Proof. Let m be the dimension of X; we can assume $m > 0$. Choose a simplicial complex K with $|K| = X$ and

$$\mu(K) < \min\left(\frac{\gamma(\varphi)}{2(1+2m)}, \frac{\varepsilon}{4m}\right),$$

and use Lemma 3 to obtain an n-valued simplicial multifunction

$$\varphi': |K'| \to |K| \quad \text{with} \quad \gamma(\varphi') \geq \gamma(\varphi) - 2\mu(K) \quad \text{and} \quad \bar{d}(\varphi, \varphi') < \mu(K).$$

Now proceed as in the proof of Theorem 2, pp. 118–119, of [2], i.e. apply the Hopf construction of Lemma 5 repeatedly on simplexes of increasing dimension until a simplicial multifunction $\psi: |K'| \to |K|$ is obtained, where K' is a refinement of K so that ψ is fixed point free on all non-maximal simplexes. An argument parallel to the one in [2], p. 119 implies that the image of each point is changed at most m times. Hence Lemma 5 iii) shows that

$$\gamma(\psi) \geq \gamma(\varphi') - 4m\mu(K) \geq \gamma(\varphi) - 2\mu(K) - 4m\mu(K) > 0,$$

so ψ is n-valued. Similarly, we see that each intermediate simplicial multifunction φ'' which is fixed point free on all p-simplexes ($p < m$) satisfies the assumption $\gamma(\varphi'') > 4\mu(K)$ of Lemma 5. It follows that

$$\bar{d}(\varphi, \psi) \leq \bar{d}(\varphi, \varphi') + \bar{d}(\varphi', \psi) < \tfrac{1}{4}\varepsilon + m\cdot\frac{\varepsilon}{2m} < \varepsilon.$$

The verification that ψ satisfies i) and ii) is analogous to the one in [2], pp. 118–119.

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A universal metacompact developable T_1-space of weight $\mathfrak{m}$

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Abstract. For each cardinal number $\mathfrak{m}$ we construct a metacompact developable T_1-space $T(\mathfrak{m})$. If $\mathfrak{m}$ is infinite, then $T(\mathfrak{m})^{\aleph_0}$ is universal for all metacompact developable T_1-spaces of weight $\mathfrak{m}$. The space $T(0)$ is the set of irrational numbers with a weaker topology and $T(0)^{\aleph_0}$ is universal for all perfect T_1-spaces of countable weight. Each $T(\mathfrak{m})$ is built of $\mathfrak{m}$ copies of $T(0)$. Moreover, each mapping of a closed subset of a perfect space X into $T(0)$ can be extended to a mapping of X into $T(0)$.

In [3] we have introduced a method of constructing mappings into metacompact developable T_1-spaces. More precisely, we have constructed, for a point-finite open cover $\mathcal{U}$ of a perfect space X, a continuous mapping p of X onto a metacompact developable T_1-space Z such that each element of $\mathcal{U}$ is an inverse image of an open subset of Z.

The examination of this construction shows that the space Z can be regarded as a subspace of a metacompact developable T_1-space T which depends only on the cardinality of $\mathcal{U}$.

In the first section of this paper we give a modification of the construction from [3]. The ideas of the first section are used in the second section to construct, for each cardinal number $\mathfrak{m}$, a metacompact developable T_1-space $T(\mathfrak{m})$ and a point-finite collection $\mathcal{G}$ of open subsets of $T(\mathfrak{m})$ such that, for any perfect space X and any point-finite collection $\mathcal{U}$ of cardinality $\mathfrak{m}$ consisting of open subsets of X, there exists a mapping $f: X \to T(\mathfrak{m})$ satisfying $\mathcal{U} = \{f^{-1}(G): G \in \mathcal{G}\}$.

The weight of $T(\mathfrak{m})$ is $\mathfrak{m} + \aleph_0$ and it follows that, for infinite $\mathfrak{m}$, $T(\mathfrak{m})^{\aleph_0}$ is universal for all metacompact developable T_1-spaces of weight $\mathfrak{m}$.

A space with properties similar to the properties of $T(0)$ is constructed in [6]. Our construction of $T(0)$ is more direct and can be regarded as a simplification of the construction in [6]. We prove an extension theorem for mappings into $T(\mathfrak{m})$ (Theorem 2) and obtain a number of corollaries showing that $T(0)$ can be considered to be a D-line (see Remarks 3 and 4).

