Cylinder problem

by

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Abstract. S. Ulam in the Scottish Book (see [Ma2]) posed the so-called rectangle problem. As a generalization F. Galvin (compare [Ga]) formulated the n-dimensional cylinder problem $P_{n-1}(\varkappa)$ where $\varkappa$ is a cardinal, $n \geqslant 1$. The 2-dimensional case is original Ulam's problem. In this paper we consider the question for which cardinals $\varkappa$ the problem $P_n(\varkappa)$ has a positive solution.

We use standard set theoretic notation. The reference for forcing is Kunen [Ku2]. For the σ-algebra $[F]$ generated by a family F of subsets of a set E, closed under complements, we define the following hierarchy: $[F]_0 = F$ and for each $\alpha < \omega_1$ $(\alpha \neq 0)$, $[F]_\alpha$ is the family of all countable unions (intersections) of sets from $\bigcup_{\beta<\alpha} [F]_\beta$ if α is odd (even). Finally, $[F] = \bigcup_{\alpha<\omega_1} [F]_\alpha$.

If $n \leqslant m < \omega$ and $i_0 < i_1 < \ldots < i_{n-1} < m$, let $C^m_{\{i_0,\ldots,i_{n-1}\}}(X)$ denote the family of all sets of the form

$$\{\langle x_0, \ldots, x_{m-1}\rangle \in {}^mX:\ \langle x_{i_0}, \ldots, x_{i_{n-1}}\rangle \in S\}$$

where $S \subset {}^nX$, and let

$$C^m_n(X) = \bigcup\{C^m_{\{i_0,\ldots,i_{n-1}\}}(X):\ i_0 < \ldots < i_{n-1} < m\}.$$

For $1 \leqslant n \leqslant m < \omega$, let $P^m_n(X)$ denote the sentence

$$\mathcal{P}({}^mX) = [C^m_n(X)]$$

and let $P_n(X)$ stand for $P^{n+1}_n(X)$. The sentence $P_n(X)$ is called the $(n+1)$-*dimensional cylinder problem* for X.

Let us note the following simple facts:

PROPOSITION 1. *Let X be a set, let $\omega \leqslant \lambda \leqslant \varkappa$ be cardinals and let $1 \leqslant n < m < \omega$. Then*

(1) $P^m_n(X)$ *iff* $P^m_n(|X|)$,
(2) $P^n_n(\varkappa)$,
(3) *if* $P_n(\varkappa)$ *then* $P_n(\lambda)$,
(4) $P^m_n(\varkappa)$ *iff* $P_n(\varkappa)$,

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(5) *if* $P_n(\varkappa)$ *then* $P_m(\varkappa)$,

(6) *if* $P_n(\varkappa)$ *then there exists* $\xi < \omega_1$ *s.t.*

$$\mathscr{P}({}^{n+1}\varkappa) = [C_n^{n+1}(\varkappa)]_\xi .$$

Proof. (1), (2), and (3) are obvious.

(4) Let $P_n^m(\kappa)$ and let $A \in P({}^{n+1}\varkappa)$. Then $A \times \{0\}^{m-n-1} \in P({}^m\varkappa) = [C_n^m(\varkappa)]$. So $A \in [C_n^{n+1}(\varkappa)]$, and hence $P_n(\varkappa)$.

For the reverse implication it is enough to show that for every $m > n$, $P_n^m(\varkappa)$ implies $P_n^{m+1}(\varkappa)$. By $P_n^m(\varkappa)$ we have

$$[C_n^m(\varkappa)] = \mathscr{P}({}^m\varkappa) = \mathscr{P}({}^{m-1}\varkappa \times \varkappa) .$$

Thus, identifying $\varkappa$ with $\varkappa \times \varkappa$ we obtain

$$[C_{n+1}^{m+1}(\varkappa)] = \mathscr{P}({}^{m-1}\varkappa \times \varkappa \times \varkappa) = \mathscr{P}({}^{m+1}\varkappa) .$$

However, by $P_n^m(\varkappa)$, $[C_{n+1}^{m+1}(\varkappa)] \subset [C_m^{m+1}(\varkappa)] \subset [C_n^{m+1}(\varkappa)]$. Hence $C_n^{m+1}(\varkappa) = \mathscr{P}({}^{m+1}\varkappa)$, i.e. $P_n^{m+1}(\varkappa)$.

