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Countable subsets of Suslinian continua

by

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Abstract. An example of a Suslinian continuum Y with no countable set intersecting all nondegenerate subcontinua of Y is given.

All spaces are considered to be metric. A *continuum* is a connected and compact space. A continuum is *Suslinian* if it does not contain uncountably many mutually exclusive nondegenerate subcontinua.

In 1971 A. Lelek posed the following question: If Y is a Suslinian continuum, does there exist a countable set A in Y such that A intersects every nondegenerate subcontinuum of Y? ([2], Problem 10, P 726). A partial positive answer was given by A. Lelek in the case where Y is hereditarily unicoherent ([3], Th. 2.2, p. 133). The aim of this note is to describe an example which gives a negative answer to this question.

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CONSTRUCTION OF THE EXAMPLE. Denote by I the unit interval $[0,1]$. Let $h\colon I\to I$ be a mapping defined by the following formula:

$$h(t)=\begin{cases}2t & \text{for } 0\leqslant t\leqslant \frac{1}{2},\\ 2-2t & \text{for } \frac{1}{2}\leqslant t\leqslant 1.\end{cases}$$

For an arbitrary finite collection A of subintervals of I, we will say that A has the property (*) provided that for every $J_1, J_2\in A$ either $J_1=J_2$ or $J_1\subset \operatorname{int} J_2$ or $J_2\subset \operatorname{int} J_1$ or $J_1\cap J_2=\emptyset$.

For any collection A with the property (*) let us adopt the following notation. $L(A)$ is the set of all left ends of intervals from A, and $R(A)$ is the set of all right ends of intervals from A. For a point $p\in L(A)\cup R(A)$ let $d(p)$ denote the length of the interval from A having p as an endpoint.

Set $r(A)=\frac{1}{4}\min\{|a-b|\colon a\neq b,\ a,b\in L(A)\cup R(A)\}$.

For $n=1,2,\ldots$ let us define a mapping $g_n[A]\colon I\to I$ by the following formula:

$$g_n[A](t) = \begin{cases} p+d(p)h^n\left(\frac{p-t}{r(A)}\right) & \text{for } p-r(A)\leqslant t\leqslant p \text{ and } p\in L(A), \\ 2t+2r(A)-p & \text{for } p-2r(A)\leqslant t\leqslant p-r(A) \text{ and } p\in L(A), \\ p-d(p)h^n\left(\frac{t-p}{r(A)}\right) & \text{for } p\leqslant t\leqslant p+r(A) \text{ and } p\in R(A), \\ 2t-2r-p & \text{for } p+r(A)\leqslant t\leqslant p+2r(A) \text{ and } p\in R(A), \\ t & \text{for the remaining } t. \end{cases}$$

Set $g_0[A] = \mathrm{id}_I$.

Observe that $g_n[A]$ is a continuous mapping such that the preimage of any point from I is finite.

1. PROPOSITION. *$g_n[A](p) = p$ for $p\in L(A)\cup R(A)$ and $g_n[A](J) = J$ for $J\in A$.*

2. PROPOSITION. *If $J\in A$, J is not contained in any other element of A and $J = [a, b]$, then $g_n[A]([0, b]) = [0, b]$ and $g_n[A]([a, 1]) = [a, 1]$.*

If $J_1 = [a_1, b_1]$ and $J_2 = [a_2, b_2]$ are elements of A such that J_2 is the minimal element of A containing J_1 in its interior, then

$$g_n[A]([a_2, b_1]) = [a_2, b_1] \quad\text{and}\quad g_n[A]([a_1, b_2]) = [a_1, b_2].$$

Let $B_0, B_1, \ldots$ be a sequence of finite collections of mutually exclusive subintervals of I. Let $A_n = \bigcup_{k=0}^{n} B_k$. Let us assume that for every $n = 0, 1, \ldots$ the collection A_n has the property (*). For each pair $n>m$ of positive integers let us define a mapping $g_{n,m}\colon I\to I$ by a formula

$$g_{n,m} = g_m[A_m]g_{m+1}[A_{m+1}]\ldots g_{n-1}[A_{n-1}].$$

Set $g_{n,n} = \mathrm{id}_I$.

