On the category of n-groups

by

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Abstract. Let $n = s \cdot k$. We introduce the notions of a free covering $(k+1)$-group and a covering $(k+1)$-group of an $(n+1)$-group, which are generalizations of analogous notions used by Post. This enables us to construct a functor Φ_s: $Gr_{n+1} \to Gr_{k+1}$ (Gr_n — the category of n-groups) which is left adjoint to the forgetful functor Ψ_s: $Gr_{k+1} \to Gr_{n+1}$. We obtain several theorems on commutativity of those functors with inductive and projective limits. We also prove a general theorem on the form of inductive limits of covering $(k+1)$-groups of $(n+1)$-groups.

1. Introduction. Several authors have investigated n-groups, but not much attention has been paid so far to the category of n-groups. This paper is an attempt to put together and systematize certain facts concerning categorical properties of n-groups. Although in most notions and theorems where n-groups and their homomorphisms occur they are considered from the external, categorical view-point, in some cases, of necessity, the internal approach is used, which means considering n-groups as sets with a certain structure and their homomorphisms as functions. This causes a certain inconsistency of notation. Namely, an n-group as a set with operations will be denoted by $\mathfrak{G} = (G, f)$ (or shortly (G, f)), whereas for the same n-group considered as an object in the category of n-groups simply the abbreviation G will be used. To avoid numerous repetitions, we assume that f and g always denote $(n+1)$-group and $(k+1)$-group operations, respectively, and we write (G, f) and (G, g) only to avoid a possible misunderstanding. The terms homomorphisms and morphisms will be used interchangeably, depending on which properties, internal or external, are to be emphasized.

The symbol α: $A \to B$, where A and B are objects of the same category, usually means a morphism of that category. The identity morphism will be denoted by e_A: $A \to A$, or shortly e, if it is not misleading.

In this paper the term functor always means a covariant one. We shall use interchangeably the following terms: a small category and a diagram scheme; a functor from a small category and a diagram. The latter terms will be used especially in dealing with limits. The symbol $\mathscr{D}$ will always denote a small category and the symbol F a functor from that category $\mathscr{D}$ (i.e., F will denote a diagram).

For operations in n-groups the same notation as in [9] is adopted. Let us recall only the most common conventions:

$$f(\ldots, \overset{r}{x}, \ldots) = f(\ldots, \underbrace{x, \ldots, x}_{r}, \ldots);$$

$$f(\ldots, x_{i-1}, \overset{0}{x_i}, x_{i+1}, \ldots) = f(\ldots, x_{i-1}, x_{i+1}, \ldots).$$

Also most of the notions and theorems to which we refer here can be found in [9]. The notions of category theory used here can be found in any monograph in this field (e.g. [2], [14]).

Part of the results here presented have been announced in [7] and [8].

2. The functors Ψ_s and Φ_s. The category consisting of n-groups as objects and their homomorphisms as morphisms will be denoted (as in [7] and [8]) by $\boldsymbol{Gr}_n$. Consequently, the category of groups will be denoted by $\boldsymbol{Gr}_2$. The definition of n-group given by Dörnte in [4] assumed that its carrier was nonempty. In considering the category of n-groups for $n>2$ it is convenient to admit the empty n-group. The empty n-group is an initial object in $\boldsymbol{Gr}_n$. The category $\boldsymbol{Gr}_n$ always has final objects. They are one-element n-groups. In $\boldsymbol{Gr}_2$ there exist zero objects: one-element groups.

The class of n-groups is a variety (cf. [5] and [12]); thus in $\boldsymbol{Gr}_n$ monomorphisms and injective homomorphisms coincide (cf. [2]).

Let $\mathfrak{G} = (G, g)$ be a $(k+1)$-group. Then the $(sk+1)$-groupoid $\mathfrak{G}_{(s)} = (G, g_{(s)})$ where $g_{(s)}$ is an $(sk+1)$-ary operation which is a simple iteration of the $(k+1)$-ary operation g, i.e.,

$$g_{(s)}(x_1, \ldots, x_{sk+1}) = \underbrace{g\big(g(\ldots g\big(g}_{s}(x_1, \ldots, x_{k+1}), x_{k+2}, \ldots, x_{2k+1}\big), \ldots\big), x_{(s-1)k+2}, \ldots, x_{sk+1}\big)$$

is already an $(sk+1)$-group (cf. [4], [9]). In certain situations we will write $g_{(\cdot)}$ to mean $g_{(u)}$ for some $u = 1, 2, \ldots$ In [5] it was shown that the resulting $(sk+1)$-group $\mathfrak{G}_{(s)}$ is a reduct (in the sense of [6]; note that in n-group theory the term reduct is also used in another sense) of the $(k+1)$-group $\mathfrak{G}$, i.e., every term operation in $\mathfrak{G}_{(s)}$ is a term operation in $\mathfrak{G}$. This leads to the forgetful functor Ψ_s: $\boldsymbol{Gr}_{k+1} \to \boldsymbol{Gr}_{sk+1}$. Note that for the case of $s = 1$ the functor Ψ_1 is simply the identity functor. When it is not misleading we will write simply Ψ in place of Ψ_s. Like every forgetful functor, Ψ turns out to be a faithful functor. The functor Ψ thus reflects monomorphisms and epimorphisms. On the other hand, like every forgetful functor, Ψ preserves and reflects injective and surjective homomorphisms. Hence Ψ preserves monomorphisms.

In [9] the notion of a free covering k-group and covering k-group has been introduced. The definitions of these notions given here differ from the corresponding definitions of [9]. However, one can prove that in the case of nonempty n-groups (and in fact only such n-groups were considered in [9]) the two definitions are equivalent.

For the sake of description of the construction of the free covering k-group and for the investigation of the functors Ψ and Φ it will be convenient to treat $(k+1)$-groups and $(n+1)$-groups, rather than k-groups and n-groups. Henceforth, throughout the paper, we always assume $n = sk$ (admitting $k = 1$, $s = 1$), $s = mq$; furthermore m and q may have the same indices.