(5) follows immediately from (4).

(6) We may assume $\varkappa > \omega$.

If there is no ξ with the property asserted, then for each $\xi < \omega_1$ there exists $A_\xi \in [C_n^{n+1}(\varkappa)]_{\xi+1} \setminus [C_n^{n+1}(\varkappa)]_\xi$. Thus, using the natural bijection between $\varkappa$ and $(\xi+1)\cdot\varkappa \setminus \xi\cdot\varkappa$, we may assume that for $\xi < \omega_1$ we have $A_\xi \in {}^{n+1}((\xi+1)\cdot\varkappa \setminus \xi\cdot\varkappa)$. Then $\bigcup_{\xi<\omega_1} A_\xi \notin [C_n^{n+1}(\omega_1\cdot\varkappa)]$, i.e. $\neg P_n(\omega_1\cdot\varkappa)$, which contradicts $P_n(\varkappa)$.

From the above proposition it follows, in particular, that in order to check whether $P_n^m(X)$ holds it is enough to verify a suitable cylinder problem $P_n(\varkappa)$ where $\varkappa = |X|$.

Now we are going to study the interrelation between the $(n+1)$- and $(n+2)$-dimensional cylinder problems. This will yield the positive solutions for $P_n(\varkappa)$.

THEOREM 1. *Let* $1 \leqslant n < \omega$ *and let* λ *be a cardinal s.t.* $\operatorname{cf}\lambda \neq \omega_1$. *If* $P_n(\alpha)$ *holds for every* $\alpha < \lambda$ *then* $P_{n+1}(\lambda)$.

Proof. By Proposition 1 (6) and our assumption on the cofinality of λ there exists an ordinal $\xi < \omega_1$ s.t.

$$(*) \qquad \mathscr{P}({}^{n+1}\alpha) = [C_n^{n+1}(\alpha)]_\xi \text{ for every } \alpha < \lambda .$$

Let $T \in \mathscr{P}({}^{n+2}\lambda)$. We show that $T \in [C_{n+1}^{n+2}(\lambda)]$.

For any $i < n+2$ write

$$F_i = \{\langle x_0, \ldots, x_{n+1}\rangle \in {}^{n+2}\lambda \colon x_j \leqslant x_i \quad \text{for } j < n+2\} .$$

Then ${}^{n+2}\lambda = \bigcup_{i<n+2} F_i$. So it is enough to show that, for any $i < n+2$

$$T^i = T \cap F_i \in [C_{n+1}^{n+2}(\lambda)] .$$

By symmetry the proof can be reduced to the case of $i = n+1$.

Let us define for each $\zeta < \omega_1$ a set-theoretical operation φ_ζ describing the inductive definition of the hierarchy $[\ ]_\zeta$: φ_0 is the identity operation; for $0 < \zeta < \omega_1$ let $j_\zeta \colon \zeta \times \omega \times \omega \to \omega$ be a bijection and let

$$\varphi_\zeta(\langle B_k^i \colon i < n+1, k < \omega\rangle) = \begin{cases} \bigcup_{\alpha<\zeta} \bigcup_{l<\omega} \varphi_\alpha\{\langle B^i_{j_\zeta(\alpha,l,k)} \colon i<n+1, \varkappa<\omega\rangle\} & \text{if } \zeta \text{ is odd},\\ \bigcup_{\alpha<\zeta} \bigcup_{l<\omega} \varphi_\alpha\{\langle B^i_{j_\zeta(\alpha,l,k)} \colon i<n+1, \varkappa<\omega\rangle\} & \text{if } \zeta \text{ is even}. \end{cases}$$

Then, for every $\zeta < \omega_1$ and every sequence $\langle \mathscr{F}_i \colon i < n+1\rangle$ of families of subsets of a set E, closed under complements, φ_ζ defines a function from $\prod_{i<n+1} {}^\omega\mathscr{F}_i$ onto $[\bigcup_{i<n+1} \mathscr{F}_i]_\zeta$.