We will construct by induction a sequence $B_0, B_1, \ldots$ such that

0) $B_0 = \{I\}$,

$1)_n$ the collection $\{g_{n,m}(J)|\ J\in A_n\}$ has the property (*) for every $m\leqslant n$,

$2)_n$ $\operatorname{diam} g_{n,0}(J)\leqslant \frac{1}{n+1}$ for $J\in B_n$,

$3)_n$ $\operatorname{diam}(S)<\frac{1}{2}r(A_{n-1})$ and $\operatorname{diam} g_{n,0}(S)\leqslant\frac{1}{n+1}$ for every component S of $I-\bigcup_{J\in B_n} J$,

$4)_n$ $\operatorname{diam}(J)\leqslant\frac{1}{2}r(A_{n-1})$ for $J\in B_n$ and

$5)_n$ for $J_1\in B_n$ and $J_2\in A_{n-1}$ either

$$g_{n,0}(J_1)\subset g_{n,0}(J_2) \quad\text{or}\quad g_{n,0}(J_1)\cap g_{n,0}(J_2) = \emptyset.$$

Let us suppose that sets $B_0, \ldots, B_{n-1}$ satisfying these conditions are constructed. It suffices to construct B_n. By $1)_{n-1}$ there exist functions $g_{n,m}$ for $m<n$. Consider the set $Z = g_{n,0}^{-1}\big(g_{n,0}(L(A_{n-1})\cup R(A_{n-1}))\big)$. Note that Z is a finite set such that $g_{n,0}$ is a local homeomorphism on $I-Z$. Take a finite set $P\subset I-Z$ such that $\operatorname{diam} g_{n,0}(S)<\frac{1}{n+1}$ and $\operatorname{diam}(S)<\frac{1}{2}r(A_{n-1})$ for every component S of $I-P$. Now, in order to obtain a collection B_n it suffices to take a collection of sufficiently small intervals about points from P. (Condition $1)_n$ is easy to fulfil because $g_{n,m}$ is a local homeomorphism on $I-Z$).

Let us denote by T the inverse limit $\varprojlim\{I_n, g_{n,m}\}$, where $I_n = I$. Let g_n be the projection of T onto I_n. For every $J\in B_k$ $(k = 0, 1, \ldots)$ the sequence $\{J_n = J, g_{n,m}, n\geqslant m\geqslant k\}$ is an inverse system, denote its limit by $T(J)$.

3. LEMMA. *If E is a subcontinuum of T such that $T(J)\cap E\neq\emptyset\neq E-T(J)$ for a certain $J\in B_k$ $(k = 1, 2, \ldots)$ then $T(J)\subset E$.*

Proof. There exists an integer $m>k$ such that $g_n(E)-J\neq\emptyset$ for $n\geqslant m$. Therefore there is an $a\in g_{n+1}(E)$ such that $g_{n+1,n}(a)\notin J$. Since $g_{n+1}(E)\cap J\neq\emptyset$, it follows that $g_n(E) = g_{n+1,n}g_{n+1}(E)$ contains J for every $n\geqslant m$. Hence $T(J)\subset E$.

4. LEMMA. *Let E be a subcontinuum of $T(J_1)$ (for $J_1\in B_{k_1}$) such that, for every k and every $J\in B_k$, if $E\subset T(J)$ then $J_1\subset J$. Let $J_0\in B_{k_0}$ be such that $J_0\subset J_1$ and such that, for every k and every $J\in B_k$, if $J_0\subset J$ then either $J_0 = J$ or $J_1\subset J$. Then if $J_0\subset\operatorname{int} g_n(E)$ for a certain $n\geqslant k_0$, then $T(J_0)\subset E$.*

Proof. By 2 it follows that $J_0\subset\operatorname{int} g_{n+1}(E)$. Thus $J_0\subset\operatorname{int} g_m(E)$ for every $m\geqslant n$. Hence $T(J_0)\subset E$.

