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On a class of topological spaces with a Scott sentence

by

Juan Carlos Martínez (Madrid)

Abstract. By means of a topological game with two players we study the expressive power of the topological language \( L_{\omega_1} \) for \( T_3 \) spaces. The main result of this paper is a partial characterization of homeomorphic type of the space by means of certain topological properties which are expressible in this language. In this way, we find a class of topological spaces with a Scott sentence which includes every countable ordinal with order topology.

§ 0. Introduction. The infinitary language \( L_{\omega_1} \) is obtained from the first order language \( L_{\omega} \) (in the classical sense) by adding the following formation rule: If \( \varphi \) is a countable set of formulas, \( \forall \varphi \) and \( \exists \varphi \) are formulas.

\( (L_{\omega_1}) \) is the topological analog of the language \( L_{\omega} \). It is a formal language in the study of topological structures. \( (L_{\omega_1}) \) is obtained from \( L_{\omega} \) by adding the symbol \( e \) and set variables \( X, Y, \ldots \). The atomic formulas of \( (L_{\omega_1}) \) are of the form \( x = y \) and \( x \in X \). The formation rules of \( (L_{\omega_1}) \) are those of \( L_{\omega} \) and the following two rules:

(i) If a formula \( \varphi \) is positive in \( X \), then \( \forall X (x \in X \rightarrow \varphi) \) is a formula.
(ii) If a formula \( \varphi \) is negative in \( X \), then \( \exists X (x \in X \land \varphi) \) is a formula.

A formula \( \varphi \) is positive (negative) in \( X \) if each free occurrence of \( X \) in \( \varphi \) is within the scope of an even (odd) number of negation symbols. The set variables range over the class of open sets of the space and, intuitively, quantifications over sets in \( (L_{\omega_1}) \) are quantifications over small enough neighborhoods of a point.

It is shown in [1] that in many cases it is possible to give a parallel treatment of classical and topological model theory.

Every space considered here is assumed to be \( T_3 \) (i.e., Hausdorff and regular).

We denote \( T_3 \) spaces by \( A, B, \ldots \). It is an immediate consequence of the Löwenheim–Skolem theorem for \( (L_{\omega_1}) \), that, for every sentence \( \varphi \) of \( (L_{\omega_1}) \), if \( \varphi \) is satisfied in a \( T_3 \) space then \( \varphi \) is satisfied in a countable metrizable space. This says that the class of countable metrizable spaces is, from the point of view of \( (L_{\omega_1}) \), dense in the class of all \( T_3 \) spaces. Let \( A \) be a countable metrizable space. A sentence \( \varphi \) of \( (L_{\omega_1}) \) is said to be a Scott sentence of \( A \) if \( A \models \varphi \) and every countable metrizable space which satisfies \( \varphi \) is homeomorphic to \( A \). In the present paper we find a class

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of countable metrizable spaces with a Scott sentence. It is not known whether there exists a countable metrizable space without such a sentence.

We study the set of accumulation points of a space by means of a certain topological game with two players. In this way, we partition the space into classes of points of the same type. The main result of this paper is a characterization of homeomorphism types, for a certain class of $T_3$ spaces which includes every countable ordinal with order topology, by means of certain topological properties which are expressible in $(L_{	ext{unc}})$. Intuitively, a $T_3$ space $A$ belongs to our class if for every $a \in A$ and every neighborhood $U$ of $a$ we can find a neighborhood $U'_a$ of $a$ with $U'_a \cap U$ in such a way that $U'_a$ is sufficiently small and we can determine which types of points are in $U'_a$. The main theorem permits us to characterize the $(L_{	ext{unc}})$-theory of any space of our class. Then as a corollary we deduce that every countable ordinal with order topology has a Scott sentence.

The results of the present paper are an improvement of the results of [2]. A classification of the $(L_{	ext{unc}})$-theories of $T_3$ spaces is given in [1].

§ 1. Accessible sets and types of convergence. If $A^*$ is a subset of a space $A$ and $a$ is an accumulation point of $A^*$, we say that $A^*$ converges to $a$ and write $A^* \rightarrow a$. $A^*_1, A^*_2$ are subsets of $A$ such that $A^*_1 \rightarrow a$ for every $a \in A_2$, we write $A^*_1 \rightarrow A^*_2$. In order to study that convergence, we introduce the notion of accessible set.

We have given a $T_3$ space $A$, a subset $A^*$ of $A$ and two players I and II. In the game $G(A^*, A)$ each player makes infinitely many moves. In his $i$th move player I first chooses an arbitrary finite sequence $a_1, a_2, \ldots, a_i$ of points of $A$ and then in his $i$th move player II chooses a sequence of $n$ neighborhoods $U_1, a_1, U_2, a_2, \ldots, U_{2i}$. Let $U_1, U_2$ be all the neighborhoods chosen by II in the moves $1, 2, \ldots, i$. $A^*$ is covered in move $i$ if $A^* \subseteq U_1 \cup \cdots \cup U_i$. Then I wins in the game $G(A^*, A)$ if $A^*$ is covered in move $i$ for some $i$. We say that the set $A^*$ is accessible if I has a winning strategy in $G(A^*, A)$. Otherwise, we say that $A^*$ is an inaccessible set.

For each ordinal $\xi$ we introduce the notion of $\xi$-accessible set as follows. $A^*$ is 0-accessible if $A^* \subseteq \varnothing$. If $\xi = \mu + 1$ we say that $A^*$ is $\xi$-accessible if, for some $n \in \omega$, there exist $a_1, a_2, \ldots, a_n \in A$ such that for all neighborhoods $U_1, a_1, U_2, a_2, \ldots, U_n$ of $a_n$, $A^* \subseteq (U_1 \cup \cdots \cup U_n)$ is $\mu$-accessible. If $\xi$ is a limit ordinal, then $A^*$ is $\xi$-accessible if $A^*$ is $\mu$-accessible for some $\mu < \xi$. The notion of $\omega$-accessible set is the crucial notion we use in [2].

Remark. After the publication of [2], R. Telgársky has pointed out that the game we use to define the notion of accessible set is a refinement of the point-open game, which was introduced by F. Galvin and R. Telgársky and which has also been studied by other authors. This game is presented in [3].