Let for $\eta < \lambda$

$$T_\eta = T^{n+1} \cap ({}^{n+1}\lambda \times \{\eta\} \subset {}^{n+1}(\eta+1) \times \{\eta\} .$$

Hence, by $P_n(\eta+1)$, for each $i < n+1$ there exists a sequence $\langle A_{k,i}^\eta \colon k < \omega\rangle$ of elements of $C_{n+1 \setminus \{i\}}^{n+1}(\eta+1)$ s.t.

$$T_\eta = \varphi_\xi(\langle A_{k,i}^\eta \colon i < n+1, k < \omega\rangle) \times \{\eta\} .$$

Let

$$A_{k,i} = \bigcup_{\eta<\lambda} (A_{k,i}^\eta \times \{\eta\}) .$$

Hence $A_{k,i} \in C_{n+2\setminus\{i\}}^{n+2}(\lambda)$ and

$$\begin{aligned} T^{n+1} &= \bigcup_{\eta<\lambda} T_\eta = \bigcup_{\eta<\lambda} (\varphi_\xi(\langle A_{k,i}^\eta \colon i<n+1,\ k<\omega\rangle) \times \{\eta\}) \\ &= \bigcup_{\eta<\lambda} (\varphi_\xi(\langle A_{k,i}^\eta \times \{\eta\} \colon i<n+1,\ k<\omega\rangle) \\ &= \bigcup_{\eta<\lambda} (\varphi_\xi(\langle A_{k,i} \cap ({}^{n+1}\lambda \times \{\eta\}) \colon i<n+1\ k<\omega\rangle) \\ &= \bigcup_{\eta<\lambda} \varphi_\xi((\langle A_{k,i} \colon i<n+1,\ k<\omega\rangle) \cap ({}^{n+1}\lambda \times \{\eta\}) \\ &= \varphi_\xi(\langle A_{k,i} \colon i<n+1\cdot\ k<\omega\rangle) \in [\bigcup_{i<n+1} C_{n+2\setminus\{i\}}^{n+2}(\lambda)] \\ &\subset [C_{n+1}^{n+2}(\lambda)] , \end{aligned}$$

which finishes the proof.

COROLLARY 1. *For any cardinal* $\varkappa$ *and any* n, $1 \leqslant n < \omega$ *if* $P_n(\varkappa)$ *then* $P_{n+1}(\varkappa^+)$. *In particular,* $P_n(\omega_n)$.

Proof. This follows from the fact that the rectangle problem holds for ω_1, i.e. we have $P_1(\omega_1)$ (see [Ku1]).

From the fact that MA implies $P_1(\mathfrak{c})$ (see [Ku1]) we obtain also

COROLLARY 2. *If* MA *then* $P_2(\mathfrak{c}^+)$.

The upper bound for κ such that $P_n(\kappa)$ holds is given by the following

THEOREM 2. *Let* $1 \leqslant n < \omega$. *If* $P_n(\kappa)$ *holds then* $\kappa \leqslant \beth_n$.

Proof. We use two partition properties, $\beth_n^+ \to (n+2)_c^n$ and $\beth_n^+ \not\to (n+2)_c^{n+1}$.

The first of them is a weak statement of the Erdös–Radó theorem: $(\beth_{n-1}(\varkappa))^+ \to (\varkappa^+)_\varkappa^n$ for $1 \leqslant n \leqslant \varkappa$ (see [Ku3, Theorem 6.4. p. 392]). The second one follows from the theorem: $\beth_n(\varkappa) \not\to (n+2)_\varkappa^{n+1}$ for $1 \leqslant n < \omega \leqslant \varkappa$ (see [Ku3; Theorem 6.5, p. 393]).

Let us suppose that $P_n(\varkappa)$ holds for $\varkappa = \beth_n^+$ and choose a partition $f\colon [\varkappa]^{n+1} \to {}^{\omega}2$ witnessing $\varkappa \not\to (n+2)_c^{n+1}$.