5. LEMMA. *If E is a nondegenerate subcontinuum of T then there are k and $J_0\in B_k$ such that*

$$T(J_0)\subset E \quad\text{and}\quad E-T(J_0)\neq\emptyset.$$

Proof. Observe that $g_0(E)$ is nondegenerate; therefore there is an n such that $\operatorname{diam} g_0(E)>\frac{4}{n+1}$.

Let $J_1\in B_{k_1}$ be such that $E\subset T(J_1)$ and for every k and every $J\in B_k$, if $E\subset T(J)$ then $J_1\subset J$. By condition $2)_{k_1}$ we have $k_1<n$.

By condition $3)_n$ there is a $J_2\in B_n$ such that $J_2\cap g_n(E)\neq\emptyset$. Let J_3 be an element of A_n, distinct from J_1 containing J_2 and such that no other element of A_n but J_1 contains J_3.

If $g_m(E)\cap J_3\neq\emptyset$ for every $m\geqslant n$ then the lemma follows by 3. So we may assume that there is an $m>n$ such that $g_m(E)\cap J_3 = \emptyset$. We may also assume that $g_{m-1}(E)\cap J_3\neq\emptyset$. Let $J_3 = [a, b]$. Without loss of generality we may assume that $g_m(E)\subset[0, a]$.

We have

$$g_m(E)\cap[a-r(A_{m-1}), a]\neq\emptyset.$$

By conditions $2)_m$, $3)_m$ and $4)_m$, there exists a $J_0 \in A_m$ such that $J_0 \subset \operatorname{int} g_m(E) \cap$ $\cap [a-4r(A_{m-1}), a]$. Observe that by the choice of $r(A_{m-1})$ no element of A_{m-1} but J_1 contains J_0. Now the lemma follows by 4.

Let X_n be a space resulting from $I\times\{0,1,\ldots,2^n-1\}$ by the following identification: a pair (x, i) is identified with a pair (y, j) provided that there exists a $k\leqslant n$ such that $x = y \in L(B_k) \cup R(B_k)$ and $E(i2^{k-n-1}) = E(j2^{k-n-1})$, where $E(a)$ denotes the integral part of the real number a.

Points of X_n will be denoted in the same way as points of $I\times\{0,1,\ldots,2^n-1\}$.

Let us denote by p_n the projection of X_n onto I.

For every $n = 1, 2, \ldots$, let $f_{n,n-1}\colon X_n \to X_{n-1}$ be defined by the formula

$$f_{n,n-1}(x,i) = \begin{cases} \big(g_{n,n-1}(x), 2^{n-k}E(i2^{k-n-1})+j\big) \text{ for} \\ \quad p-\frac{j+1}{2^{n-k}}r(A_{n-1})\leqslant x\leqslant p-\frac{j}{2^{n-k}}r(A_{n-1}), \\ \quad j=0,1,\ldots,2^{n-k}-1,\ k=1,2,\ldots,n-1 \text{ and } p\in L(B_k), \\ \big(g_{n,n-1}(x), 2^{n-k}E(i2^{k-n-1})+j\big) \text{ for} \\ \quad p+\frac{j}{2^{n-k}}r(A_{n-1})\leqslant x\leqslant p+\frac{j+1}{2^{n-k}}r(A_{n-1}), \\ \quad j=0,1,\ldots,2^{n-k}-1,\ k=1,2,\ldots,n-1 \text{ and } p\in R(B_k), \\ \left(g_{n,n-1}(x), E\left(\frac{i}{2}\right)\right) \text{ for the remaining } x. \end{cases}$$

Observe that $f_{n,n-1}$ is a continuous function and the preimages of points are finite.