DEFINITION 1. A pair $\langle A', \tau_A\rangle$ where τ_A: $A \to \Psi_s(A')$, $A \in \boldsymbol{Gr}_{n+1}$, $A' \in \boldsymbol{Gr}_{k+1}$ is said to be a *free covering $(k+1)$-group of an $(n+1)$-group A* if for each h: $A \to \Psi_s(B)$, where $B \in \boldsymbol{Gr}_{k+1}$, there exists a unique morphism h^*: $A' \to B$ such that $\Psi_s(h^*)\tau_A = h$.

The construction of a free covering group is due to Post (cf. [13]). Here we cite the construction of free covering $(k+1)$-groups as it was given in [7].

The set $Z_s = \{0, 1, \ldots, s-1\}$ where $s = 2, 3, \ldots$ together with the $(k+1)$-ary operation $\varphi(l_1, \ldots, l_{k+1}) \equiv l_1 + \ldots + l_{k+1} + 1 \,(\mathrm{mod}\, s)$ is a cyclic $(k+1)$-group of order s. We will denote it by $\mathfrak{C}_{s,k+1}$. Additionally, by $\mathfrak{C}_{1,k+1}$ we will denote the one-element $(k+1)$-group.

Let $\mathfrak{G} = (G, f)$ be an arbitrary nonempty $(n+1)$-group and let $c \in G$ be an arbitrary but fixed element of G. Form the set $G^{*s} = G \times Z_s$ and define a $(k+1)$-ary operation f^* in G^{*s} in the following way:

$$f^*((x_1, l_1), \ldots, (x_{k+1}, l_{k+1})) = \big(f_{(\cdot)}(x_1, \overset{l_1 k}{c}, \ldots, x_{k+1}, \overset{l_{k+1} k}{c}, \bar{c}, \overset{n-1-\varphi(l_1, \ldots, l_{k+1})k}{c}), \varphi(l_1, \ldots, l_{k+1})\big)$$

for $x_1, \ldots, x_{k+1} \in G$, $l_1, \ldots, l_{k+1} \in Z_s$.

The $(k+1)$-groupoid $\mathfrak{G}^{*s} = (G^{*s}, f^*)$ together with a mapping τ_G: $G \to \Psi_s(G^{*s})$ given by $\tau_G(x) = (x, 0)$ is a free covering $(k+1)$-group of the $(n+1)$-group $\mathfrak{G}$ in the sense of Definition 2 of [9] (cf. [9], Theorem 1) and hence also in the sense of Definition 1. Note that when $s = 1$ the $(n+1)$-group $\mathfrak{G}$ is isomorphic to the $(n+1)$-group $\mathfrak{G}^{*1}$ and τ_G is an isomorphism.

PROPOSITION 1. *If a pair $\langle A', \lambda_A\rangle$ is a free covering $(k+1)$-group of an $(n+1)$-group A, then the morphism λ_A: $A \to \Psi_s(A')$ is a monomorphism and the set $\lambda_A(A)$ generates the $(k+1)$-group A'.*

With each free covering $(k+1)$-group $\langle A', \tau_A\rangle$ of the $(n+1)$-group A we can connect some morphism ζ_A: $A' \to \mathfrak{C}_{s,k+1}$. For a nonempty $(n+1)$-group A the morphism ζ_A is determined as in Coset Theorem (cf. [13] and Theorem 2 of [9]). It is easy to check that for the empty $(n+1)$-group A a free covering $(k+1)$-group $\langle A', \tau_A\rangle$ is, depending on k, the empty one (for $k>1$) or the one-element group (for $k = 1$), whence ζ_A: $A' \to \mathfrak{C}_{s,k+1}$ is uniquely determined.

DEFINITION 2. A triple $\langle A', \lambda_A, \zeta_A\rangle$ where λ_A: $A \to \Psi_s(A')$, ζ_A: $A' \to \mathfrak{C}_{q,k+1}$, $A \in \boldsymbol{Gr}_{n+1}$, $A' \in \boldsymbol{Gr}_{k+1}$, is said to be a *covering $(k+1)$-group of index q of the $(n+1)$-group A* if there exists a $(qk+1)$-group $\tilde{A}$ together with an embedding $\tau_{\tilde{A}}$: $\tilde{A} \to \Psi_q(A')$ such that $\langle A', \tau_{\tilde{A}}\rangle$ is a free covering $(k+1)$-group of the $(qk+1)$-group $\tilde{A}$, $\Psi_m(\tilde{A}) = A$, $\Psi_m(\tau_{\tilde{A}}) = \lambda_A$, and the morphism ζ_A is determined by $\langle A', \tau_{\tilde{A}}\rangle$.

Note that if the $(n+1)$-group A is nonempty, the number q and the morphism ζ_A are uniquely determined by the pair $\langle A', \lambda_A\rangle$ and, conversely, the morphism λ_A is uniquely determined by the pair $\langle A', \zeta_A\rangle$ (cf. [9], Theorem 2 and Definition 3). If the $(n+1)$-group A is empty, each number q with $q|s$ can be treated as an index of $\langle A', \lambda_A\rangle$.

Proposition 2. *Let A be a nonempty $(n+1)$-group. A pair $\langle A', \lambda_A\rangle$ is a covering $(k+1)$-group of A if and only if λ_A is a monomorphism and the set $\lambda_A(A)$ generates the $(k+1)$-group A'.*

The pair $\langle G^{*s}, \tau_G\rangle$ is a quasireflect of the object $G \in \boldsymbol{Gr}_{n+1}$ with respect to the forgetful functor $\Psi_s\colon \boldsymbol{Gr}_{k+1} \to \boldsymbol{Gr}_{n+1}$ (cf. [14]). This enables us to define the functor $\Phi_s\colon \boldsymbol{Gr}_{n+1} \to \boldsymbol{Gr}_{k+1}$ by $\Phi_s(G) = G^{*s}$ for $G \in \boldsymbol{Gr}_{n+1}$, which is a left adjoint functor to Ψ_s. Note that to guarantee the correctness of this definition we choose in each nonempty $(n+1)$-group (G,f) an element $c_G \in G$ and then by $\Phi_s(G)$ we mean the $(k+1)$-group (G^{*s}, f^*) constructed as in [7] for a previously given element c_G. The procedure of choice is not essential, since all functors obtained by this way are naturally equivalent. Therefore when investigating the preservation properties of Φ_s we may choose the elements c_G in the most convenient way. As in the case of Ψ, we also now admit writing Φ in place of Φ_s. And similarly to the case of Ψ, the functor Φ_1 is the identity functor.