Let $A$ be a $T_3$ space. Let $A^*$ be a subset of $A$. In the sequel, the following basic properties (i)-(viii) will be used without explicit mention.

(i) If $B^*$ is a subset of $A^*$, then $A^*$ accessible ($\xi$-accessible) implies $B^*$ accessible ($\xi$-accessible).

(ii) For every ordinals $\xi, \eta$ with $\xi < \eta$, $A^* \xi$-accessible implies $A^* \eta$-accessible.

Note that if $x$ is a regular cardinal and the topology of $A$ has a basis of cardinal $\leq x$, then $A^* (x+1)$-accessible implies $A^* x$-accessible. Hence we infer that if $A^*$ is not $x$-accessible, player II has a winning strategy in the game $G(A^*, A)$. Therefore $A^*$ accessible implies $A^* x$-accessible. Thus we obtain:

(iii) $A^*$ is accessible if and only if $A^*$ is $\xi$-accessible for some ordinal $\xi$.

The following are easy generalizations of the basic properties of $\omega$-accessibility given in [2].

(iv) $A^* = A^*_1 \cup \cdots \cup A^*_n$ is accessible ($\xi$-accessible) if and only if $A^*_1, \ldots, A^*_n$ are accessible ($\xi$-accessible).

(v) If $A^*$ is accessible ($\xi$-accessible), then the set of accumulation points of $A^*$ is accessible ($\xi$-accessible).

(vi) If $A^*$ is an infinite subset of $A$ and no point of $A$ is an accumulation point of $A^*$, then $A^*$ is inaccessible.

Note also:

(vii) If $A^* \neq \varnothing$ and $A^* \rightarrow A^*$, then $A^*$ is inaccessible.

Suppose that $A$ is a $T_3$ space and $A^*$ is a subset of $A$. For every ordinal $\xi$, we define by transfinite induction the $\xi$-derivative of $A^*$, $(A^*)^\xi$, as follows:

$$(A^*)^0 = A^*,$$

$$(A^*)^\xi + 1 = \{a \in A : (A^*)^\xi \rightarrow a\},$$

$$(A^*)^\xi = \bigcap_{0 \leq \eta < \xi} (A^*)^\eta$$

if $\xi$ is a limit ordinal.

We shall need the following result.

Lemma 1.1. Suppose that $A^*$ is an accessible subset of $A$. Then, for every ordinal $\xi$, $A^*$ is $\xi$-accessible if and only if $(A^*)^\xi = \varnothing$.

Proof. We show that if $U_1, \ldots, U_n$ are open sets, then $(A^*)^\xi - (U_1 \cup \cdots \cup U_n) = \varnothing$ implies $A^* - (U_1 \cup \cdots \cup U_n)$ $\xi$-accessible, for each $\xi$. The case $\xi = 0$ is immediate. If $\xi = \mu + 1$ and $(A^*)^\mu - (U_1 \cup \cdots \cup U_n) = \varnothing$, it is easy to infer that

$$(A^*)^{\mu + 1} - (U_1 \cup \cdots \cup U_n)$$

is finite (otherwise, $(A^*)^\mu$ would not be accessible). Now assume that $\xi$ is a limit ordinal and $(A^*)^\xi - (U_1 \cup \cdots \cup U_n) = \bigcap_{0 \leq \eta < \xi} ((A^*)^\eta - (U_1 \cup \cdots \cup U_n)) = \varnothing$. Consider $\tilde{A} = (A^*)^\xi - (U_1 \cup \cdots \cup U_n)$. Since $\tilde{A}$ is accessible and closed, $\tilde{A}$ is compact. Therefore $A^* - (U_1 \cup \cdots \cup U_n)$ is $\mu$-accessible for some $\mu < \xi$, and by the induction hypothesis, $A^* - (U_1 \cup \cdots \cup U_n)$ is $\mu$-accessible.

On the other hand, for closed sets $U_1, \ldots, U_n$, one can check that if $(A^*)^\xi - (U_1 \cup \cdots \cup U_n) \neq \varnothing$ then $A^* - (U_1 \cup \cdots \cup U_n)$ is not $\xi$-accessible.
Suppose that $A$ is a $T_2$ space, $A^*$ is a subset of $A$ and $a \in A$. If $A^* \to a$ we consider the following two types of convergence:

(a) $A^* \to a$ if there is a neighborhood $U$ of a such that $A^* \cap U$ is accessible,

(b) $A^* \not\to a$ otherwise.

Assume that $A$ is a $T_2$ space and $\mathcal{V}$ is a nonempty set of subsets of $A$. The game of infinitely many moves $M(A, A)$ is defined in the same way as $G(A^*, A)$ for a subset $A^*$ of $A$. We say that player I wins in $M(A, A)$ if, for some natural number $i$, there exists an $A^* \in \mathcal{V}$ such that $A^*$ is covered in move $i$. We say that $\mathcal{V}$ is accessible if I has a winning strategy in $M(A, A)$, otherwise it is inaccessible. If $A$ is an open set (or a closed set) in $A$, we write $\mathcal{V} \uparrow U = \{A^* \cap U : A^* \in \mathcal{V}\}$. We say that $\mathcal{V}$ converges to $a \in A$, $\mathcal{V} \to a$, if for every $A^* \in \mathcal{V}$ we have $A^* \to a$. If $\mathcal{V} \to a$ we consider the following two types of convergence:

(a) $\mathcal{V} \to a$ if there is a neighborhood $U$ of $a$ such that $\mathcal{V} \uparrow U$ is accessible,

(b) $\mathcal{V} \not\to a$ otherwise.

We define by transfinite induction when a nonempty set $\mathcal{V}$ of subsets of $A$ is $\xi$-accessible. $\mathcal{V}$ is $\alpha$-accessible if $\mathcal{V} \in \mathcal{V}$. If $\xi = \alpha + 1$, we say that $\mathcal{V}$ is $\xi$-accessible if, for some $n \in \omega$, there exists $\alpha_1, \ldots, \alpha_n \in A$ such that for all neighborhoods $U_1, U_2, \ldots, U_n$ of $\alpha_1, \ldots, \alpha_n$, $\mathcal{V} \cap (U_1 \cup \cdots \cup U_n)$ is $\mu$-accessible. If $\xi$ is a limit ordinal, then $\mathcal{V}$ is $\xi$-accessible if $\mathcal{V}$ is accessible for some $\mu < \xi$.