For $n>m$ let us define $f_{n,m}\colon X_n \to X_m$ by the formula

$$f_{n,m} = f_{m+1,m} f_{m+2,m+1} \ldots f_{n,n-1}.$$

Set $X = \varprojlim\{X_n, f_{n,m}\}$, and denote by f_n the projection of X onto X_n.

6. Proposition. *The diagram*

$$\begin{array}{ccc} X_m & \xleftarrow{f_{n,m}} & X_n \\ \downarrow{\scriptstyle p_m} & & \downarrow{\scriptstyle p_n} \\ I & \xleftarrow{g_{n,m}} & I \end{array}$$

is commutative.

It follows that mappings p_n induce a mapping $p\colon X\to T$ such that $p_n f_n = g_n p$.

For every $n = 1, 2, \ldots$, $k\leqslant n$, $J\in B_k$, and $j = 0, 1, \ldots, 2^{k-1}-1$, let $C_j^n(J)$ denote the component of $p_n^{-1}(J)$ which contains $J\times\{j2^{n-k+1}\}$.

Observe that if $k\leqslant m\leqslant n$, then $f_{n,m}(C_j^n(J)) = C_j^m(J)$.

Let $C_j(J)$ be the inverse limit of the sequence $C_j^k(J), C_j^{k+1}(J), \ldots$, where $J\in B_k$ and $j = 0, \ldots, 2^{k-1}-1$.

7. Proposition. *The diameters of $C_j(J)$'s tend to zero when k tends to infinity and $J\in B_k$.*

The proof of the following proposition is the same as that of 3.

8. Proposition. *If E is a subcontinuum of X such that $E\cap C_j(J)\neq\emptyset\neq E-C_j(J)$ for a certain $J\in B_k$ and a certain $j = 0, \ldots, 2^{k-1}-1$ then $C_j(J)\subset E$.*

9. Proposition. *The set $p^{-1}(t)$ is zero-dimensional for every $t\in T$.*

$$p^{-1}(T(J)) = \bigcup_{j=0}^{2^{k-1}-1} C_j(J) \quad \text{for} \quad j\in B_k.$$

By Theorem 6 § 47, III from [1], the definition of the property (*) and conditions $1)_n$ and $5)_n$ of the construction of the sequences $B_0, B_1, \ldots$, in the interior of $g_0(T(J))$, where $J\in B_k$, there exists a point $t(J)$ which does not belong to any $g_0(T(J'))$ for $J'\in B_{k'}$ and $k'>k$.

Set $(g_0 p)^{-1}(t(J)) \cap C_j(J) = K_j(J)$. Observe that this set is homeomorphic to the Cantor set.

Let Z be the ternary Cantor set on the unit interval I. For every $k = 1, 2, \ldots$, let $Z_k(0), Z_k(1), \ldots, Z_k(2^{k-1}-1)$ be the natural partition of Z into a collection of open-closed sets such that

$$Z_k(i) = Z_{k+1}(2i)\cup Z_{k+1}(2i+1),$$

Note that $Z = Z_1(0)$.

Consider the Cartesian product $X\times Z$. Let R be an equivalence relation on this set such that the class of abstraction of a point (x, z) is a one-point set unless $(x, z)\in K_j(J)\times Z_k(i)$ for certain $k = 1, 2, \ldots, J\in B_k$ and $i, j = 0, 1, \ldots, 2^{k-1}-1$. On each set $K_j(J)\times Z_k(i)$ the relation R induces an upper semi-continuous decomposition such that the quotient space is homeomorphic to the Cantor set, and is such that the classes of abstraction are at most two point sets and for every $z_1, z_2\in Z_k(i)$ there exist $x_1, x_2\in K_j(J)$ such that $(x_1, z_1)R(x_2, z_2)$. Such a relation may be obtained by a homeomorphism identifying $K_j(J)\times Z_k(i)$ with $Z\times Z$ and the relation $(z_1, z_2)\sim(z_2, z_1)$ on $Z\times Z$.

