains an element of condensation of itself. It easily follows that there exists an uncountable subset G of H containing all the elements of H, with the possible exception of a countable number, and such that every element of G is an element of condensation of G. The truth of our lemma readily follows.

Definition. A continuum M will be said to be *triodic* if it contains three continua such that the common part of all three of them is both a non vacuous proper subcontinuum of each of them and the common part of every two of them. A continuum which is not triodic will be said to be *atriodic*.

Theorem. *If, in the plane, G is an uncountable set of bounded triodic continua there exists an uncountable subset H of G such that every two continua of the set H have a point in common.*

Proof. Suppose first that the set G contains an uncountable subset Q such that every continuum of the set Q contains a continuum that separates the plane. Every triodic continuum contains two continua a and b such that $a.b$ is a proper subcontinuum both of a and of b. Hence, by a theorem of Janiszewski's [4], if a bounded triodic continuum contains a subcontinuum that separates the plane it contains a *proper* subcontinuum that separates the plane. It follows, by a theorem of Kuratowski's [1], that the continua of the set Q are not mutually exclusive and, indeed, Kuratowski's argument, with little or no modification, suffices to show that Q contains an uncountable subset such that every two continua of this subset have a point in common.

[1]) Cf. my *Report on continuous curves from the viewpoint of analysis situs*, Bulletin of the American Mathematical Society, vol. 29 (1923), p. 297, footnote.

[2]) Cf. F. Hausdorff, *Mengenlehre*, Berlin and Leipzig, 1927, § 28.

[3]) Cf. W. Gross, *Zur Theorie der Mengen, in denen ein Distanzbegriff definiert ist*, Sitzungsberichte d. k. Akad. der Wissenschaften, Math. Naturw. Kl., Wien, Bd. 123 (1914), pp. 801—809.

[4]) S. Janiszewski, *Sur les coupures du plan faites par des continus*, Prace matem.-fizyczne, tom XXVI (1913).

Suppose, on the other hand, that no continuum of the set G cotains a subcontinuum that separates the plane. There exists a positive number e and an uncountable subset G_1 of G such that each continuum g of the set G_1 contains a continuum K_g and three continua a_g, b_g and c_g such that (1) $a_g . b_g = b_g . c_g = c_g . a_g = K_g$, (2) each of the continua a_g, b_g and c_g contains a point at a distance greater than e from every point of K_g. By the above lemma there exists an uncountable subset G_2 of G_1 such that if x is any continuum of the set G_2 then for each positive number there exist uncountably many continua y of the set G_2 such that neither of the two continua K_x and K_y contains a point which is at a distance equal to or greater than that number from every point of the other one. Let g_0 denote some definite continuum of the set G_2. Since K_{g_0} does not separate the plane, there exists [2] a simple closed curve J enclosing K_{g_0} and such that every point within J is at a distance less than $e/2$ from some point of K_{g_0}. Let d denote the shortest distance from J to K_{g_0}. There exists an uncountable subset G_3 of G_2 such that if g is any continuum of the set G_3 then every point of K_g is at a distance less than d from some point of K_{g_0} and every point of K_{g_0} is at a distance less than d from some point of K_g. If g is any continuum of the set G_3, the three continua a_g, b_g and c_g have in common with J three points A_g, B_g and C_g respectively. There exist three mutually exclusive arcs t_a, t_b and t_c on J and a subset H of G_3 such that if h is any continuum of the set H then the points A_h, B_h and C_h lie on the arcs t_a, t_b and t_c respectively. It may be seen that every two continua of the set H have at least one point in common.

[1]) C. Kuratowski, *Sur les coupures irreductibles du plan*, Fundamenta Mathematicae, vol. VI (1924), Lemme 2, p. 144.

[2]) Cf. my paper *Concerning the separation of point sets by curves*, Proceedings of the National Academy of Sciences, vol. 11 (1925), Theorem 1, p. 469.