We shall use the terminology and notation from [5]. By a mapping we always mean a continuous function. Metacompact spaces are not necessarily Hausdorff but all spaces we consider are T_1-spaces. If $\mathcal{D}$ is a family of subsets of X and $x \in X$, then $\mathscr{D}(x) = \{D \in \mathscr{D}\colon x \in D\}$. The set of non-negative integers will be denoted by N and the set of positive integers by N_+.

1. Mappings onto metacompact developable spaces. The results of this section are implicitly contained in Section 2 of [3]. We include them here in order to clarify the construction of spaces $T(\mathfrak{m})$.

PROPOSITION. *Let $\mathscr{U}$ be a point-finite open cover of a perfect space X. There exists a metacompact developable space Z and a mapping p of X onto Z such that each element of $\mathscr{U}$ is an inverse image of an open subset of Z.*

Proof. The sets $U(i) = \{x\colon |\mathscr{U}(x)| \geqslant i\}$ are open in X. Since X is a perfect space, we can define, by induction on $n \geqslant 1$, for each $\tau \in N^n$, an open subset $U(\tau)$ of X such that

$$U(\tau) = U(i) \quad \text{for} \quad \tau = (i) \in N^1, \tag{1}$$

$$X \setminus U(\tau) = \bigcap \{U((\tau, j))\colon j \in N\}, \tag{2}$$

where (τ, j) denotes the extension of τ by j. We shall often write $U(\tau, j)$ instead of $U((\tau, j))$.

Put $\mathscr{B} = \mathscr{U} \cup \{U(\tau)\colon \tau \in N^n, n \geqslant 1\}$ and consider a relation $x \sim x'$ iff $\mathscr{B}(x) = \mathscr{B}(x')$. Clearly, $\sim$ is an equivalence relation.

We define Z to be the set of the equivalence classes of $\sim$ and p to be the natural function from X onto Z. We generate a topology of Z by taking $p(\mathscr{B}) = \{p(B)\colon B \in \mathscr{B}\}$ to be a subbase. From the definition of $\sim$ it follows that Z is a T_0-space and $p^{-1}(p(B)) = B$ for $B \in \mathscr{B}$. Thus p is a continuous function and each element of $\mathscr{U}$ is an inverse image of an open subset of Z.

In order to show that Z is a metacompact developable T_1-space, we need a lemma.

LEMMA. *A T_0-space Z is a metacompact developable T_1-space if and only if Z has a subbase $\mathscr{P} = \bigcup_{k \geqslant 0} \mathscr{P}_k$, where each $\mathscr{P}_k$ is a point-finite collection and $z \in P \in \mathscr{P}$ implies that, for a certain $k \geqslant 0$, $Z \setminus \bigcup (\mathscr{P}_k \setminus \mathscr{P}_k(z)) \subset P$.*

Before proving the lemma, we shall show that the space Z and its subbase $p(\mathscr{B})$ satisfy the conditions of the lemma. This will reduce the proof of our proposition to the proof of the lemma.

We shall carry out our reasoning in X with $p(\mathscr{B})$ replaced by $\mathscr{B}$. This will simplify the notation and is justified by the definition of Z and p.

Consider the countable family consisting of the following point-finite collections of subsets of X: $\mathscr{U}$, $\mathscr{U}_{i,j} = \mathscr{U} \cup \{U(i, j)\}$ for $i, j \geqslant 0$ and $\mathscr{U}(\tau) = \{U(\tau)\}$ for $\tau \in N^n$ and $n \geqslant 1$. Clearly, $\mathscr{B}$ is the union of this family.

Assume that $x \in B \in \mathscr{B}$. We have to distinguish two cases. If $B \in \mathscr{U}$, then we take $i = |\mathscr{U}(x)|$ and a $j \geqslant 0$ satisfying $x \notin U(i, j)$. The existence of such a j is assured by (2). It is easy to see that

$$X \setminus \bigcup (\mathscr{U}_{i,j} \setminus \mathscr{U}_{i,j}(x)) = X \setminus (U(i, j) \cup \bigcup (\mathscr{U} \setminus \mathscr{U}(x))) \subset B.$$

If $B = U(\tau)$, then, by virtue of (2), $x \notin U(\tau, j)$ for a certain $j \geqslant 0$ and

$$X \setminus \bigcup (\mathscr{U}(\tau, j) \setminus \mathscr{U}(\tau, j)(x)) = X \setminus U(\tau, j) \subset B.$$

Thus the lemma implies that Z is a metacompact developable T_1-space.