Let $\mathfrak{A} = (A,f)$, $\mathfrak{B} = (B,f)$ be $(n+1)$-groups and $h\colon \mathfrak{A} \to \mathfrak{B}$. Fix elements $c_A \in A$ and $c_B \in B$ to be used in the construction of the free covering $(k+1)$-groups. Then (cf. [9]) $\Phi_s(h) = (\tau_B h)^*$, whence

$$\Phi_s(h)(x,l) = \big(f(h(x), \underbrace{h(c_A), \ldots, h(c_A)}_{lk}, \bar{c}_B, \underbrace{c_B, \ldots, c_B}_{n-1-lk}), l\big).$$

Observe that if we take $c_B = h(c_A)$, the above formula is simplified. In fact, for the functor Φ_s determined by such a choice we have $\Phi_s(h)(x,l) = (h(x), l)$. Hence

Proposition 3. *The functor Φ preserves and reflects injective and surjective homomorphisms.*

In $\boldsymbol{Gr}_n$ the class of injective homomorphisms is equal to the class of monomorphisms. Thus, in view of Proposition 3, Φ preserves and reflects monomorphisms (cf. [1], [3]). Note that the last remark results also from the fact that Φ is a faithful functor (since the first canonical transformation $\tau_G\colon G \to \Psi_s\Phi_s(G)$ is a monomorphism, cf. [2]). Being such a functor, Φ reflects epimorphisms. On the other hand, being a left adjoint functor, Φ preserves epimorphisms (cf. [3]). Hence and by Proposition 3 we obtain

Corollary 1. *In the category $\boldsymbol{Gr}_n$ the class of surjective homomorphisms is equal to the class of epimorphisms.*

Recall that a monomorphism $\mu\colon U \to A$ is said to be a *regular monomorphism* (cf. [2], p. 41) if, for any mo phism $\gamma\colon X \to A$ such that for each pair $\alpha\colon A \to Y$, $\beta\colon A \to Y$ the equality $\alpha\mu = \beta\mu$ implies the equality $\alpha\gamma = \beta\gamma$, there exists a morphism $\gamma'\colon X \to U$ such that $\mu\gamma' = \gamma$. In a dual way we define the notion of a regular epimorphism.

As in each variety, in $\boldsymbol{Gr}_n$ every surjective homomorphism is a regular epimorphism (cf. [2]). Hence one can obtain

Corollary 2. *In the category $\boldsymbol{Gr}_n$ every epimorphism is a regular epimorphism.*

Proposition 4. *The functor Φ reflects regular monomorphisms.*

Proof. Let a morphism $\Phi_s(\mu)\colon \Phi_s(A) \to \Phi_s(B)$, where $A, B \in \boldsymbol{Gr}_{n+1}$, be regular. The morphism $\mu\colon A \to B$ is a monomorphism. Take an arbitrary $\gamma\colon X \to B$, where $X \in \boldsymbol{Gr}_{n+1}$, having the following property: for every pair $\alpha\colon B \to Y$, $\beta\colon B \to Y$ from the equality $\alpha\mu = \beta\mu$ it follows that $\alpha\gamma = \beta\gamma$. One can show that $\Phi_s(\gamma)$ also has this property. Hence there exists a morphism $\eta\colon \Phi_s(X) \to \Phi_s(A)$ such that $\Phi_s(\mu)\eta = \Phi_s(\gamma)$. According to Corollary 6 of [9] it follows that $\eta = \Phi_s(\delta)$ where $\delta\colon X \to A$ and $\mu\delta = \gamma$. This proves that μ is a regular monomorphism.

Corollary 3. *In the category $\boldsymbol{Gr}_n$ every monomorphism is a regular monomorphism.*

3. The relations of the functor Φ_s to inductive and projective limits. As we mentioned above, the class of n-groups is a variety. The category $\boldsymbol{Gr}_n$ is therefore a complete category with respect to inductive and projective limits for all diagram schemes including the empty diagram scheme (cf. [2]). The functor Φ, being a left adjoint functor, preserves inductive limits. We show even more, namely that Φ also reflects inductive limits.

Lemma 1. *Let categories $\mathscr{K}_1$ and $\mathscr{K}_2$ be complete with respect to inductive limits. If a faithful functor $\Lambda\colon \mathscr{K}_1 \to \mathscr{K}_2$ preserves inductive limits and reflects isomorphisms, then Λ reflects inductive limits.*

Proposition 5. $[L; \{\alpha_D\colon F(D) \to L\}_{D\in\mathscr{D}}]$ *is the inductive limit of $F\colon \mathscr{D} \to \boldsymbol{Gr}_{n+1}$ if and only if $[\Phi_s(L); \{\Phi_s(\alpha_D)\colon \Phi_sF(D) \to \Phi_s(L)\}_{D\in\mathscr{D}}]$ is the inductive limit of $\Phi_sF\colon \mathscr{D} \to \boldsymbol{Gr}_{k+1}$.*

Corollary 4. *If $[L'; \{\gamma_D\colon \Phi_sF(D) \to L'\}_{D\in\mathscr{D}}]$ is the inductive limit of $\Phi_sF\colon \mathscr{D} \to \boldsymbol{Gr}_{k+1}$, then there exists an object $L \in \boldsymbol{Gr}_{n+1}$ unique up to an isomorphism, a family $\{\alpha_D\colon F(D) \to L\}_{D\in\mathscr{D}}$ and an isomorphism $\eta\colon L' \to \Phi_s(L)$ such that $\Phi_s(\alpha_D) = \eta\gamma_D$ for every $D \in \mathscr{D}$. Moreover, $[L; \{\alpha_D\colon F(D) \to L\}_{D\in\mathscr{D}}]$ is then the inductive limit of $F\colon \mathscr{D} \to \boldsymbol{Gr}_{n+1}$.*

The above corollary shows that the inductive limit of free covering $(k+1)$-groups is a free covering $(k+1)$-group, i.e., the class of free covering $(k+1)$-groups of $(n+1)$-groups is closed with respect to inductive limits. Note that we now regard free covering $(k+1)$-groups of $(n+1)$-groups as certain $(k+1)$-groups (to be exact, such $(k+1)$-groups G for which there exist epimorphisms $\zeta\colon G \to \mathfrak{C}_{s,k+1}$). Being a free covering $(k+1)$-group of $(n+1)$-group is then an inner property of the $(k+1)$-group itself but not of the pair (the form of the epimorphism is not essential here).