Let us say that a subset $A^*$ of $A$ is $(\mathcal{L}_{\text{count}})$-definable (or, simply, definable) if there is a $(\mathcal{L}_{\text{count}})$-formula $\phi(x)$ such that, for all $a \in A$, $a \in A^*$ if and only if $\phi(a)$. Note that if $A^*$ is $(\mathcal{L}_{\text{count}})$-definable subset of $A$ and $\xi$ is a countable ordinal, then the condition $A^* \to a$ is $\xi$-accessible is definable. If $\xi$ is a limit ordinal and, for $\mu < \xi$, $A^*_\mu$ is a definable subset of $A$, let us say that $\{A^*_\mu : \mu < \xi\}$ is a sequence in $A^*$ if $A^*_\mu \to A^*_\xi$ for $\mu < \xi$. Then, if $\xi$ and $\eta$ are countable, the condition $\{A^*_\mu : \mu < \eta\}$ is $\xi$-accessible is definable. Note that if $\eta$ is not a limit ordinal and $A^*_\eta \to A^*_\xi$, for $\mu < \eta$, then $\{A^*_\mu : \mu < \eta\}$ is accessible (not $\xi$-accessible) if $A^*_\eta \to A^*_\xi$ is accessible (not $\xi$-accessible). In this paper we introduce a class of $T_2$ spaces in which the notion of accessibility can be treated as a $(\mathcal{L}_{\text{count}})$-definable notion. For any space $A$ of this class there will exist a countable ordinal $\xi$ such that the terms "accessible" and "$\xi$-accessible" will be equivalent for definable subsets and sequences of $A$. For example, in any space of $\omega$-finite type (in the sense of [2]), "accessible" and "$\omega$-accessible" will be equivalent.

**Examples.** To characterize the $(\mathcal{L}_{\text{count}})$-theory of a countable ordinal $\Omega$ with order topology we consider, for every ordinal $\xi$, the definable set $\Omega^\xi = (\Omega^0)^\xi$. We have $\Omega^0 = \{\text{isolated points of } \Omega \}$ and, for $\xi > 0$, $\Omega^{\xi+1} = \{\text{accumulation points of } \Omega^\xi \}$. For every $\mu < \xi$ (for example, if $\xi$ is $\omega^n$, then $\omega \in \Omega^1$, $\omega \cdot \omega \in \Omega^2$, $\omega \cdot \omega \cdot \omega \in \Omega^3$, ..., $\omega^\omega \in \Omega^\omega$, ... and $\Omega^\omega = \Omega$ if $\xi = \omega$). Then we have to check whether the sets $\Omega^\xi$ and the sequences $\{\mu^\xi : \mu < \xi\}$ are accessible or not. If $\xi > 0$ and $a \in \Omega^\xi$ we have to look how the sets $\Omega^\xi$ with $\mu < \xi$ and the sequences $\{\mu^\xi : \mu < \eta\}$ with $\eta < \xi$ converge to $a$. In this case, these convergences are of type 1.

Now let us consider the spaces we presented in [2, Example 2.5]. We consider the ordinals $\omega+1$ and $\omega^\omega$, each with order topology, and for every $n \in \omega$ a homeomorphically copy $(\omega^n)$ of $\omega^n$. In $\omega+1$, each replace each $n \in \omega$ by $\omega^n$. Let us denote by $\Omega_n$ the resulting space and by $\omega_n$ the only point of $\omega+1$ not replaced. If $\eta < \omega^n$ we denote the corresponding point in $(\omega^n)$ by $\eta_n$. We add a new point $\omega_n$ to the topological sum $\sum \omega_n$ and take as a neighborhood basis of $\omega_n$ the sets of the form $\{\eta_n \cup \omega_n : \eta < \omega\}$. Let us denote by $\Omega_n$ the resulting space. For $n = 0, 1$ we consider the sets $\Omega_n^0 (\xi)$ defined as before. Then $\Omega_n^0 = \{\omega_n\}$ and $\Omega_n^0 = \emptyset$ if $\xi > \omega (i = 0, 1)$. We find that $\Omega_n^0$ is inaccessible and $\Omega_n^0 \to \omega_n$ for every $n \in \omega$ ($i = 0, 1$). However:

(a) $\{\xi \in \Omega_n^0 : n \geq 0\}$ is inaccessible and $\{\xi \in \Omega_n^0 : n > 0\} \to \omega_n$,

(b) $\{\xi \in \Omega_n^0 : n > 0\}$ is 1-accessible and therefore $\{\xi \in \Omega_n^0 : n > 0\} \to \omega_n$.

We can then infer that $\Omega_n$ and $\Omega_n$ are not $(\mathcal{L}_{\text{count}})$-equivalent. Now let us consider the topological sum $\omega^\omega + \omega_n$ and let us denote this space by $\Omega_n$. In this case, $\Omega_n^0 = n \geq 0$ is inaccessible and $\{\xi \in \Omega_n^0 : n > 0\} \to \omega_n$.

**2. The notion of spectrum.** Suppose that $E$ is a nonempty set and $\prec$ is a binary transitive relation on $E$ (possibly $\prec = \emptyset$). If the set $\{x \in E : x \prec a\}$ is finite, we say that $(E, \prec)$ is normal.

Let $(E, \prec)$ be a normal relation. Suppose that $a \in E$. We say that $a$ is comparable in $E$ if there is a $b \in E$ with $a < b$ or $b < a$. Let $a$ be a nonempty subset of $E$. We say that $a$ is a chain in $E$ if, for some ordinal $\eta \geq 1$, we can write $a = \{a_\eta : \mu < \eta\}$ with $a_\mu < a_{\mu+1}$ and $a_\eta \not= a_\eta$ for $\mu < \mu < \eta$; then we say that $a$ is the length of $a$. Since $\{x \in E : x \prec a\}$ is finite, it is easy to see that $\eta$ is the length of $a$. If $a$ is a nonempty subset of $E$ and $a$ is a chain in $E$, then $a$ is a maximal chain in $E$. We say that $a$ is a maximal chain in $E$ if, for every $b \in E$, $a < b$ implies $b = a$. Note that if $a$ is a noncomparable element of $E$ then $a$ is a maximal chain in $E$. We say that $a$ is a chain $\gamma = \{a_\chi : \mu < \chi\}$ is open if $\eta$ is a limit ordinal; otherwise, we say that $a$ is closed.