For each $k < \omega$ let

$$A_k = \{\langle x_0, \dots, x_n\rangle \in {}^{n+1}\varkappa\colon x_0 < x_1 < \dots < x_n \;\&\; f(\{x_0, \dots, x_n\})(k) = 0\}.$$

By $P_n(\varkappa)$ there exist sets $S_i \subset {}^n\varkappa$ $(i < \omega)$ s.t. A_k's belong to the σ-algebra generated by the family of cylinders:

$$\{\{\langle x_0, \dots, x_n\rangle \in {}^{n+1}\varkappa\colon \langle x_0, \dots, x_{j-1}, x_{j+1}, x_{j+1}, \dots, x_n\rangle \in S_i\}\colon j<n+1,\ i<\omega\}.$$

For $x_0 < \dots < x_{n-1}$ define $g(\{x_0, \dots, x_{n-1}\}) = \{i < \omega\colon \langle x_0, \dots, x_{n-1}\rangle \in S_i\}$. By $\varkappa \to (n+2)_c^n$ there is a subset $X \subset \varkappa$, $|X| = n+2$, s.t. g is constant on $[X]^n$. However, f witnesses $\varkappa \not\to (n+2)_c^{n+1}$, and so f is not constant on $[X]^{n+1}$. Let $\{x_0, \dots, x_n\}$, $\{y_0, \dots, y_n\} \in [X]^{n+1}$ be such that $x_0 < \dots < x_n$, $y_0 < \dots < y_n$ and

$$f(\{x_0, \dots, x_n\}) \neq f(\{y_0, \dots, y_n\}).$$

Hence there exists an integer k s.t. exactly one of the points $\langle x_0, \dots, x_n\rangle$ and $\langle y_0, \dots, y_n\rangle$ is an element of A_k. So, there exist $i < \omega$ and $j < n+1$ s.t. S_i distinguishes the points

$$\langle x_0, \dots, x_{j-1}, x_{j+1}, \dots, x_n\rangle \quad \text{and} \quad \langle y_0, \dots, x_{j-1}, y_{j+1}, \dots, y_n\rangle,$$

which contradicts the fact that g is constant on $[X]^n$.

The above theorem and Corollary 1 give

COROLLARY 3. *If* GCH *holds then for* $1 \leqslant n < \omega$ $P_n(\kappa)$ *iff* $\kappa \leqslant \omega_n$.

It seems natural to ask whether $P_n(\beth_n)$ is a theorem of ZFC. The following generalization of a theorem due to Kunen [Ku1] provides a negative answer to this question.

THEOREM 3. *If M is a model of* ZFC *and f is a Cohen generic function over M adding at least ω_{n+1} reals $(0 < n < \omega)$ then* $M[f] \models$ "$\neg P_n(\omega_{n+1})$".

Proof. We show that for every model M of ZFC and every Cohen generic function $f\colon \omega_1 \times \omega_2 \times \dots \times \omega_{n+1} \to 2$, if $A_f = \{x \in {}^{n+1}\omega_{n+1}\colon f(x) = 0\}$ then $M[f] \models$ "$A_f \notin [C_n^{n+1}(\omega_{n+1})]$".

Let us asume that for some n the above statement is not true. Then there exists a minimal $m<\omega$ $(m>0)$ s.t. for some model M of ZFC and a Cohen generic function $f\colon \omega_1 \times \dots \times \omega_{m+1} \to 2$ over M,

$$M[f] \models \text{“}A_f \in [C_m^{m+1}(\omega_{m+1}) \cup (\mathscr{P}({}^{m+1}\omega_{m+1}) \cap M)\text{”}.$$

Hence there exists (in $M[f]$) a sequence $\langle S_i\colon i<\omega\rangle$ of subsets of $\omega_1 \times \dots \times \omega_m$ s.t. if

$$S = \{\{\langle x_0, \dots, x_m\rangle \in {}^{m+1}\omega_{m+1}\colon \langle x_0, \dots, x_{m-1}\rangle \in S_i\}\colon i<\omega\}$$

then

$$M[f] \models \text{“}A_f \in [S \cup \bigcup_{i<m} C_{m+1\setminus\{i\}}^{m+1}(\omega_{m+1}) \cup (\mathscr{P}({}^{m+1}\omega_{m+1}) \cap M)]\text{”}.$$

