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DRUKARNIA UNIWERSYTETU JAGIELLOŃSKIEGO W KRAKOWIE

Coding that preserves Ramseyness

by

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Abstract. We show how a transitive model, M, of ZFC + GCH + "$\varkappa$ is Ramsey" can be coded by a subset $C \subseteq \varkappa$ in a generic extension $M[G]$, a model of ZFC + "$\varkappa$ is Ramsey + $V = K^C$". (This is an analogue of a theorem of Jensen with Ramseyness replacing measurability and K^C replacing $L[\mu, C]$.)

§ 1. Introduction. In [CU] Jensen showed how a transitive model V of ZF + GCH could be "coded" by a subset r of ω in a generic extension $N = V[G]$ so that $N \models$ "$V = L[r]$" and further that the cardinality and cofinality structure of V was that of N. He further showed that most "large cardinal" properties were preserved by his forcing conditions; that is those consistent with V being L of a real. We consider the question of forcing to obtain a coding extension in the same spirit but preserving a particular property: that of Ramseyness. Clearly for no $r \subseteq \omega$ can a cardinal in $L[r]$ be Ramsey; further if $\varkappa$ is a cardinal in a model of the form $L[C]$ for $C \subseteq \varkappa$, then still $\varkappa$ is not Ramsey: it is easy to see that $C^{\#}$ must exist if $\varkappa$ is to be Ramsey. But we could consider forcing M to be a model N of the form K^C where by K^C we mean the class of C-mice (that is, the notion of mouse, cf. [CM; 9.25], relativised to a predicate $C \subseteq \varkappa$.)

This we do here, subject to a proviso (*) below. (Remark 3 indicates why such a requirement may be needed.) The preservation of the Ramsey property is an example of some rather general conditions on forcing which preserves Erdös properties which Jensen formulated in the unpublished [J]. We use his techniques to prove preservation when we have used the method of [CU; 9.9] to set up our coding conditions.

The more difficult question of coding V by a *real* so that $V[G] \models$ "$V = K^r$" and still preserving a cardinal's Ramsey property looks impossible with the coding techniques at present available, although the kind of considerations we use here, that our coding structures (the $\mathscr{M}_\xi$ below) contain enough suitably large mice does illustrate a prime ingredient of the K-coding technique. We refer to [CU] for coding techniques and a familiarity with its Ch. 1 will be very useful. For background on mice the reader should refer to [CM].

I would like to thank Dieter Donder for many discussions on these preservation questions. This presentation owes much to his insistence that the question be answered and to his suggestions as to how to go about it.

§ 2. Coding that preserves Ramseyness. Suppose that V is a model of ZF and GCH and "$\varkappa$ is a Ramsey Cardinal". Suppose further that $A \subseteq \mathrm{On}$ is such that $H_\tau = L_\tau[A \cap \tau]$ for all cardinals τ. (There is no harm in this cf. [CU; p. 26].)

THEOREM. *Let V, A and $\varkappa$ be as above, and that*

$$(*) \qquad V \models \forall \xi \in [\varkappa, \varkappa^+)\ K^{A \cap \varkappa}[A \cap \xi] \models \text{“}\varkappa \text{ is Ramsey”}$$

then there is a definable class of forcing conditions, $\boldsymbol{P}$, so that if G is $\boldsymbol{P}$-generic over V then

$$V[G] \models \text{“}\exists C \subseteq \varkappa (V = K^C \wedge \varkappa \text{ is Ramsey})\text{”}.$$

Some remarks. 1. This, as for standard coding techniques, is a cardinal preserving extension.

2. By $K^A_\mu[B]$ we mean $\langle L_\mu[D^A \cap \mu^2, A, B], \in, D^A \cap \mu^2, A, B\rangle$, where the predicate D^A codes mice "over" the predicate A. We assume a knowledge of [CM] throughout and that the reader can relativise all the appropriate definitions. So, such mice have their measurable cardinals $> \sup A$, but their projecta are allowed to drop into A itself.