Proposition 6. *If $[L'; \{\pi_D\colon L' \to \Phi_sF(D)\}_{D\in\mathscr{D}}]$ is the projective limit of $\Phi_sF\colon \mathscr{D} \to \boldsymbol{Gr}_{k+1}$, where $\mathscr{D}$ is a nonempty category or $k = 1$, then there exists an object $L \in \boldsymbol{Gr}_{n+1}$ such that $\Phi_s(L)$ is isomorphic to L'.*

Proof. Let $[G, \{\gamma_D\colon G \to F(D)\}_{D\in\mathscr{D}}]$ be the projective limit of $F\colon \mathscr{D} \to \boldsymbol{Gr}_{n+1}$ where $\mathscr{D}$ is nonempty. The family $\{\Phi_s(\gamma_D)\}_{D\in\mathscr{D}}$ is compatible with $\Phi_s F\colon \mathscr{D} \to \boldsymbol{Gr}_{k+1}$. Then there exists a unique morphism $\mu\colon \Phi_s(G) \to L'$ with $\pi_D \mu = \Phi_s(\gamma_D)$ for $D\in\mathscr{D}$. By Corollary 6 of [9] the object L' is a free covering $(k+1)$-group of an $(n+1)$-group L. Hence L' is isomorphic to $\Phi_s(L)$.

Now, let the object $L' \in \boldsymbol{Gr}_2$ be the projective limit of $\Phi_s F\colon \mathscr{D} \to \boldsymbol{Gr}_2$ where $\mathscr{D}$ is the empty category. Then L' (as a final object in $\boldsymbol{Gr}_2$) is a one-element group. But a one-element group is a free covering group of the empty $(n+1)$-group. This completes the proof.

Thus the class of free covering $(k+1)$-groups of $(n+1)$-groups is closed with respect to projective limits (where free covering $(k+1)$-groups of $(n+1)$-groups are understood simply as a subclass of the class of $(k+1)$-groups, as in the remark to Corollary 4). It is worthwhile to add that the object L is not uniquely determined and depends on the choice of the morphism γ_D (cf. Corollary 6 of [9]). Proposition 6 is (regarding projective limits) essentially weaker than Corollary 4 (regarding inductive limits). The functor Φ need not preserve a cartesian product. For example, let $\mathfrak{A} = (A, f_A)$, $\mathfrak{B} = (B, f_B)$ be finite $(n+1)$-groups. Then $\mathfrak{A}^{*s} = (A\times Z_s, f_A^*)$, ${}^{*s}\mathfrak{B} = (B\times Z_s, f_B^*)$, whence $\mathfrak{A}^{*s}\times B^{*s} = (A\times Z_s\times B\times Z_s, f_A^*\times f_B^*)$. On the other hand, $(\mathfrak{A}\times\mathfrak{B})^{*s} = (A\times B\times Z_s, f_{A\times B}^*)$, which shows that the $(k+1)$-groups $\mathfrak{A}^{*s}\times\mathfrak{B}^{*s}$ and $(\mathfrak{A}\times\mathfrak{B})^{*s}$ are not isomorphic. However, under additional conditions set upon the diagram scheme $\mathscr{D}$ one can obtain

THEOREM 1. *Let $\mathscr{D}$ have a final object. Then $[L; \{\pi_D\colon L \to F(D)\}_{D\in\mathscr{D}}]$ is the projective limit of $F\colon \mathscr{D} \to \boldsymbol{Gr}_{n+1}$ if and only if $[\Phi_s(L); \{\Phi_s(\pi_D)\colon \Phi_s(L) \to \Phi_s F(D)\}_{D\in\mathscr{D}}]$ is the projective limit of $\Phi_s F$.*

Proof. Let E be a final object of $\mathscr{D}$, i.e., for each object $D\in\mathscr{D}$ there exists a unique morphism $\sigma_D\colon D \to E$. We first prove that Φ_s preserves projective limits.

Let $[L; \{\pi_D\colon L \to F(D)\}_{D\in\mathscr{D}}]$ and $[L'; \{\gamma_D\colon L' \to \Phi_s F(D)\}_{D\in\mathscr{D}}]$ be the projective limits of $F\colon \mathscr{D} \to \boldsymbol{Gr}_{n+1}$ and $\Phi_s F\colon \mathscr{D} \to \boldsymbol{Gr}_{k+1}$, respectively. The family $\{\Phi_s(\pi_D)\}_{D\in\mathscr{D}}$ is compatible with $\Phi_s F$; thus there exists a unique morphism $\delta\colon \Phi_s(L) \to L'$ with $\gamma_D \delta = \Phi_s(\pi_D)$ for $D\in\mathscr{D}$. Since $\Phi_s(L)$ and $\{\Phi_s F(D)\}_{D\in\mathscr{D}}$ are free covering $(k+1)$-groups, there exist epimorphisms $\zeta_L\colon \Phi_s(L) \to \mathfrak{C}_{s,k+1}$ and $\zeta_D\colon \Phi_s F(D) \to \mathfrak{C}_{s,k+1}$ for $D\in\mathscr{D}$ (cf. [9], Theorem 2). Note that $\zeta_D \Phi_s(\pi_D) = \zeta_L$ and $\zeta_E \Phi_s F(\sigma_D) = \zeta_D$, $\zeta_Y \Phi_s F(\alpha) = \zeta_X$ if $\alpha\colon X \to Y$, for every $D, X, Y \in\mathscr{D}$. Hence $\zeta_D\gamma_D = \zeta_E\gamma_E$, which proves that the morphisms $\zeta_D\gamma_D$ are equal to each other for every $D\in\mathscr{D}$. Denote these morphisms by ζ', i.e., $\zeta' = \zeta_D\gamma_D$. The morphism $\zeta'\colon L' \to \mathfrak{C}_{s,k+1}$ is an epimorphism since $\zeta'\delta = \zeta_L$. Thus $\langle L', \zeta'\rangle$ is a free covering $(k+1)$-group of the $(n+1)$-group $G = \zeta'^{-1}(0)$ (cf. [9]). Furthermore, there exists an isomorphism $\eta\colon \Phi_s(G) \to L'$ such that $\gamma_D\eta = \Phi_s(\alpha_D)$ where $\alpha_D\colon G \to F(D)$ for $D\in\mathscr{D}$. Hence $\Phi_s(F(\alpha)\alpha_X) = \Phi_s(\alpha_Y)$. The last equality shows that the family $\{\alpha_D\}_{D\in\mathscr{D}}$ is compatible with F. Then there exists a unique morphism $\beta\colon G \to L$ such that $\pi_D\beta = \alpha_D$ for $D\in\mathscr{D}$. Thus $\Phi_s(\pi_D)\Phi_s(\beta)\eta^{-1}\delta = \Phi_s(\pi_D)$ for $D\in\mathscr{D}$, which shows that $\Phi_s(\beta)\eta^{-1}\delta = e_{\Phi_s(L)}$. On the other hand, the equality $\zeta'\delta = \zeta_L$ implies the existence of a morphism $\mu\colon L \to G$ such that $\eta\Phi_s(\mu) = \delta$ (cf. [9], Theorem 4). Hence $\Phi_s(\alpha_D\mu\beta) = \Phi_s(\alpha_D)$. By the faithfulness of Φ_s we get $\alpha_D\mu\beta = \alpha_D$ for $D\in\mathscr{D}$, whence $\mu\beta = e_G$. Thus $\delta\Phi_s(\beta)\eta^{-1} = e_{L'}$. So δ is an isomorphism, which proves that