However, $|S_i| \leqslant \omega_m$ for each $i<\omega$, so there exists a set $B \subset \omega_{m+1}$, $|B| \leqslant \omega_m$ s.t. $S \in M[f \upharpoonright (\omega_1 \times \dots \times \omega_m \times B)]$. Thus if $\eta \in \omega_{m+1} \setminus B$, $f_1 = f \upharpoonright (\omega_1 \times \dots \times \omega_m \times (\omega_{m+1} \setminus \{\eta\}))$, and $g\colon \omega_1 \times \dots \times \omega_m \to 2$ is s.t. $g(x_1, \dots, x_m) = f(x_1, \dots, x_m, \eta)$, then

$$M[f] \models \text{“}A_{f \upharpoonright (\omega_1 \times \dots \times \omega_m \times \{\eta\})} \in [\bigcup_{i<m} C_{m+1\setminus\{i\}}^{m+1}(\omega_{m+1})(\mathscr{P}({}^{m+1}\omega_{m+1}) \cap M[f_1])]\text{”}$$

and $M[f] = M[f_1][g]$. Hence

$$M[f_1][g] \models \text{“}A_g \in [C_{m-1}^m(\omega_m) \cup (\mathscr{P}({}^m\omega_m) \cap M[f_1])]\text{”}.$$

By the minimality of m and the fact that g is Cohen generic over $M[f_1]$, we have $m = 1$. So

$$g\colon \omega_1 \to 2 \text{ and } M[f_1][g] \models \text{“}A_g \in [\mathscr{P}(\omega_1) \cap M[f_1]]\text{”}.$$

Thus there exists a real number $r \in M[f_1][g]$ s.t. for some countable $F \in M[f_1]$, $F \subset \mathscr{P}(\omega_1)$ the pair $\langle F, r\rangle$ codes A_g. It follows that for some countable $D \in M[f_1]$, $D \subset \omega_1$, we have $r \in M[f_1][g \upharpoonright D]$. Hence $A_g \in M[f_1][g \upharpoonright D]$ and so $g \upharpoonright \omega_1 \setminus D \in M[f_1][g \upharpoonright D]$, which is impossible, because $g \upharpoonright \omega_1 \setminus D$ is Cohen generic over $M[f_1][g \upharpoonright D]$.

COROLLARY 4. *If M is a model of* ZFC *and f is a Cohen generic function over M adding at least ω_ω reals then for $1 \leqslant n < \omega$*

$$M[f] \models \text{“}P_n(\varkappa) \text{ iff } \varkappa \leqslant \omega_n\text{”}.$$

We do not know any model of set theory in which there exist n and $\varkappa$ s.t. $P_{n+1}(\varkappa^+)$ and $\neg P_n(\varkappa)$. In particular, the following problem seems to be interesting.

PROBLEM. *Is P_2 $(\mathfrak{c}^{++})$ consistent?*

Let us now introduce a generalization of the cylinder (see [Ma1]). For $n < \omega$ let $Q_n(\varkappa)$ denote the statement: "for every $S \subset \mathscr{P}({}^{n+1}\varkappa)$ s.t. $|S| \leqslant \varkappa$ there exists $\mathscr{D} \subset \mathscr{P}({}^{n+1}\varkappa)$ s.t. $|\mathscr{D}| < \omega$ and $S \subset [C_n^{n+1}(\kappa) \cup \mathscr{D}]$".

PROPOSITION 2. *For any cardinal $\varkappa$ and $n < \omega$ we have*

$$P_{n+1}(\varkappa) \Rightarrow Q_n(\varkappa) \Rightarrow Q_{n+1}(\varkappa).$$

Proof. The first implication is an immediate consequence of the definitions. The proof of the other one is similar to that of Proposition 1 (4).

The above proposition gives us the positive solution: $Q_n(\omega_{n+1})$ for every $n < \omega$. For the negative part we have only a partial solution.

THEOREM 4. *If* $2^{\varkappa}>2^{\omega}$ *then* $\neg Q_1((2^{\varkappa})^+)$.

Proof. Let $\lambda=(2^{\varkappa})^+$. For each $\alpha<\lambda$ choose a one-to-one function $h_\alpha\colon \alpha\to\mathscr{P}(\varkappa)$ and define

$$S_\alpha=\{\langle\xi,\eta\rangle\colon\ \eta<\alpha\ \&\ \xi\in h_\alpha(\eta)\}\subset\varkappa\times\lambda.$$