3. $\varkappa$ Ramsey $\not\Leftrightarrow \varkappa$ Ramsey in K^C for $C \subseteq \varkappa$ in general. Thus we require the second condition to enable us to find sufficient indiscernibles in small enough structures. The statement that $\varkappa$ is Ramsey is taken to mean that models whose domain includes $\varkappa$ and have similarity type of size $< \varkappa$, have "good" sets of indiscernibles of order type $\varkappa$. This is known to be equivalent to the $\varkappa$-Erdös property: that if C is a closed and unbounded (cub) subset of $\varkappa$, $f\colon [C]^{<\omega} \to \mathrm{On}$ is regressive, i.e. is such that for all a $f(a) < \min a$, then there is a subset C' of C of order type $\varkappa$ that is homogeneous for f.

For simplicity we first perform a coding extension to code $A \cap [\varkappa^+, \infty)$ by a subset A_0 of $[\varkappa, \varkappa^+)$ — using the L-coding conditions of [CU]. This is a $\varkappa$-distributive coding and thus in $L[A \cap \varkappa^+, A_0]$ $\varkappa$ is still Ramsey. We now set $A = A \cap \varkappa$ and $B \subseteq [\varkappa, \varkappa^+)$ recursively coding A_0 and $A \cap [\varkappa, \varkappa^+)$.

We note that if we now take the 'V' of our theorem as $L[A, B]$ that each $K^A[B \cap \xi]$ (for $\xi \in [\varkappa, \varkappa^+)$) still thinks that $\varkappa$ is Ramsey, simply due to the fact that no new $\varkappa$-sequences have been added.

We code B by a $C_0 \subseteq \varkappa$ much as in [CU; Thm. 9.9]. Our final C will recursively code C_0 and A. To this end:

DEFINITION. We define by recursion on $\xi \in [\varkappa, \varkappa^+)$, μ_ξ, $\mathcal{M}_\xi$

$\mu_\xi =$ least $\mu > \sup_{\zeta < \xi} \mu_\zeta$ so that

$\mathcal{M}_\xi = K^A_\mu[B \cap \xi] \models$ "$\varkappa$ is Ramsey $\wedge$ ZF$^-$ $\wedge$ $L[A, B \cap \xi] \models \overline{\overline{\xi}} = \varkappa$."

CLAIM 1. *μ_ξ exists for $\xi < \varkappa^+$.*

Proof. This is clear from our requirement on $K^A[B \cap \xi]$ (taking $\varkappa$ being Ramsey in the sense already described) once we note that the final coding extension yields that

$$L[A \cap \varkappa, B \cap \xi] \models \overline{\overline{\xi}} = \varkappa$$

for all $\xi < \varkappa^+$ (cf. [CU; 2.4.3]).

CLAIM 2. *Every $x \in |\mathcal{M}_\xi|$ is $\mathcal{M}_\xi$-definable with parameters from $\varkappa$.*

Proof. $\langle \mu_\zeta |\ \zeta < \xi\rangle$ is definable in $\mathcal{M}_\xi$ in the same way it is definable in $K^A[B]$. So ξ is definable as the least point for which μ_ξ does not exist. If $X \prec \mathcal{M}_\xi$ so that $\varkappa \subseteq X$ it is easy to see, by the simple minimality argument that X is transitive and equals $K^A_{\mu_\xi}[B \cap \xi]$.

DEFINITION. $X_{\xi,\tau}$ the smallest $X \prec \mathcal{M}_\xi$ such that $\tau + 1 \subseteq X$, $\pi_{\xi\tau}\colon \mathcal{M}_{\xi\tau} \overset{\sim}{\leftrightarrow} X_{\xi,\tau}$ the transitive collapse and $\varrho_\xi(\tau)$ the ordinal code of $\mathcal{M}_{\xi\tau}$ in some definable well-order of $K^A[B]$.

$\boldsymbol{P}$ is the set of $p\colon \mathrm{dom}(p) \to 2$ such that

(a) $\mathrm{dom}(p) \subseteq [\omega, \varkappa)$;
(b) For all cardinals τ, $\mathrm{card}(\mathrm{dom}(p) \cap [\tau, \tau^+)) \leqslant \tau$.

Set $|p| =$ the least $\xi \in [\varkappa, \varkappa^+)$ such that $p \in \mathcal{M}_\xi$.