Observe that the relation R induces an upper semi-continuous decomposition of $X\times Z$ (see 7). Let $Y = X\times Z/R$ and let $s\colon X\times Z\to Y$ be the quotient map. Note that Y is a continuum.

10. Lemma. *For every countable set $W\subset Y$ there exists a nondegenerate subcontinuum of Y which misses W.*

Proof. The set $s^{-1}(W)$ is countable. Thus there exists a $z\in Z$ such that $X\times\{z\}\cap s^{-1}(W) = \emptyset$. Hence the nondegenerate continuum $s(X\times\{z\})$ does not intersect W.

Observe that the function $u\colon Y\to T$ defined by the formula $u(s(x, z)) = p(z)$ is a continuous function.

11. Proposition. *The set $u^{-1}(t)$ is zero-dimensional for every $t\in T$.*

12. Lemma. *For every nondegenerate subcontinuum E of Y there exist $z\in Z$, $k = 1, 2, \ldots$, $J\in B_k$ and $j = 0, \ldots, 2^{k-1}-1$ such that $s(C_j(J)\times\{z\})\subset E$.*

Proof. By 11, the set $u(E)$ is a nondegenerate subcontinuum of T. There exist $k = 1, 2, \ldots$, and $J \in B_k$ such that

$$T(J) \subset u(E) \quad \text{and} \quad u(E) - T(J) \neq \emptyset$$

(see 5).

Let K be the union of all the sets $K_j(J)$, $j \in 0, \ldots, 2^{k-1}-1$, $k = 1, 2, \ldots$ Denote by B the set $s(K \times Z)$. Note that $Y - B$ can be considered as a subset of $X \times Z$. Denote by v the projection of $X \times Z$ onto X. Observe that $pv|_{Y-B} = u|_{Y-B}$ and $u(B)$ is a zero-dimensional set. Thus $pv(E-B) \subset u(E)$ and $u(E) - pv(E-B) \subset u(B)$ is zero-dimensional. Therefore $pv(E-B) \cap T(J) \neq \emptyset \neq pv(E-B) - T(J)$. By 9 it follows that there is a $j = 0, \ldots, 2^{k-1}-1$ such that

$$v(E-B) \cap C_j(J) \neq \emptyset \neq v(E-B) - C_j(J).$$

Hence

$$E \cap s\big(C_j(J) \times Z\big) \neq \emptyset \neq E - s\big(C_j(J) \times Z\big).$$

We will prove the following claim.

CLAIM. *For every $n = 1, 2, \ldots$ there exists an $i = 0, 1, \ldots, 2^{n-1}-1$ such that*

$$C_j(J) \subset \operatorname{cl} v\big(E \cap s(X \times Z_n(i)) - B\big).$$

Assume that the claim is false. So there is an n such that, for every $i = 0, \ldots, 2^{n-1}-1$, there is an open in X set V_i such that $V_i \cap C_j(J) \neq \emptyset$ and $V_i \cap v(E \cap s(X \times Z_n(i)) - B) = \emptyset$.

For every $i = 0, \ldots, 2^{n-1}-1$, let V_i' be an open set intersecting $C_j(J)$ and such that $V_i' \subset \operatorname{cl} V_i' \subset V_i$. Let U be a neighborhood of $C_j(J)$ in X such that

(i) for every $k' \geqslant n$, $J' \in B_{k'}$ and $j' = 0, \ldots, 2^{k'-1}-1$ if $\operatorname{cl} U \cap K_{j'}(J') \neq \emptyset$, then $C_{j'}(J') \subset C_j(J)$ and

(ii) for every $k' \geqslant n$, $J' \in B_{k'}$ $(J' \neq J)$, $j' = 0, \ldots, 2^{k'-1}-1$, and $i = 0, \ldots, 2^{n-1}-1$, if $C_{j'}(J') \cap \operatorname{cl} U - V_i \neq \emptyset$ then $C_{j'}(J') \cap V_i' = \emptyset$.