Now suppose that there is a countable family $\mathscr{D}=\langle D_n\colon n<\omega\rangle$ of subsets of $\kappa\times\lambda$ s.t. $\{S_\alpha\colon \alpha<\lambda\}\subset[C_1^2(\lambda)\cup\mathscr{D}]$. Choose $X\subset\lambda$ s.t. $|X|=(2^{\omega})^+$ and for each $n<\omega$ $D_n\cap(\varkappa\times X)=D_n'\times X$ for some $D_n'\subset\varkappa$ (X is a subset of a counter-image of the point $g\colon\lambda\to{}^{\varkappa\times\omega}2$ defined by $g(\eta)(\xi,n)=0$ iff $\langle\xi,\eta\rangle\in D_n$). Thus

$$\{S_\alpha\cap(\varkappa\times X)\colon\ \alpha<\lambda\}\in[C_1^2(\lambda)].$$

Fix $\alpha<\lambda$ s.t. $X\subset\alpha$. Now, if $\eta_1\eta_2\in X$ and $\eta_1\neq\eta_2$ then

$$h_\alpha(\eta_1)\neq h_\alpha(\eta_2),\quad \text{i.e.}\quad \{\xi\colon\langle\xi,\eta_1\rangle\in S_\alpha\}\neq\{\xi\colon\langle\xi,\eta_2\rangle\in S_\alpha\}.$$

Since $|X|>2^{\omega}$, it is easy to see that $S_\alpha\cap(\kappa\times X)\notin[C_1^2(\lambda)]$, because otherwise each set $\{\xi\colon\langle\xi,\eta\rangle\in S_\alpha\}$ would be determined by some real number. This gives contradiction.

An easy corollary to this theorem is that $2^{\omega}=\omega_2$, $2^{\omega_1}=\omega_3$ and $2^{\omega_2}\geqslant\omega_4$ imply $\neg Q_1((2^{\mathfrak{c}})^+)$. Another consequence is that $\neg Q_1((2^{\mathfrak{c}})^+)$. Let us also notice that an easy modification of the proof of Theorem 4 (using the fact that $\neg P_n(\beth_n^+)$) gives also that, for any $n<\omega$, $Q_n(\kappa)$ implies $\kappa\leqslant 2^{\beth_n^+}$. So the following problem might be mentioned in this context:

"does $Q_n(\varkappa)$ imply $\varkappa\leqslant\beth_{n+1}$ for $2\leqslant n<\omega$?"

Let us finally note that in a model of ZFC obtained by adding at least ω_ω Cohen reals, for every $n<\omega$ we have $Q_n(\kappa)$ iff $\kappa\leqslant\omega_{n+1}$.

The proof is similar to that of Theorem 3.

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Terminal continua and the homogeneity *

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Abstract. In the paper we prove the following statements: (1) every hereditarily indecomposable and continuously homogeneous continuum is one-dimensional; (2) every proper terminal subcontinuum of a homogeneous curve is tree-like; (3) every homogeneous hereditary θ-continuum is atriodic.

1. Terminal continua. Definitions which are not recalled here can be found in [13]. All spaces in this paper are metric.

A compact space X has *Kelley's property at* $x\in X$ if for every continuum $Y\subset X$ containing x and for every sequence x_n of points of X converging to x, there exists a sequence of continua $Y_n\subset X$ converging to Y such that $x_n\in Y_n$. A space X has *Kelley's property* if it has Kelley's property at each point (see [21]).

A space is said to be *homogeneous with respect to the class M of mappings* if for every two points p and q of X, there exists a continuous surjection f from X onto itself such that $f\in M$ and $f(p)=q$. A continuum homogeneous with respect to homeomorphisms (continuous maps) will be simply called *homogeneous* (*continuously homogeneous*).

Charatonik has observed in [2] that

(1.1) *Continua which are homogeneous with respect to open mappings have Kelley's property.*

A subcontinuum Q of X is called *terminal* if $K\in C(X)$ and $K\cap Q\neq\emptyset$ imply either $K\subset Q$ or $Q\subset K$, where $C(X)$ denotes as usually the space of all subcontinua of X with the Hausdorff distance. We will denote the collection of all terminal subcontinua of X by $T(X)$ and the collection of all indecomposable subcontinua of X by $IN(X)$. The following proposition is an immediate consequence of above definitions.

(1.2) *If a continuum X has Kelley's property, then $T(X)$ is closed in $C(X)$.*

We have (see [10])

(1.3) *If f is a continuous mapping from a continuum X onto Y, $K\in T(Y)$ and C is a component of $f^{-1}(K)$, then $f(C)=K$.*

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