Let $(E, \prec)$ be a normal relation. We write $E = P(\bigcup \{\{x, y\} : a \in E\})$ (where $P$ denotes the powerset operation). Let $a$ be a chain in $E$ and $a \in E$. We say that $a$ is a chain in $E$ if, for every $b \in a$, $b \prec a$ and $a \prec a$, or $a \prec a$ and $a \prec a$, or $a \prec a$ and $a \prec a$. We say that $(\{S_\lambda : \lambda \in \Omega\} : \xi$ ordinal) is a complex of types if, for every ordinal $\xi$, $(S_\lambda, \prec \lambda)$ is normal and $S_\lambda$ satisfies the following:

(a) $S_\lambda = \emptyset$.

(b) If $\xi = \mu + 1$, then $S_\lambda$ is a countable nonempty subset of the set $\{x, y : \delta \in S_\delta \} \cap \delta$ and $\delta$ is a set of pairs $(\gamma, \lambda)$ such that $(\gamma, \lambda) \in \delta$ for $\lambda = 0$ or $\lambda = 1$ if $\gamma$ is an open chain in $\delta$.
(iii) If $\xi$ is a limit ordinal, then $S_\xi$ is a countable nonempty subset of the set $\{b_{a, b} \mid a \in S_\eta\}$.

Thus if we want to construct a complex of types we define $S_0 = \{\ast\}$ and take $<_0$ then we define $S_1$ and take $<_1$, and so on.

If $\langle S_\xi, <_\xi \rangle$ is a complex of types, the members of $S_\xi$ will be called $\xi$-types. Note that, for every natural number $n$, $S_{n+1}$ does not depend on $<_n$. Note that we denote chains by $\gamma, \gamma', \ldots$, and types by $\alpha, \beta, \ldots$.

Let $S = \langle S_\xi, <_\xi \rangle$ be a complex of types. For every ordinal $\xi$, we define the $\xi$-type of $a$ in $A$ with respect to $S$, $s_\xi(a, A)$, for every $T_\xi$ space $A$ and $a \in A$. If $a \in S_\xi$, we write $A_a = \{a \in A \mid s_\xi(a, A) = a\}$ and if $\gamma$ is a chain in $S_\xi$, we write $A_\gamma = \{a \in A \mid a \in \gamma\}$. We define $s_\xi(a, A)$ by transfinite induction on $\xi$ as follows:

(i) $s_0(a, A) = \ast$.

(ii) If $\xi = \mu + 1$, let $\delta_0 = \bigcup_{\delta < \xi} \{\{\delta, \lambda\} : \delta \in S_\delta \text{ and } A_\delta \rightarrow a\}$ and $\delta_0 = \bigcup_{\delta < \xi} \{\{\delta, \lambda\} : \delta \in S_\delta \text{ and } A_\delta \rightarrow a\}$. Then $s_\xi(a, A) = \langle \delta_0, \delta_0 \rangle$.

(iii) If $\xi$ is a limit ordinal, then $s_\xi(a, A) = [s_\eta(a, A)]_{\eta < \xi}$.

To see an example, let $\Omega$ be a countable ordinal with order topology. Let $\xi_0$ be the least ordinal $\xi$ such that $(\Omega)^\xi = \Omega$. We will construct a complex of types $S_\xi = \langle S_\xi, <_\xi \rangle$ in such a way that $S_{\xi_0}$ and $<_\xi$ have the following form:

\[
S_{\xi_0} = \{s_\xi : q < \xi \} < \xi_0 \}
\]

(b) $a <_\xi b$ if $a = s_\xi, b = s_\xi' \text{ and } q_1 <_\xi q_2 < \xi_0$. Furthermore, the types $s_\xi, b_\xi$ will satisfy $\Delta_\xi = (\xi)^\xi - (\xi)^{\xi+1}$ for every $q < \xi, q_0$ and $\Delta_\xi = (\xi)^\xi$. We define the types $s_\eta, b_\eta$ by transfinite induction on $\xi$ as follows. We put $b_0 = \ast$. Suppose that $\xi = \mu + 1$. Let $q$ be an ordinal such that $q < \xi, \xi_0$. To define $s_\xi$ we consider $s_\xi = \{s_\xi, 1\} : q < q$ and put $s_\xi = \{s_\xi, (\gamma, 1) : \gamma \text{ is an open chain in } s_\xi\}$. If $q < \xi_0$, we set $s_\xi = \{s_\xi, \chi, 1\} \text{ for some open chain } s_\xi$. Now suppose that $\xi$ is a limit ordinal. If $q < \xi, \xi_0$, we put $s_\xi = \{s_\xi, q\} \text{ such that } s_\xi = s_\eta$ if $\mu < q$ and $s_\eta = s_\eta$ if $\mu > q$. If $\xi < \xi_0$, we set $s_\xi = \{s_\xi, b_\eta\} < \xi_0$.

If $S = \langle S_\xi, <_\xi \rangle$ is a complex of types, $A$ is a $T_\xi$ space and $a \in A$, then it is possible that $s_\xi(a, A) \notin S_\xi$. To see this, consider, for any ordinal $\xi_0$, the complex of types $S_\xi = S_{\xi_0} = \langle S_{\xi_0}, <_\xi \rangle$ defined before. Let $R$ be the space of real numbers with the usual topology. Then for every $x \in R, s_\xi(x, R) = \{(x, 0), \xi\} \notin S_{\xi_0}^\xi$.

Let $S = \langle S_\xi, <_\xi \rangle$ be a complex of types. Let $A$ be a $T_\xi$ space. A type $a \in S_\xi$ is satisfiable in $A$ if $s_\xi(a, A) = a$ for some $a \in A$. We say that $A$ satisfies $S$ if for every ordinal $\xi$ the following conditions hold:

(i) $s_\xi(a, A) \in S_\xi$ for all $a \in A$.

(ii) For all comparable types $\alpha, \beta$ in $S_\xi$ which are satisfiable in $A$, $a <_\xi \beta$ iff $A_a \rightarrow A_\beta$.