(c) If $\xi < |p|$, then there is a cardinal τ such that for all larger cardinals ν

$$p(\varrho_\xi(\nu)) = \begin{cases} 1 & \text{if } \xi \in B, \\ 0 & \text{if not}. \end{cases}$$

DEFINITION. $\boldsymbol{P}_\tau$ is defined exactly as $\boldsymbol{P}$ using τ in the place of ω. We set $\boldsymbol{P}^\tau$ for τ a cardinal to be the set of p, $p\colon \mathrm{dom}(p) \to 2$, such that

(a) $\mathrm{dom}(p) \subseteq [\omega, \tau)$;
(b) For ν a cardinal, $\mathrm{card}(\mathrm{dom}(p) \cap [\nu, \nu^+)) < \nu$.

For $p \in \boldsymbol{P}$, ν a cardinal, set

$$(p)_\nu = p \restriction [\nu, \varkappa) \quad \text{and} \quad (p)^\nu = p \restriction [\omega, \nu).$$

With the above $\boldsymbol{P}_\nu = \{(p)_\nu\colon p \in \boldsymbol{P}\}$, $\boldsymbol{P}^\nu = \{(p)^\nu\colon p \in \boldsymbol{P}\}$ and $\boldsymbol{P} = \boldsymbol{P}_\nu \times \boldsymbol{P}^\nu$.

For the extendability of any condition $p \in \boldsymbol{P}$ we may use the proof of [CU; 1.4] using the fact that $L[A, B \cap \xi] \models$ "$\overline{\overline{\xi}} = \varkappa$" yields $\square_\varkappa$ which was used there for proving the "limit stage" of such extensions was possible. Similarly the fact that $\boldsymbol{P}_\tau$ was τ-distributive for $\tau < \varkappa$ is as in 1.7 there and is too similar to the proof of the preservation of Ramseyness which is to follow to warrant being written out here again in full.

Then as before

LEMMA. *If G is $\boldsymbol{P}$-generic over V then in $V[G]$ we can find C coding G and A such that $V[G] \models V = K^C$.*

Proof. Clearly $B \in K^A[G]$ doing the previous inductive decoding of $B \cap \xi$ for $\xi < \varkappa^+$. Let C code G and A. Then

$$V[G] \models V = K^A[G] = K^A[C] = K^C .$$

We are then only left to show that Ramseyness is preserved. The proof is modelled on that of [CU; 9.9] and that of [J] where some rather general conditions for the preservation of the Ramsey property are given.

DEFINITION. Suppose $C \subseteq \varkappa$ is cub. For $a, b \in [C]^{<\omega}$ define

$$d(a, b) = \max\{\nu \mid \chi_a(\nu) \neq \chi_b(\nu)\} ;$$
$$a \prec b \quad \text{iff} \quad \chi_a(d(a, b)) < \chi_b(d(a, b)) .$$

Then it is easy to see that $\prec$ is a well-order of type $\varkappa$.
In the above we have set $\max \emptyset = 0$.

DEFINITION. $e(a, b) = \max\{b \cap d(a, b)\}$.

Suppose f is a function in the extension $V[G]$ such that f is regressive on $C \subseteq \varkappa$ cub. We suppose without loss of generality the domain (f) is actually $[C]^{<\omega}$ where $C =$ the set of infinite cardinals less than $\varkappa$. Suppose $p_0 \in G$ is such that

$$p_0 \Vdash \text{“}\mathring{f}\colon [\check{C}]^{<\check{\omega}} \to \text{On regressively”}$$

where $\mathring{f}$ names f. We show that there is a $q \leqslant p$ and $I \in V$ such that $q \Vdash$ “$\check{I}$ is homogeneous for $\mathring{f}$”. This then suffices by standard arguments. We inductively define $p_a \leqslant p$ such that

(1) $p_a \Vdash$ “$\exists \xi\ \mathring{f}(\check{a}) = \check{\xi}$”;

(2) $p_a \supseteq \bigcup_{b \prec a} (p_b)_{e(a,b)} \underset{\text{def}}{=} q_a$.