$$[\Phi_s(L); \{\Phi_s(\pi_D)\colon \Phi_s(L) \to \Phi_s F(D)\}_{D\in\mathscr{D}}]$$

is the projective limit of $\Phi_s F\colon \mathscr{D} \to \boldsymbol{Gr}_{k+1}$.

The proof of the converse theorem is similar to that of Proposition 5. The only facts needed are the completeness of $\boldsymbol{Gr}_{n+1}$ and faithfulness of Φ and the preservation of projective limits, as shown. This completes the proof of Theorem 1.

COROLLARY 5. *If $[L'; \{\gamma_D\colon L' \to \Phi_s F(D)\}_{D\in\mathscr{D}}]$ is the projective limit of $\Phi_s F\colon \mathscr{D} \to \boldsymbol{Gr}_{k+1}$ and $\mathscr{D}$ is a small category with a final object, then there exists an object $L\in\boldsymbol{Gr}_{n+1}$ unique up to an isomorphism, a family $\{\pi_D\colon L \to F(D)\}_{D\in\mathscr{D}}$ and an isomorphism $\varrho\colon \Phi_s(L) \to L'$ such that $\Phi_s(\pi_D) = \gamma_D\varrho$ for each $D\in\mathscr{D}$. Moreover,*

$$[L; \{\pi_D\colon L \to F(D)\}_{D\in\mathscr{D}}]$$

is then the projective limit of $F\colon \mathscr{D} \to \boldsymbol{Gr}_{n+1}$.

It turns out that Proposition 6 is a particular case of a more general statement on inductive limits of covering $(k+1)$-groups of $(n+1)$-groups.

Let $\mathscr{D}$ be an arbitrary, nonempty diagram scheme.

THEOREM 2. *If $[L'; \{\gamma_D\colon F'(D) \to L'\}_{D\in\mathscr{D}}]$ is the inductive limit of $F'\colon \mathscr{D} \to \boldsymbol{Gr}_{k+1}$, where $\langle F'(D), \lambda_D, \zeta_D\rangle$ are covering $(k+1)$-groups of indices q_D of the $(n+1)$-groups $F(D)$ and $\Psi_s F'(\alpha)\lambda_X = \lambda_Y F(\alpha)$ for each morphism $\alpha\colon X \to Y$, then L' is also a covering $(k+1)$-group of index $q = \text{g.c.d.}\,\{q_D\}_{D\in\mathscr{D}}$ of the $(n+1)$-group $\delta(L)$ where $[L; \{\alpha_D\colon F(D) \to L\}_{D\in\mathscr{D}}]$ is the inductive limit of $F\colon \mathscr{D} \to \boldsymbol{Gr}_{n+1}$, and δ is the morphism induced by $\{\Psi_s(\gamma_D)\lambda_D\}_{D\in\mathscr{D}}$.*

Proof. Let $[L; \{\alpha_D\colon F(D) \to L\}_{D\in\mathscr{D}}]$ be the inductive limit of F. The family $\{\Psi_s(\gamma_D)\lambda_D\}_{D\in\mathscr{D}}$ is compatible with F, and so there exists a unique morphism $\delta\colon L \to \Psi_s L'$ such that $\delta\alpha_D = \Psi_s(\gamma_D)\lambda_D$ for $D\in\mathscr{D}$. Since $\langle F'(D), \lambda_D, \zeta_D\rangle$ are, by assumption, covering $(k+1)$-groups of indices q_D of $F(D)$, it follows that there exists a unique family of epimorphisms $\{\xi_\alpha\colon \mathfrak{C}_{q_X,k+1} \to \mathfrak{C}_{q_Y,k+1}\}_{\alpha\in\mathscr{D}}$, where $\alpha\colon X \to Y$, with $\xi_\alpha(0) = 0$ and $\xi_\alpha\zeta_X = \zeta_Y F'(\alpha)$ (cf. [9], Theorem 4). Let $q = \text{g.c.d.}\,\{q_D\}_{D\in\mathscr{D}}$. Then for each $D\in\mathscr{D}$ there exists one (and only one) epimorphism $\xi_D\colon \mathfrak{C}_{q_D,k+1} \to \mathfrak{C}_{q,k+1}$ with $\xi_D(0) = 0$. From the definition of ξ_D it follows that for every morphism $\alpha\colon X \to Y$ the equality $\xi_Y\xi_\alpha = \xi_X$ holds. Then the family $\{\xi_D\zeta_D\}_{D\in\mathscr{D}}$ is compatible with $F'\colon \mathscr{D} \to \boldsymbol{Gr}_{k+1}$. Hence there exists a unique morphism $\zeta'\colon L' \to \mathfrak{C}_{q,k+1}$ such that $\zeta\gamma_D = \xi_D\zeta_D$ for $D\in\mathscr{D}$. This equality shows that ζ is an epimorphism.

