Correction to: Adding a random or a Cohen real: topological consequences and the effect on Martin's axiom

by

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This paper appeared in Fundamenta Mathematicae 103 (1979), 47–60 pp. and Shelah has recently written to me that there is a serious problem with Theorem 5.3, p. 57. This states that if $MA_{\Sigma\text{-linked}}$ holds in a model M then it still holds in $M[x]$ where x is a Cohen or random real over M; and if $MA_{\Sigma\text{-centered}}$ holds in a model M then it still holds in $M[x]$ where x is a Cohen real over M. The statement about $MA_{\Sigma\text{-linked}}$ is false: Todorčević noticed that when x is Cohen the statement conflicts with a result of Shelah's that appears in his paper on taking the inaccessible away from Solovay's proof that all sets are Lebesgue measurable (Israel Journal of Mathematics 48 (1984) 1–47 pp.). Shelah then noticed that his result can be modified to show that the statement about $MA_{\Sigma\text{-linked}}$ is false when x is random. The problems with the proof of this false theorem are, in the Cohen case, that the auxilliary partial order Q^* relies on maximal finite antichains being able to decide nearly everything, when, in fact, they seldom do; in the random case Q^* was not carefully defined and, in fact, fails to be transitive.

On the other hand, the second part of Theorem 5.3 — if $MA_{\Sigma\text{-centered}}$ holds in M then it holds in $M[x]$ where x is Cohen over M — is true. Perhaps the easiest proof was noticed several years ago by Baumgartner and Tall, and is sketched here.

Reall that $MA_{\Sigma\text{-centered}}$ is equivalent to the statement $P(C)$: for every centered family $\mathscr{B}$ on ω of size less than C there is some infinite $A \subset \omega$ with $A \subset B$ mod finite for all $B \in \mathscr{B}$.

So assume $\mathring{\mathscr{B}} = \{\mathring{B}_i\colon i \in I\}$ is a Cohen forcing name for a centered family on ω of size less than C. We may assume that $\mathring{\mathscr{B}}$ is forced to be closed under finite intersections. Let Q be the set of all triples $\langle s, t, \mathring{B}_i\rangle$ where s is a finite Cohen condition, t is a finite subset of ω, and $i \in I$. The order on Q is: $\langle s, t, \mathring{B}\rangle \leqslant \langle s', t', \mathring{B}'\rangle$ iff $s \subset s'$, $t \subset t'$, and $s \Vdash$ if $n \in t - t'$ then $n \in \mathring{B}'$. Q is easily seen to be Σ-centered and if G is Q-generic for the obvious dense sets and x is Cohen over M then $A = \bigcup \{t\colon \exists s \in x\, \exists \mathring{B}_i$ with $\langle s, t, \mathring{B}_i\rangle \in G\}$ is the required set.

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