Set Δ_a to be the set of $r \leqslant p_0$ such that r “decides” $f(a)$, i.e. there exists ξ such that $r \Vdash \mathring{f}(\check{a}) = \check{\xi}$. Then each Δ_a is open dense below p_0. We define recursively X_a, $\boldsymbol{b}_a$, δ_a, α_a, and σ_a as follows:

$$X_{\{\omega_0\}} = \text{least } X \prec L_{\varkappa^{++}}[A, B] \text{ such that } \mathring{f}, p_0\ \langle \Delta_a \mid a \in [C]^{<\omega}\rangle \in X$$
$$\text{and such that } \varkappa \subseteq X .$$

Suppose a^+ is the $\prec$-successor of a

$$X_{a^+} = \text{least } X \prec L_{\varkappa^{++}}[A, B] \text{ such that } X_a \cup \{X_a\} \subseteq X .$$

If a is a $\prec$-limit set $X_a = \bigcup_{b \prec a} X_b$.

$\boldsymbol{b}_a$ is the transitive collapse of X_a via σ_a, thus

$$\sigma_a\colon \boldsymbol{b}_a = L_{\delta_a}[A, B \cap \alpha_a] \overset{\sim}{\leftrightarrow} X_a .$$

Now we define p_a by induction on $\prec$ satisfying (1) & (2) and showing that q_a is always well-defined as a condition and that $p_a \in \boldsymbol{b}_{a^+}$, as is q_a.

p_0 is defined.

$p_{\{\omega_0\}} = L[A, B]$ — least $p \leqslant p_0$ so that $|p| \geqslant \alpha_{\{\omega_0\}} \wedge p \in \Delta_{\{\omega_0\}}$.

Set $q_{\{\omega_0\}} = p_0$; then $p_{\{\omega_0\}} \in \boldsymbol{b}_{\{\omega_0\}^+}$.

If p_a and q_a are defined satisfying the above requirements then

$$p_{a^+} = L[A, B]\text{-least } p \leqslant p_a \text{ so that } |p| \geqslant \alpha_a, p \in \Delta_{a^+} ,$$
$$p \leqslant \bigcup_{b \prec a^+} (p_b)_{e(a^+,b)} \underset{\text{def}}{=} q_{a^+} .$$

We claim that q_{a^+} as defined above is a condition: we note that

$$q_{a^+} \subseteq \bigcup_{b \prec a} (p_b)_{e(a\, b)} \cup (p_a)_{e(a^+,a)} = q_a \cup (p_a)_{e(a^+,a)} .$$

Now the only reason that the first inclusion is not an equality is that for finitely many $b \prec a$ we may have $e(a^+, b) > e(a, b)$; actually a simple argument shows that this can only happen when $e(a, b) = \omega_0$ i.e. when either $b \setminus \{\omega_0\}$ or b itself is a final segment of the sequence a. Since then no cardinality violations can occur and that as functions q_{a^+} and $q_a \cup (p_a)_{e(a^+,a)}$ are the same on a final segment we have that q_{a^+} is a proper condition. And so p_{a^+}, and q_{a^+} are members of $\boldsymbol{b}_{a^{++}}$.

Lastly if a is a $\prec$-limit set $q = \bigcup_{b \prec a} (p_b)_{e(a^+,a)}$. Then $\langle p_b \mid b \prec a\rangle$, and $\langle q_b \mid b \in a\rangle$ are definable in X_a and hence in the same way in $\boldsymbol{b}_a$ from

$$\sigma^{-1}(\langle \mathring{f}, <\Delta_a \mid a \in [C]^{>\omega}>, p_0\rangle)$$

as they were from $\mathring{f}, p_0, \langle \Delta_a \mid a \in [C]^{<\omega}\rangle$ in $L_{\varkappa^{++}}[A, B]$.

We note that $\langle \alpha_a \mid a \in [C]^{<\omega}\rangle$ is a normal sequence.

CLAIM 3. $\boldsymbol{b}_a \in \mathscr{M}_{\alpha_a}$.