In what follows we assume that every complex of types $S = \langle S_\xi, <_\xi \rangle$ has an associate function which assigns to each comparable type $a \in S_\xi$ and to each $\eta \geq \xi$ a comparable type $a(\eta)$ in $S_\eta$ in such a way that if $a, \beta$ is comparable types in $S_\eta$ and $\eta \geq \xi$ then $a <_\beta$ iff $a(\eta) <_\beta(\eta)$. If $\gamma$ is an open chain in $S_\xi$ and $\eta \geq \xi$, we write $\gamma(\eta) = \{a(\eta) : a \in a(\eta)\}$.

Let $S = \langle S_\xi, <_\xi \rangle$ be a complex of types. We say that a $T_\xi$ space $A$ is associated with $S$ if $A$ satisfies $S$ and for every ordinal $\xi$ the following conditions hold:

(i) For every $a \in S_\xi$ there is an $a \in A$ such that $s_\xi(a, A) = a$.

(ii) For every $\xi$-type $\alpha$ comparable in $S_\xi$ and every ordinal $\eta > \xi$ we have $A_\alpha = A_\alpha(\eta)$.

Now we define the central notion of this section. Let $S = \langle S_\xi, <_\xi \rangle$ be a complex of types. We say that $S$ is a spectrum if the following four conditions hold:

(i) For every ordinal $\xi$, every chain in $S_\xi$ is contained in a maximal chain in $S_\xi$.

(ii) For every ordinal $\xi$, the set of maximal chains in $S_\xi$ is finite.

(iii) There exists a countable ordinal $\eta$ such that every member of $S_\eta$ is comparable in $S_\eta$.

(iv) There exists a countable metrisable space associated with $S$.

The least ordinal $\eta$ satisfying (iii) will be denoted by $\mu(S)$.

Let $\Omega$ be a countable ordinal greater than $\alpha$ with order topology, and $\xi_0$ the least ordinal $\xi$ such that $(\Omega)^\xi = \Omega$. Consider the complex of types $S_{\xi_0} = \langle S_{\xi_0}, <_\xi \rangle$ defined before. Note that if $a$ is comparable in $S_{\xi_0}$, then for every $q < \xi_0$, we put $a(\eta) = a$. Now it is easy to check that $S_{\xi_0}$ is a spectrum, $\Omega$ is associated with $S_{\xi_0}$ and $\xi_0 = \mu(S_{\xi_0})$. Furthermore, every ordinal less than $\Omega$ with order topology satisfies $S_{\xi_0}$. We see that $S_{\xi_0}$ is just one maximal chain. If we consider the spaces $S_\theta$ and $S_\xi$, presented in §1, we infer that the topological sum $S_\theta + S_\xi$ is associated with a spectrum $S$ in such a way that $\mu(S) = \alpha + 1$ and in $S_{\xi+1}$ there are just two maximal chains. We leave it to the reader to find for each $\alpha$ a spectrum $S$ such that in $S_{\xi_0}^\alpha$, there are $\alpha$ maximal chains.

Now suppose that $S = \langle S_\xi, <_\xi \rangle$ is a spectrum, $a \in S_\xi$ and $\gamma$ is a chain in $\delta$. We say that $\gamma$ is a $\lambda$-chain in $\delta$ if $\beta, \lambda \in a$ for all $\beta, \lambda \in a(\lambda)$. And $\gamma$ is a $\lambda$-maximal chain in $\delta$ if moreover for every $\lambda$-chain $\lambda', \gamma' \in \delta, \gamma'$ implies $\gamma' = \gamma' (\lambda', \gamma = 0, 1)$. The following lemma follows easily from the definition of spectrum.

**Lemma 2.1.** Let $S = \langle S_\xi, <_\xi \rangle$ be a spectrum. For any $\{\delta, \theta\} \in S_{\xi+1}$ we have:
(a) Every \( \lambda \)-chain in \( \mathcal{A} \) is contained in a \( \lambda \)-maximal chain in \( \mathcal{A} \) (\( \lambda = 0, 1 \)).
(b) The set of \( \lambda \)-maximal chains in \( \mathcal{A} \) is finite (\( \lambda = 0, 1 \)).

Let \( S = \langle S_\xi, < \rangle : \xi \text{ ordinal} \rangle \) be a spectrum. Suppose that \( A \) satisfies \( S, U \) is an open set in \( A \) and \( \alpha, \beta \) are comparable types in \( S_\xi \) which are satisfiable in \( A \).

\textbf{Lemma 2.2.} Let \( S = \langle S_\xi, < \rangle : \xi \text{ ordinal} \rangle \) be a spectrum. Suppose that \( A \) satisfies \( S, \mathcal{A}(A, \xi) = \bigcup \{ \langle \beta, \lambda \rangle : \beta \in S_\alpha, A_\alpha \to A \} \).

Then \( \mathcal{A}(A, A) \) is determined by \( \mathcal{A} \) and the convergences of the open \( \lambda \)-maximal chains in \( \mathcal{A} \).

\textbf{Proof.} We have to keep in mind that if \( \gamma \) is an open chain in \( \mathcal{A} \), we have:

(i) If \( \gamma \) is not a \( 0 \)-chain in \( \mathcal{A} \), then \( A_\gamma \to A \).

(ii) If \( \gamma \) is a \( 0 \)-chain in \( \mathcal{A} \) and there is a \( \beta \in S_\alpha \) with \( \alpha < \beta \) for all \( \gamma \in \mathcal{A} \) and \( A_\alpha \to A \), then \( A_\gamma \to A \).

The desired conclusion now follows from (i) and (ii).

In the next lemma, whose proof is immediate, we give a basic property of spectra.

\textbf{Lemma 2.3.} Let \( S = \langle S_\xi, < \rangle : \xi \text{ ordinal} \rangle \) be a spectrum. For every \( p \in S_\xi \), there exists a \( p_0 \subseteq P \) with \( p_0 \) finite such that for each \( a \in p \) there is a \( p_0 \in P \) with \( \beta < \alpha \).

Let \( S = \langle S_\xi, < \rangle : \xi \text{ ordinal} \rangle \) be a spectrum. Suppose that \( A \) satisfies \( S, \alpha \in A, \mathcal{A}(A, \xi) = \bigcup \{ \langle \beta, \lambda \rangle : \beta \in S_\alpha, A_\alpha \to A \} \).