Let $a' \in \delta(L)$, i.e., $a' = \delta(a)$ where $a\in L$. The $(n+1)$-group L, being the inductive limit of F, is generated by the set $\bigcup_{D\in\mathscr{D}} \alpha_D(F(D))$, whence

$$a = f_{(\cdot)}(\alpha_{D_1}(a_1), \ldots, \alpha_{D_l}(a_l))$$

where $a_i \in F(D_i)$ for $i = 1, \ldots, l$ and $l \equiv 1 \pmod n$. Hence

$$\begin{aligned}\zeta(a') &= \zeta\delta\big(f_{(\cdot)}(\alpha_{D_1}(a_1), \ldots, \alpha_{D_l}(a_l))\big)\\ &= \zeta\big(f_{(\cdot)}(\Psi_s(\gamma_{D_1})\lambda_{D_1}(a_1), \ldots, \Psi_s(\gamma_{D_l})\lambda_{D_l}(a_l))\big)\\ &= \varphi\big(\xi_{D_1}\zeta_{D_1}(\lambda_{D_1}(a_1)), \ldots, \xi_{D_l}\zeta_{D_l}(\lambda_{D_l}(a_l))\big) = \varphi(0, \ldots, 0) = 0,\end{aligned}$$

where f denotes an $(n+1)$-group operation on L and also on $\Psi_s(L')$ and φ denotes (as usual) the $(k+1)$-group operation on $\mathfrak{C}_{s,k+1}$. Thus $\zeta(\delta(L)) = \{0\}$, i.e., $\delta(L) \subset \subset \zeta^{-1}(0)$. Let $b' \in L'$. The $(k+1)$-group L', as the inductive limit of F, is generated by the set $\bigcup_{D\in\mathscr{D}} \gamma_D(F'(D))$, whence $b' = g_{(\cdot)}(\gamma_{D_1}(a'_1), \ldots, \gamma_{D_l}(a'_l))$ where $a'_i \in F'(D_i)$ and $l \equiv 1 \pmod k$. Moreover, the $(k+1)$-groups $F'(D_i)$ are generated by the sets $\lambda_{D_i}(F(D_i))$, and so we get $a'_i = g_{(\cdot)}(\lambda_{D_i}(a_{i1}), \ldots, \lambda_{D_i}(a_{ij_i}))$ where $j_i \equiv 1 \pmod k$ for $i = 1, \ldots, l$. Hence

$$\begin{aligned}b' &= g_{(\cdot)}(\gamma_{D_1}(a_1), \ldots, \gamma_{D_l}(a_l))\\ &= g_{(\cdot)}\big(\gamma_{D_1}(g_{(\cdot)}(\lambda_{D_1}(a_{11}), \ldots, \lambda_{D_1}(a_{1j_1}))), \ldots, \gamma_{D_l}(g_{(\cdot)}(\lambda_{D_l}(a_{l1}), \ldots, \lambda_{D_l}(a_{lj_l})))\big)\\ &= g_{(\cdot)}(\delta\alpha_{D_1}(a_{11}), \ldots, \delta\alpha_{D_l}(a_{lj_l})),\end{aligned}$$

where g denotes a $(k+1)$-group operation on L' and also on $F'(D)$. Then the $(k+1)$-group L' is generated by the set $\delta(L)$, which proves that the pair $\langle L', \varepsilon\rangle$, where ε is the inclusion of $\delta(L)$ into $\Psi_s(L')$, is a covering $(k+1)$-group of the $(n+1)$-group $\delta(L)$. Let q' be the index of that covering $(k+1)$-group. Then there exists an epimorphism $\zeta'\colon L' \to \mathfrak{C}_{q',k+1}$ such that $\zeta'^{-1}(0) = \delta(L)$. Hence $\zeta'^{-1}(0) \subset \zeta^{-1}(0)$ (since $\delta(L) \subset \zeta^{-1}(0)$), which shows that there exists an epimorphism $\mu\colon \mathfrak{C}_{q',k+1} \to \to \mathfrak{C}_{q,k+1}$ such that $\mu\zeta' = \zeta$ (cf. [9], Lemma 1). From this equality it follows that $q|q'$. Now, let $a' \in \zeta_D^{-1}(0)$ for a certain $D \in \mathscr{D}$. Then $a' = \lambda_D(a)$ for a certain $a \in F(D)$. The morphism $\zeta'\gamma_D\colon F'(D) \to \mathfrak{C}_{q',k+1}$ is an epimorphism, since $\zeta'\gamma_D(\lambda_D(F(D))) = 0$. From the equality $\zeta'\gamma_D(a') = \zeta'\delta\alpha_D(a) = 0$ it follows that $\zeta_D^{-1}(0) \subset (\zeta'\gamma_D)^{-1}(0)$. This proves by Lemma 1 of [9] the existence of an epimorphism $\mu_D\colon \mathfrak{C}_{q_D,k+1} \to \to \mathfrak{C}_{q',k+1}$ such that $\mu_D\zeta_D = \zeta'\gamma_D$. Thus $q'|q_D$ for each $D \in \mathscr{D}$, whence $q'|q$. This proves that $q' = q$.

4. The relations of the functor Ψ_s to inductive and projective limits. The functor Ψ, being a right adjoint functor, preserves projective limits. On the other hand, Ψ reflects projective limits, since $\mathbf{Gr}_{k+1}$ is a complete category, Ψ is a faithful functor and Ψ reflects isomorphisms.