Let E_u be a component of $E \cap s(U \times Z)$ which intersects $s\big(C_j(J) \times Z\big)$. By Theorem 1 § 47, III from [1] it follows that

$$E_u - s\big(C_j(J) \times Z\big) \neq \emptyset.$$

Let E' be an arbitrary component of $E_u - s\big(C_j(J) \times Z\big)$. Again by Theorem 1 § 47, III from [1]), $\operatorname{cl} E' \cap s\big(C_j(J) \times Z\big) \neq \emptyset$. By the choice of U, there is an $i = 0, \ldots, 2^{n-1}-1$ such that $E' \subset s\big(X \times Z_n(i)\big)$.

Let E'' be the union of $v(E'-B)$ and of all continua $C_{j'}(J')$ $(k' > n, J' \in B_{k'}$ and $j' = 0, 1, \ldots, 2^{k'-1}-1)$ such that $E' \subset s\big(C_{j'}(J') \times Z\big) \neq \emptyset$. Observe that E'' is connected and $\operatorname{cl} E'' - C_j(J) = E''$. On the other hand, $\operatorname{cl} E'' \cap V_i' = \emptyset$ (see (ii)) and $E'' - C_j(J) \neq \emptyset$; thus by 8 we have

$$\operatorname{cl} E'' \cap C_j(J) = \emptyset.$$

Hence $s(\operatorname{cl} E'' \times Z) \cap s\big(C_j(J) \times Z\big) = \emptyset$. But $\operatorname{cl} E' \subset s(\operatorname{cl} E'' \times Z)$, which contradicts the fact that $\operatorname{cl} E' \cap s\big(C_j(J) \times Z\big) \neq \emptyset$. This contradiction proves the claim.

There is a $z \in Z$ such that if $z \in Z_n(i)$ $(n = 1, 2, \ldots, i = 0, \ldots, 2^{n-1}-1)$ then

$$C_j(J) \subset \operatorname{cl} v\big(E \cap s(X \times Z_n(i)) - B\big).$$

It suffices to prove that $s\big(C_j(J) \times \{z\}\big) \subset E$. Otherwise there exists a point $x \in C_j(J)$ such that $s(x, z) \notin E$. Thus there exists a neighborhood V of x in X and a set $Z_n(i)$ containing z such that $s\big(V \times Z_n(i)\big) \cap E = \emptyset$. It follows that $V \cap v\big(E \cap s(X \times Z_n(i)) - B\big) = \emptyset$ which contradicts the property of z. This contradiction completes the proof of the lemma.

13. THEOREM. *Y is a Suslinian continuum such that for every countable set $W \subset Y$ there is a nondegenerate continuum which does not intersect W.*

Proof. It suffices to prove that Y is Suslinian (see 10).

Let $\{E_\alpha\}_{\alpha \in G}$ be an uncountable collection of nondegenerate subcontinua of Y. Since continua $C_j(J)$'s form a countable collection, by 12 there is a $C_j(J)$, a $J \in B_k$, and an uncountable set $G' \subset G$ such that for every $\alpha \in G'$ there is a $z_\alpha \in Z$ such that $s\big(C_j(J) \times \{z_\alpha\}\big) \subset E_\alpha$. There are $i = 0, \ldots, 2^{k-1}-1$ and $\alpha_1, \alpha_2 \in G'$ such that $z_{\alpha_1}, z_{\alpha_2} \in Z_k(i)$. By the construction the intersection of $s\big(C_j(J) \times \{z_{\alpha_1}\}\big)$ and $s\big(C_j(J) \times \{z_{\alpha_2}\}\big)$ is non-void. Hence $E_{\alpha_1} \cap E_{\alpha_2} \neq \emptyset$, which completes the proof.

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