Then \( \mathcal{A}(A, A) \) is accessible for every \( \alpha \in A_0 \), and \( \bigcup_{\alpha \in A_0} \mathcal{A}_\alpha \) is accessible.

\textbf{Lemma 3.1.} If \( A \) is a \( T_\delta \) space which satisfies \( S \) and \( a \in A \), then \( \mathcal{A}(A, A) \) is accessible.

Let \( \xi \in \eta \) and \( \alpha \in S_\xi \), we define the \( \xi \)-type \( \langle \alpha \rangle^{S_\xi} \) in such a way that, proceeding by transfinite induction on \( \xi \), one can prove:

\textbf{Lemma 3.2.} Suppose that \( S \) is a spectrum, \( A \) is a \( T_\delta \) space satisfying \( S \), \( a \in A \) and \( U \) is an open neighborhood of \( a \) with the relative topology of \( A \). Then \( \mathcal{A}(A, A) = \mathcal{A}(A, U) \) for every ordinal \( \xi \), and \( U \) satisfies \( S \).

Let \( S = \langle S_\xi, < \rangle : \xi \text{ ordinal} \rangle \) be a spectrum, \( S_\xi = \mu(S) \) and \( \xi^* = \max \{ \xi : \text{ the length of some maximal chain in } S_\xi \} \). Suppose that \( A \) is a \( T_\delta \) space which satisfies \( S \), \( a \in S_\xi \), and \( A_\alpha \) is accessible. Then \( A_\alpha \) is \( \xi^* \)-accessible.

\textbf{Proof.} By using Lemma 2.3 and the hypothesis that \( S_\xi = \mu(S) \), it is easy to prove by transfinite induction that for every \( \xi \geq 1 \) there is a subset \( P \) of \( S_\xi \), with \( (A_\xi)^P = \bigcup_{\alpha \in P} A_\alpha \). Note, for every \( \eta \geq 1 \), we can obtain:

(i) If \( \langle \alpha \rangle^P = \emptyset \) then for every \( a \in (A_\xi)^P \) there exists a chain \( (a_n : a \in A_\xi^P) \) with \( a_n = a \) and \( a_n \to a \) for each \( \xi \). We can prove (i) by transfinite induction on \( \xi \). The condition is trivial if \( \xi = 1 \). If \( 1 < \xi = \mu + 1 \), consider \( P \subseteq S_\xi \) such that \( (A_\xi)^P = \bigcup_{\alpha \in P} A_\alpha \), and then make use of Lemma 2.3. If \( A \) is a \( T_\delta \) space which satisfies \( S \), then \( (A_\xi)^P \neq \emptyset \). By (i) there exists a chain of length \( \xi \) in \( S_\xi \), whence \( \xi^* < \xi \). Therefore \( A_\xi \) is \( \xi^* \)-accessible.

Suppose that \( S = \langle S_\xi, < \rangle : \xi \text{ ordinal} \rangle \) is a spectrum. If \( A \) satisfies \( S \) we define, for every ordinal \( \xi \), the function \( E^*_\xi : S_\xi \to \omega \cup \{ \emptyset \} \) by \( E^*_\xi(a) \) is the number of \( a \in A \) with \( s_\xi(a, A) = a \). Assume that \( A \) and \( B \) satisfy \( S \). We say that \( A \) and \( B \) are \( S \)-equivalent, \( A \equiv B \), if for any \( \xi \) the following two conditions hold:

(a) If \( a \in S_\xi \), then \( E^*_\xi(a) = E^*_\xi(a) \), and \( A_\alpha \) is accessible if and only if \( B_\alpha \) is accessible.
(b) If \( \gamma \) is an open chain in \( S_2 \), then \( A_\gamma \) is accessible if and only if \( B_\gamma \) is accessible.

From now on we work with countable metrizable spaces. We shall tacitly use the well-known fact that the topology of any countable \( T_3 \) space has a clopen basis. Let \( S = \langle S_\xi, <\xi; \xi \text{ ordinal} \rangle \) be a spectrum, \( \xi_0 = \mu(S) \) and \( \zeta_0 = \max \{ \eta; \eta \text{ is the length of some maximal chain in } S_\eta \} \). Suppose that \( A \) is a countable metrizable space satisfying \( S \) and \( \xi \) is an ordinal. From Lemmas 2.3, 3.1 and 3.3 we can verify the following:

\[
(\ast) \quad \text{If } a \in S_\eta, \text{ then } A_{\eta, a} \text{ accessible implies } A_{\eta, a} \text{ is } \xi-\text{accessible.}
\]

\[
(\ast\ast) \quad \text{If } a \in S_\eta \text{ and } a \in A \text{ have } A_{\eta, a} \text{ is } \xi-\text{accessible implies } A_{\eta, a} \text{ is } \xi-\text{accessible.}
\]

Now suppose that we modify the definitions of "\( A \xrightarrow{\xi} a \)" and "\( a \xrightarrow{\eta} A \)" (see §I) by setting "\( a \xrightarrow{\xi} A \)" instead of "\( a \xrightarrow{\eta} A \)" and denote by \( s^{((\eta)}(A, a) \) the corresponding \( \xi \)-type of \( a \) in \( A \) which respect to those new definitions. In the same way as we have worked with the notion of \( \xi_0(a, A) \), we can also work with the notion of \( s^{((\eta)}(a, A) \). Proceeding by transfinite induction on \( \xi \), it is easy to check by Lemma 3.2 and \( (\ast) \), (b) (and by using the fact that the topology of \( A \) has a clopen basis) that for every \( \eta \in S_\xi \) and \( a \in A \) we have

\[
(\ast\ast) \quad s^{((\eta)}(a, A) = a \quad \text{iff} \quad s^{((\eta)}(a, A) = a.
\]

Then it is not difficult to prove the following result. Consider Lemma 2.2 in order to construct the sentence \( \varphi_A \).

**Lemma 3.4.** Suppose that \( A \) is a countable metrizable space which satisfies a spectrum \( S \). Then we can find a sequence \( \varphi_a \in (L_{\omega_1}) \) such that, for every countable metrizable space \( B \), \( B \models \varphi_a \) if and only if \( (B \text{ satisfies } S \text{ and } A \models B) \).