Hence we immediately get

PROPOSITION 7. *$[L; \{\pi_D\colon L \to F(D)\}_{D\in\mathscr{D}}]$ is the projective limit of $F\colon \mathscr{D} \to \mathbf{Gr}_{k+1}$ if and only if $[\Psi_s(L); \{\Psi_s(\pi_D)\colon \Psi_s(L) \to \Psi_sF(D)\}_{D\in\mathscr{D}}]$ is the projective limit of $\Psi_sF\colon \mathscr{D} \to \mathbf{Gr}_{n+1}$.*

COROLLARY 6. *If $[L'; \{\gamma_D\colon L' \to \Psi_sF(D)\}_{D\in\mathscr{D}}]$ is the projective limit of $\Psi_sF\colon \mathscr{D} \to \mathbf{Gr}_{n+1}$, then there exists an object $L \in \mathbf{Gr}_{k+1}$ unique up to an isomorphism and a family $\{\pi_D\colon L \to F(D)\}_{D\in\mathscr{D}}$ such that $\Psi_s(L) = L'$, $\Psi_s(\pi_D) = \gamma_D$ for each $D \in \mathscr{D}$. Moreover $[L; \{\pi_D\colon L \to F(D)\}_{D\in\mathscr{D}}]$ is then the projective limit of $F\colon \mathscr{D} \to \mathbf{Gr}_{k+1}$.*

From Corollary 6 it follows that the projective limit of $(n+1)$-groups derived from $(k+1)$-groups (cf. [9]) is an $(n+1)$-group derived from a $(k+1)$-group. We shall prove a similar theorem for inductive limits, for the case of $s>1$, but under additional conditions set upon $\mathscr{D}$. For the case of $s = 1$ the functor Ψ, being the identity functor, obviously preserves and reflects all limits.

THEOREM 3. *Given a diagram $F\colon \mathscr{D} \to \mathbf{Gr}_{k+1}$, suppose $\mathscr{D}$ has an initial object I and $\Psi_sF(I)$ is not an initial object in $\mathbf{Gr}_{n+1}$ $(s>1)$. Then $[L; \{\sigma_D\colon F(D) \to L\}_{D\in\mathscr{D}}]$ is the inductive limit of $F\colon \mathscr{D} \to \mathbf{Gr}_{k+1}$ if and only if $[\Psi_s(L); \{\Psi_s(\sigma_D)\colon \Psi_sF(D) \to \to \Psi_s(L)\}_{D\in\mathscr{D}}]$ is the inductive limit of $\Psi_sF\colon \mathscr{D} \to \mathbf{Gr}_{n+1}$.*

Proof. Let $[L; \{\sigma_D\colon F(D) \to L\}_{D\in\mathscr{D}}]$ and $[L'; \{\gamma_D\colon \Psi_sF(D) \to L'\}_{D\in\mathscr{D}}]$ be the inductive limits of F and Ψ_sF, respectively. Denote (as usual) the $(k+1)$-group operations on $F(D)$ by g, and the $(n+1)$-group operation on L' by f. Let I be an initial object in $\mathscr{D}$, i.e., for each object $D \in \mathscr{D}$ there exists a unique morphism $\alpha_D\colon I \to D$. Take an arbitrary element $c_0 \in F(I)$. We shall prove that the element $d = \gamma_I(\bar{c}_0)$, where $\bar{c}_0$ is the skew element to c_0 in the $(k+1)$-group $F(I)$, is an s-skew element to $c = \gamma_I(c_0)$ (cf. [10]). In fact, take an arbitrary element $x \in L'$. The $(n+1)$-group L', as the inductive limit of Ψ_sF, is generated by the set $\bigcup_{D\in\mathscr{D}} \gamma_D(\Psi_sF(D))$, and thus $x = f_{(\cdot)}(\gamma_{D_1}(x_1), \ldots, \gamma_{D_r}(x_r))$ where $r \equiv 1 \pmod n$ and $x_i \in \Psi_sF(D_i)$ for each $i = 1, \ldots, r$. Then

$$\begin{aligned}f(\overset{s}{d}, \overset{(k-1)s}{c}, x) &= f(\overset{s}{\gamma_I(\bar{c}_0)}, \overset{(k-1)s}{\gamma_I(c_0)}, f_{(\cdot)}(\gamma_{D_1}(x_1), \ldots, \gamma_{D_r}(x_r)))\\ &= f_{(\cdot)}\big(f(\overset{s}{\gamma_{D_1}\Psi_sF(\alpha_{D_1})(\bar{c}_0)}, \overset{(k-1)s}{\gamma_{D_1}\Psi_sF(\alpha_{D_1})(c_0)}, \gamma_{D_1}(x_1)), \gamma_{D_2}(x_2), \ldots, \gamma_{D_r}(x_r)\big)\\ &= f_{(\cdot)}\big(\gamma_{D_1}(g_{(s)}(\overset{s}{F(\alpha_{D_1})(\bar{c}_0)}, \overset{(k-1)s}{F(\alpha_{D_1})(c_0)}, x_1)), \gamma_{D_2}(x_2), \ldots, \gamma_{D_r}(x_r)\big)\\ &= f_{(\cdot)}(\gamma_{D_1}(x_1), \ldots, \gamma_{D_r}(x_r)) = x,\end{aligned}$$

which shows that condition 1° from the definition of an s-skew element (cf. [10]) is fulfilled. Similarly one can prove that $f(\overset{(k-1)s}{c}, \overset{s}{d}, x) = x$.

Now, take arbitrary elements $x_1, \ldots, x_{n+1-k} \in L'$ and fixed $i = 1, \ldots, n+1-k$. Let $x_i = f_{(\cdot)}(\gamma_{D_1}(y_1), \ldots, \gamma_{D_r}(y_r))$ where $r \equiv 1 \pmod n$ and $y_j \in \Psi_sF(D_j)$ for $j = 1, \ldots, r$. Then

$$\begin{aligned}&f(x_1, \ldots, x_i, d, \overset{k-1}{c}, x_{i+1}, \ldots, x_{n+1-k})\\ &\quad= f_{(\cdot)}(\ldots, x_{i-1}, \gamma_{D_1}(y_1), \ldots, \gamma_{D_{r-1}}(y_{r-1}), f(\overset{(k-1)s}{c}, \overset{s}{d}, \gamma_{D_r}(y_r)), d, \overset{k-1}{c}, x_{i+1}, \ldots)\\ &\quad= f_{(\cdot)}(\ldots, \gamma_{D_{r-1}}(y_{r-1}), \overset{k}{c}, f(\overset{(k-1)s-k}{c}, \overset{s}{d}, \gamma_{D_r}(y_r), d, \overset{k-1}{c}), x_{i+1}, \ldots)\\ &\quad= f_{(\cdot)}(\ldots, \gamma_{D_{r-1}}(y_{r-1}), \overset{k}{c}, f(\overset{(k-1)s-k}{\gamma_{D_r}\Psi_sF(\alpha_{D_r})(c_0)}, \overset{s}{\gamma_{D_r}\Psi_sF(\alpha_{D_r})(\bar{c}_0)}, \gamma_{D_r}(y_r),\\ &\qquad\qquad \gamma_{D_r}\Psi_sF(\alpha_{D_r})(\bar{c}_0), \overset{k-1}{\gamma_{D_r}\Psi_sF(\alpha_{D_r})(c_0)}), x_{i+1}, \ldots)\end{aligned}$$