Proof. $\mathscr{M}_{\alpha_a} \models$ “$\bar{\bar{a}}_a = \varkappa\ V = K^A[B \cap \alpha_a]$.”

$$\boldsymbol{b}_a = L_{\delta_a}[A, B \cap \alpha_a] \models \text{“}\alpha_a = \varkappa^+\text{.”}$$

Clearly if $\mu_{\alpha_a} > \delta_a$ we are finished, so suppose not. It is easy to see in this case that $\mathscr{M}_{\alpha_a} \not\models$ “$V = L[A, b \cap \alpha_a]$” since α_a is collapsed inside $\mathscr{M}_{\alpha_a}$. There is thus an A-mouse, M, in $\mathscr{M}_{\alpha_a} \setminus L[A, B \cap \alpha_a]^{\mathscr{M}_{\alpha_a}}$. A standard argument shows that the $\varkappa$th iterate of such a mouse has as measurable cardinal an ordinal that is one of the Silver indiscernibles for $(A, B \cap \alpha_a)^{\#}$. Clearly such an ordinal is bigger than δ_a. But M and $\varkappa$ are members of $\mathscr{M}_{\alpha_a} \models \mathrm{ZF}^-$. Thus the $\varkappa$th iterate of M is in $\mathscr{M}_{\alpha_a}$. A contradiction. ■

Since now $\boldsymbol{b}_a \in \mathscr{M}_{\alpha_a}$ we then obtain that the sequence $\langle p_b \mid b \prec a\rangle$ is in $\mathscr{M}_{\alpha_a}$ and $|p_b| \geqslant \alpha_b$ and that $q \in \mathscr{M}_{\alpha_a}$. Also $q = \bigcup_{\gamma} \bigcup_{\substack{b \prec a \\ e(a,b)=\gamma}} (p_b)_\gamma = \bigcup_{\gamma} r_\gamma$ say.

Note that $\mathrm{card}(\{b \mid b \prec a \wedge e(a, b) = \gamma\}) \leqslant \gamma$, so for r_γ

(I) $\mathrm{dom}(r_\gamma) \cap \gamma^+$ has cardinality less than or equal to γ. Thus

(II) $q = \bigcup r_\gamma$ as a function has $\mathrm{card}(\mathrm{dom}(q) \cap \gamma^+) \leqslant \gamma$. But

(III) If $\xi \in [\varkappa, \alpha_a)$ then $\xi < |p_b|$ some $b \prec a$. So

$$\exists \eta \in C\ \forall \nu \in C\ (\nu \geqslant \eta \Rightarrow q(\varrho_\xi(\nu)) = \chi_B(\xi))$$

since $q \supseteq (p_b)_\gamma$ for some γ.

But (I)–(III), together with $q \in \mathcal{M}_{\alpha_a}$ imply that q is a condition.

Let $p_a =$ the $L[A, B]$—least $p \leqslant q_a$, $p \in \Delta_a$; p_a is thus again $\in b_{a^+}$, $|p_a| \geqslant \alpha_a$.

Thus $\langle p_a |\ a \in [C]^{<\omega}\rangle$ is defined. Let $p_a \Vdash \check{f}(\check{a}) = \xi_a$. Let $X_C = \bigcup_{a \in [C]^{<\omega}} X_a$ and analogously b_C, δ_C, α_C are also defined. Then $b_C = L_{\delta_C}[A, B \cap \alpha_C] \models$ "$\alpha_C = \varkappa^+$". As in Claim 3 $b_C \in \mathcal{M}_{\alpha_C}$ and further since $\langle p_a |\ a \in [C]^{<\omega}\rangle$, $\langle \xi_a |\ a \in [C]^{<\omega}\rangle$ are definable over b_C the same way as from $L_{\varkappa^{++}}[A, B]$, we have a good set of indiscernibles of order type $\varkappa$, I say, in $\mathcal{M}_{\alpha_C}$ for

$$\langle L_{\delta_C}[A, B \cap \alpha_C], \in, A, B \cap \alpha_C, \{\langle a, p_a, \xi_a\rangle_{a \in [C]^{<\omega}}\}\rangle .$$

Then Jensen's analysis shows that

LEMMA I. *If* $a . b \in [I]^n$ *and* $a \cap \nu = b \cap \nu$ *then* $(p_a)^\nu = (p_b)^\nu$.