It is also possible to prove \( (\ast), (\ast\ast) \) and Lemma 3.4 for uncountable \( T_3 \) spaces.

Our main result is

**Theorem 1.** Let \( S \) be a spectrum. Suppose that \( A, B \) are countable metrizable spaces with \( A \models B \). Then \( A \) and \( B \) are homeomorphic.

We can show Theorem 1 by using a back and forth argument. Put \( \xi_0 = \mu(S) \).

We define the symmetric relation \( R \) between \( T_3 \) spaces with a finite (possibly empty) set of distinguished points by

\[
(A, a_1, ..., a_m)(B, b_1, ..., b_m) \text{ if and only if } (a_1, ..., a_m)(B, b_1, ..., b_m) \text{ and } (b_1, a_1)(B, a_1) \text{ and }
\]

Note that if \( A, B \) satisfy the assumptions of Theorem 1, then \( A RB \) holds. We need to show that \( R \) satisfies the two back and forth properties, that is, the properties (1) and (2) of [2, Theorem 2.2]. To carry out the proof of the nontrivial back and forth property, we give a criterion for choosing small neighborhoods of a point.

Assume that \( A \) is a countable metrizable space which satisfies a spectrum \( S = \langle S_\xi, <\xi; \xi \text{ ordinal} \rangle \), \( a \) is an accumulation point of \( A \) and \( \xi_0 = \mu(S) \). Then consider \( \langle \beta, A \rangle = s_{\xi_0+1}(a, A) \) and \( \Gamma = \bigcup_{\lambda \leq \xi_0} \{ \gamma; \gamma \text{ is a } \lambda \text{-maximal chain in } \beta \} \). Note that, for \( \gamma \) and \( \eta \) and \( \Gamma \), if \( A_\gamma \) is inaccessible (\( A_\gamma \) is inaccessible), then there is a neighborhood \( U \) of \( a \) such that \( A_\gamma \cap (A - U) \) is inaccessible (\( A_\gamma \cap (A - U) \) is inaccessible).

For each \( \gamma \) and \( \eta \) in \( \Gamma \) we take a neighborhood \( U_\gamma \) of \( a \) by distinguishing the following five cases:

\- **Case (1):** \( \gamma \) is a closed 1-chain in \( \beta \). Put \( U_\gamma \) such that \( A_\gamma \cap U_\gamma \) is accessible. If there is a neighborhood \( U \) of \( a \) with \( A_\gamma \cap (A - U) \) infinite, then \( U_\gamma \cup U \).

\- **Case (2):** \( \gamma \) is a closed 0-chain in \( \beta \). Consider \( U \) such that \( A_\gamma \cap (A - U) \) is inaccessible.

\- **Case (3):** \( \gamma \) is an open 1-chain in \( \beta \). Take \( U \) with \( A_\gamma \cap U \) accessible. If there is a neighborhood \( U \) of \( a \) with \( A_\gamma \cap (A - U) \) infinite, then \( U_\gamma \cup U \).

\- **Case (4):** \( \gamma \) is an open 0-chain in \( \beta \) with \( A_\gamma \cap U \) accessible. If there is a neighborhood \( U \) of \( a \) such that \( A_\gamma \cap (A - U) \) is inaccessible for all \( \mu < \eta \), then \( U_\gamma \cup U \).

\- **Case (5):** \( \gamma \) is an open 0-chain in \( \beta \) with \( A_\gamma \cap U \) accessible. Take \( U \) such that \( A_\gamma \cap (A - U) \) is inaccessible.

Then if \( A \) is an open set such that \( a \in U \subset U \) and, for every \( \beta \in S_{\xi_0} \), \( A_\beta \models a \) implies \( A_\beta \cap (U - \{a\}) = \emptyset \), we say that \( U \) is a good neighborhood of \( a \). Note that, by Lemmas 2.1 and 2.3, we can always find a good neighborhood of a point, and if \( U \) is a good neighborhood of \( a \) we have:

(a) If \( a \in S_\eta \) and \( A_\eta \models a \), then \( A_\eta \cap U \) is accessible.

(b) If \( \gamma \) is an open chain in \( S_\theta \) and \( A_\gamma \models a \), then \( A_\gamma \cap U \) is accessible.

Now assume that \( S = \langle S_\xi, <\xi; \xi \text{ ordinal} \rangle \) is a spectrum, \( \xi_0 = \mu(S) \), \( A \) and \( B \) are countable metrizable spaces with \( A \models B \), \( a \in A \), \( b \in B \) and \( B, A \models s_{\xi_0+1}(a, A) = s_{\xi_0+1}(b, B) \). Under these assumptions we show the following two lemmas.

**Lemma 3.5.** Suppose that \( \gamma \) is an open chain in \( \beta \). Then:

(a) If \( \gamma \) is a neighborhood \( U \) of \( a \) such that \( A_\gamma \cap (A - U) \) \( \emptyset \) for every \( \mu < \eta \), then there is a neighborhood \( V \) of \( b \) such that \( B_\gamma \cap (B - V) \) \( \emptyset \) for every \( \mu < \eta \).

(b) If there is a neighborhood \( U \) of \( a \) such that \( A_\gamma \cap (A - U) \) inaccessible for every \( \mu < \eta \), then there is a neighborhood \( V \) of \( b \) such that \( B_\gamma \cap (B - V) \) inaccessible for every \( \mu < \eta \).

**Proof.** The lemma is trivial if \( A_\gamma \) is inaccessible. Assume that \( A_\gamma \) is accessible. To show (a), note that if \( A_\gamma \cap (A - U) \) \( \emptyset \) for every \( \mu < \eta \) and \( U \) is open then there is an \( \alpha' \in A - U \). By (a) we have that \( A_\gamma \cap (A - U) \) inaccessible for every \( \mu < \eta \). Since \( A_\gamma \) is accessible, one can check that there is an \( \alpha' \in A - U \) such that if \( \langle a_1, \delta_1 \rangle = s_{\xi_0+1}(a, A) \) then \( \gamma \) is a 0-chain in \( \delta_1 \).