$$= f_{(\cdot)}\big(\ldots, \gamma_{D_{r-1}}(y_{r-1}), \overset{k}{c}, \gamma_{D_r}\big(g_{(s)}(\overset{(k-1)s-k}{F(\alpha_{D_r})(c_0)}, F(\alpha_{D_r})(\bar{c}_0), y_r, F(\alpha_{D_r})(\bar{c}_0), \overset{k-1}{F(\alpha_{D_r})(c_0)})\big), x_{i+1}, \ldots\big)$$

$$= f_{(\cdot)}\big(\ldots, \gamma_{D_{r-1}}(y_{r-1}), \overset{k}{c}, f(\overset{(k-1)s-k}{\gamma_{D_r}\Psi_s F(\alpha_{D_r})(c_0)}, \overset{s}{\gamma_{D_r}\Psi_s F(\alpha_{D_r})(\bar{c}_0)}, \gamma_{D_r}(y_r), \overset{k-1}{\gamma_{D_r}\Psi_s F(\alpha_{D_r})(c_0)}, \gamma_{D_r}\Psi_s F(\alpha_{D_r})(\bar{c}_0)), x_{i+1}, \ldots\big)$$

$$= f_{(\cdot)}\big(\ldots, \gamma_{D_{r-1}}(y_{r-1}), \overset{k}{c}, f(\overset{(k-1)s-k}{c}, \overset{s}{d}, \gamma_{D_r}(y_r), \overset{k-1}{c}, d), x_{i+1}, \ldots\big)$$

$$= \ldots = f(x_1, \ldots, x_i, \overset{k-1}{c}, d, x_{i+1}, \ldots, x_{n+1-k}).$$

In a similar way we prove that

$$f(x_1, \ldots, x_i, d, \overset{k-1}{c}, x_{i+1}, \ldots, x_{n+1-k}) = f(\overset{k-1}{c}, d, x_1, \ldots, x_{n+1-k}).$$

This shows that the second condition from the definition of an s-skew element is also fulfilled. Thus (cf. [9], Theorem 5 and [10], Proposition 1) the $(n+1)$-group (L', f) is derived from a certain $(k+1)$-group (A, g), i.e., $\Psi_s(A) = L'$. The $(k+1)$-group operation on A is described by the formula

$$g(x_1, \ldots, x_{k+1}) = f(x_1, \ldots, x_{k+1}, \overset{s-1}{d}, \overset{(k-1)(s-1)}{c}).$$

Let $x_1, \ldots, x_{k+1} \in F(D)$. Then

$$\gamma_D\big(g(x_1, \ldots, x_{k+1})\big) = \gamma_D\big(g_{(s)}(x_1, \ldots, x_{k+1}, \overset{s-1}{F(\alpha_D)(\bar{c}_0)}, \overset{(k-1)(s-1)}{F(\alpha_D)(c_0)})\big)$$
$$= f\big(\gamma_D(x_1), \ldots, \gamma_D(x_{k+1}), \overset{s-1}{d}, \overset{(k-1)(s-1)}{c}\big) = g\big(\gamma_D(x_1), \ldots, \gamma_D(x_{k+1})\big),$$

which shows that $\gamma_D = \Psi_s(\beta_D)$, where $\beta_D\colon F(D) \to A$, for $D \in \mathcal{D}$. By the faithfulness of Ψ it follows that the family $\{\beta_D\}_{D\in\mathcal{D}}$ is compatible with $F\colon \mathcal{D} \to \boldsymbol{Gr}_{k+1}$. Then there exists a unique morphism $\delta\colon L \to A$ such that $\delta\sigma_D = \beta_D$ for $D \in \mathcal{D}$. One can prove that δ is an isomorphism. Therefore $[\Psi_s(L); \{\Psi_s(\sigma_D)\colon \Psi_s F(D) \to \Psi_s(L)\}_{D\in\mathcal{D}}]$ is the inductive limit of $\Psi_s F$.

COROLLARY 7. *Let $\mathcal{D}$ have an initial object I and let F be a diagram such that $\Psi_s F(I)$ is not an initial object in $\boldsymbol{Gr}_{n+1}$ $(s>1)$. If $[L'; \{\gamma_D\colon \Psi_s F(D) \to L'\}_{D\in\mathcal{D}}]$ is the inductive limit of $\Psi_s F$, then there exists an object $L \in \boldsymbol{Gr}_{k+1}$ unique up to an isomorphism and a family $\{\sigma_D\colon F(D) \to L\}_{D\in\mathcal{D}}$ such that $\Psi_s(L) = L'$, $\Psi_s(\sigma_D) = \gamma_D$ for each $D \in \mathcal{D}$. Moreover, $[L; \{\sigma_D\colon F(D) \to L\}_{D\in\mathcal{D}}]$ is then the inductive limit of F.*

This shows that for $s>1$ the class of $(n+1)$-groups derived from $(k+1)$-groups is closed with respect to the inductive limits of the diagrams $F\colon \mathcal{D} \to \boldsymbol{Gr}_{n+1}$ where $\mathcal{D}$ has an initial object and $F(I)$ is not an initial object in $\boldsymbol{Gr}_{n+1}$.

The functors Φ and Ψ, being adjoint functors, have numerous dual properties. Hence also free covering $(k+1)$-groups and derived $(n+1)$-groups, being objects of the form $\Phi_s(A)$ and $\Psi_s(A)$, respectively, also have many dual properties. Examples

are contained in Proposition 5 and Proposition 7, Theorem 1 and Theorem 3. But not all facts listed in Section 3 have their counterparts in Section 4. The class of free covering $(k+1)$-groups is closed with respect to projective limits, whereas the class of derived $(n+1)$-groups is not closed with respect to inductive limits. An example is given by the free product of two derived $(n+1)$-groups from $(k+1)$-groups, which is even a primitive $(n+1)$-group, i.e., is not derived from any $(k+1)$-group where k is an arbitrary divisor of n (cf. [11]).

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