II. $a, b \in [I]^n$, $\max(\alpha) \leqslant \min(b)$ *then* $p_b \leqslant p_a$.

Proof. I follows from a simple argument using the "goodness" of the indiscernibles. For II let $a = \{\nu_1, \ldots, \nu_n\}$, $b = \{\tau_1, \ldots, \tau_n\}$. Then, by I, $(p_a)^{\nu_1} = (p_b)^{\nu_1}$ and for the same reason $(p_a)^{\nu_{i+1}} = p^{\nu_{i+1}}_{\{\nu_1, \ldots, \nu_i, \tau_{i+1}, \ldots, \tau_n\}}$. By the definition of our conditions $p_b \leqslant (p_{\{\nu_1, \ldots, \nu_i, \tau_{i+1}, \ldots, \tau_n\}})_{\nu_i}$; hence $p_b \leqslant ((p_a)^{\nu_{i+1}})_{\nu_i}$ for $1 \leqslant i \leqslant n$. Again by definition $p_b \leqslant (p_a)_{\nu_n}$. All these together prove part II. ■

Thus the "goodness" of our indiscernibles ensures that we have a certain coherence property on the bottom parts of our conditions. We shall be finished if we show:

CLAIM 4. $\bigcup_{a \in [I]^{<\omega}} p_a = r$ *say, is a condition.*

Proof. We are then finished since then $r \Vdash$ "$\check{I}$ is homogeneous for $\check{f}$". Since b_C, and I are in $\mathcal{M}_{\alpha_C}$, $r \in \mathcal{M}_{\alpha_C}$. Firstly note that as $[I]^{<\omega}$ is $\prec$-cofinal in $[C]^{<\omega}$ $r \notin \mathcal{M}_{\alpha_a}$ for $a \in [C]^{<\omega}$. Likewise $[I]^n$ is also $\prec$-cofinal in $[C]^{<\omega}$. By Lemma I we may define $\bar{Q}^n_\nu = (p_a)^\nu$ where $a \in [I]^n$, $\nu = \min(a)$. Let $Q^n = \bigcup_\nu \bar{Q}^n_\nu$, the union of the lower parts. By Lemma II and I, $Q^n \leqslant p_a$ for all $a \in [I]^n$, if we establish:

SUBCLAIM 1. Q^n *is a proper condition.*

Proof. Clearly Q^n does not violate any cardinality restrictions on its domain. And Q^n is again in $\mathcal{M}_{\alpha_C}$ being definable from $\langle p_a\rangle$ and $[I]^n$. Again

$$\forall a \in [I]^n\ Q^n \notin \mathcal{M}_{\alpha_C}, \quad \text{but } \forall \xi < \alpha_C\ \exists a \in [I]^n\ (|p_a| \geqslant \xi) .$$

Thus Q^n codes such ξ correctly. So it is a condition.

SUBCLAIM 2. $\bigcup_n Q^n$ *is a condition.*

Proof. Notice if $a \in [I]^n$, $\max(a) = \nu$ and we choose $b \in [I]^n$ with $\nu < \min(b)$ then (using Lemma II for the first inequality):

$$p_a \geqslant p_b \geqslant p_{\{\nu\} \cup b} \geqslant Q^{n+1} \quad \text{since } e(b \cup \{\nu\}, b) = 0 .$$

But $\bigcup_n Q^n = r$. ■ (Claim 4 & Theorem.)

Our final model only has $H_{\varkappa^+}$ closed under the # operation; with some additions to the arguments concerning the X_a sub-structures, we could use K-coding techniques to provide an extension K^C where the universe was closed under #, had V been so.

References

[CM] A. J. Dodd, *The Core Model*, LMS Lecture Note Series 61, Cambridge University Press, 1982.

[CU] A. Beller, R. Jensen and P. Welch, *Coding the Universe*, LMS Lecture Note Series 47, Cambridge University Press, 1982.

[J] R. Jensen, *Forcing which preserves Ramsey Cardinals*, unpublished manuscript, 1966.

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