Note that, by using Lemma 3.1 and the fact that \( \xi_0 = \mu(S) \), we can infer that:

\[
\text{for } a, b \in S_\eta, \quad A_\eta \models A_\beta \iff B_\eta \models B_\beta \quad \text{(} \lambda = 0, 1 \text{).}
\]
Lemma 3.6. Consider $\Sigma = \{ \gamma; \gamma \text{ is an open \(0\)-maximal chain in } \mathfrak{A} \text{ such that for every neighborhood } U \text{ of } a \text{ there is an } a \in \gamma \text{ with } A_{a} \cap (A \setminus \overline{U}) \text{ accessible} \}$. Let $U^{0}, V^{0}$ be clopen good neighborhoods of $a, b$ respectively. Then we can find clopen sets $U, V$ with $a \in U \subset \overline{U}$, $b \in V \subset \overline{V}$ in such a way that for each $\gamma \in \Sigma$ we have:

(a) For every $\gamma \in \Sigma$, $A_{a} \cap (A \setminus U)$ is accessible iff $B_{a} \cap (B \setminus V)$ is accessible.

(b) For every open chain $\gamma' \subset \gamma$, $A_{a} \cap (A \setminus U)$ is accessible iff $B_{a} \cap (B \setminus U)$ is accessible.

Proof. Put $\mathcal{S} = \{ \beta \in S_{0}; A_{a} \rightarrow d = \{ \beta \in S_{0}; B_{d} \rightarrow b \}$.

Suppose

$\Sigma = \{ \gamma_{1}, \ldots, \gamma_{n} \}$ and $(\gamma_{1}, \ldots, \gamma_{n}) = \{ \gamma_{i}; \gamma_{i} \text{ is an } a_{i} \in \gamma_{i} \text{ such that, for every } \beta \in \mathcal{S}^{*}, A_{a_{i}} \rightarrow A_{b_{i}} \}$.

By Lemma 2.1 we can find, for $i = 1, \ldots, n$, an $a_{i} \in \gamma_{i}$ with $a_{i} \in \gamma_{i}$ such that, for every $\beta \in \mathcal{S}^{*} \setminus \gamma_{i}$, $A_{a_{i}} \rightarrow A_{b_{i}}$ implies $A_{a_{i}} \rightarrow A_{b_{i}}$. By Lemma 2.1 we can find, for $i = 1, \ldots, n$, an $a_{i} \in \gamma_{i}$ with $a_{i} \in \gamma_{i}$ such that, for every $\beta \in \mathcal{S}^{*} \setminus \gamma_{i}$, $A_{a_{i}} \rightarrow A_{b_{i}}$ implies $A_{a_{i}} \rightarrow A_{b_{i}}$.

We may assume that $A_{a_{i}} \rightarrow A_{b_{i}}$. Since $A_{a_{i}} \rightarrow A_{b_{i}}$ we deduce that, for every $\beta \in \mathcal{S}^{*}$, $A_{a_{i}} \rightarrow A_{b_{i}}$ implies $A_{a_{i}} \rightarrow A_{b_{i}}$. We may assume that $A_{a_{i}} \rightarrow A_{b_{i}}$. From the fact that $\gamma_{i}$ is a $0$-maximal chain in $\mathfrak{A}$ and $\gamma_{i}$ is the way in which $a_{i}$ is chosen we infer that, for $i \neq j$, if $A_{a_{i}} \rightarrow A_{b_{i}}$, then $a_{i} \in \gamma_{j}$. Thus we may assume that $A_{a_{i}} \rightarrow A_{b_{i}}$ for $i \neq j$.

Let $U^{0}$ be a clopen neighborhood of $a$ such that $U^{0} \subset U^{0}$ and $A_{a_{i}} \cap (U^{0} \setminus U^{0})$ is inaccessible ($i = 1, \ldots, n$). Let $U^{0} \supseteq U^{0}$ and $A_{a_{i}} \cap (U^{0} \setminus U^{0})$ is inaccessible ($i = 1, \ldots, n$). We have that $U^{0}$ is a clopen neighborhood of $a$ such that $U^{0} \subset U^{0}$ and $A_{a_{i}} \cap (U^{0} \setminus U^{0})$ is inaccessible ($i = 1, \ldots, n$).

We take a clopen neighborhood $U_{k}$ of $d'$ such that $U_{k} \subset U^{0} \setminus U^{0}$ and $U_{0}, U_{1}, U_{2}, \ldots, U_{k}$ are pairwise disjoint.

We take a clopen neighborhood $U_{k}$ of $d'$ such that $U_{k} \subset U^{0} \setminus U^{0}$ and $U_{0}, U_{1}, U_{2}, \ldots, U_{k}$ are pairwise disjoint. Consider $U = U^{0} \cup \bigcup_{i=0}^{k} U_{i}$. Proceeding in the same way we construct the corresponding neighborhood $\overline{P}$ of $b$. Then, for $1 \leq i \leq n$ and $\gamma \in \gamma_{i}$, $A_{a_{i}} \cap (A \setminus U)$ is accessible iff $B_{a_{i}} \cap (B \setminus \overline{P})$ is accessible iff $a_{i} \in \gamma_{i}$.

Now let us consider, for $\gamma = n+1, \ldots, m$, an $a_{i} \in \gamma_{i}$ such that $A_{a_{i}} \cap (A \setminus U)$ and $B_{a_{i}} \cap (B \setminus \overline{P})$ are accessible, a $B_{i} \in S^{*}$ with $A_{a_{i}} \rightarrow A_{b_{i}}$ and $a_{i} \in \gamma_{i}, b_{i} \in \overline{P}$ with $a_{i} \in S^{*}$ with $A_{a_{i}} \rightarrow A_{b_{i}}$, $b_{i} \in \overline{P}$.

We take a clopen neighborhood $U_{i}$ of $d'$ such that $U_{i} \subset U^{0} \setminus U^{0}$ and $U_{0}, U_{1}, U_{2}, \ldots, U_{i}$ are pairwise disjoint. Consider $U = U^{0} \cup \bigcup_{i=0}^{k} U_{i}$. Proceeding in the same way we construct the corresponding neighborhood $\overline{P}$ of $b$.

References


DEPARTAMENTO DE ECUACIONES PARciales

FACULTAD DE MATEMÁTICAS

UNIVERSIDAD COMPLUTENSE

28040 Madrid, Spain.

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