Proof of the Lemma. The "only if" part is obvious. In order to prove the "if" part, consider a T_0-space Z with a subbase $\mathscr{P} = \bigcup_{k \geqslant 0} \mathscr{P}_k$ satisfying the conditions of the lemma.

Put $P_k(z) = \bigcap \mathscr{P}_k(z)$, $D_k(z) = \bigcap_{k' \leqslant k} P_{k'}(z)$ and $\mathscr{D}_k = \{D_k(z)\colon z \in Z\}$. Each $\mathscr{D}_k$ is a point-finite open cover of Z and, since developable T_0-spaces are T_1, it follows that it is sufficient to prove that $\{\mathscr{D}_k\}_{k \geqslant 0}$ is a development for Z.

Let $z \in Z$ and let P be an open subset of Z containing z. We want to find a $k \geqslant 0$ such that $\mathrm{St}(z, \mathscr{D}_k) \subset P$. Since $\mathscr{D}_{k+1}$ refines $\mathscr{D}_k$ for $k \geqslant 0$, we can assume that $P \in \mathscr{P}$. Then $P \in \mathscr{P}_{k_1}$ for a $k_1 \geqslant 0$ and $Z \setminus \bigcup (\mathscr{P}_{k_2} \setminus \mathscr{P}_{k_2}(z)) \subset P$ for a $k_2 \geqslant 0$. Take $k = k_1 + k_2$. If $z \in D_k(z') \in \mathscr{D}_k$, then $z \in P_{k_2}(z')$, which implies $z' \in P$ and $D_k(z') \subset P_{k_1}(z') \subset P$. Thus, $\mathrm{St}(z, \mathscr{D}_k) \subset P$.

Observe that the space Z depends on $\mathscr{B}$ rather than on X. It is clear that in order to obtain a space $T(\mathfrak{m})$ with the properties mentioned in the introduction it is sufficient to apply the above construction to a space X with a point-finite open cover $\mathscr{U}$ of cardinality $\mathfrak{m}$ such that the relation $\sim$ described above has as many equivalence classes as possible. In particular, each i-element subcollection of $\mathscr{U}$ has to have a non-empty intersection. This intersection should intersect both $U(i', j)$ and $X \setminus U(i', j)$ for $i' \leqslant i$ (for $i' > i$ this intersection has to be contained in $U(i', j)$) and so on.

In the next section we shall construct a space with such a cover and a collection $\mathscr{B}$ such that each of the equivalence classes of $\sim$ will be a one-point set.

2. Universal spaces. The main step in the construction of our universal spaces will be the following.

THEOREM 1. *Let $\mathfrak{m}$ be a cardinal number. There exists a metacompact developable T_1-space $T(\mathfrak{m})$ of weight $\mathfrak{m} + \aleph_0$ with a point-finite collection $\mathscr{G}$ of open subsets such that any perfect space X with a point-finite open collection $\mathscr{U}$ of cardinality $\mathfrak{m}$ can be mapped into $T(\mathfrak{m})$ by a mapping f satisfying $\mathscr{U} = \{f^{-1}(G)\colon G \in \mathscr{G}\}$.*

As an immediate consequence, we obtain

COROLLARY 1. *If $\mathfrak{m}$ is an infinite cardinal number, then $T(\mathfrak{m})^{\aleph_0}$ is universal for all metacompact developable T_1-spaces of weight $\mathfrak{m}$.*

Proof of Theorem 1. Let $\mathfrak{m}$ be a fixed cardinal number and let $\mathrm{Fin}(\mathfrak{m})$ denote the set of all finite subsets of $\mathfrak{m}$.

We define the set of points of $T(\mathfrak{m})$ (since $\mathfrak{m}$ is fixed, we write T instead of $T(\mathfrak{m})$)

$$T = \mathrm{Fin}(\mathfrak{m}) \times N^{N_+}.$$

Thus each element of T is a sequence $(t(0), t(1), \ldots)$ such that $t(0)$ is a finite subset of $\mathfrak{m}$ and $t(n) \in N$ for $n \geq 1$.

For $\alpha \in \mathfrak{m}$, let $G_\alpha = \{t \in T: \alpha \in t(0)\}$ and put $\mathcal{G} = \{G_\alpha: \alpha \in \mathfrak{m}\}$.

If $a \in \mathrm{Fin}(\mathfrak{m})$ is identified with its characteristic function, then $\mathrm{Fin}(\mathfrak{m})$ with the topology generated by the projections of the sets G_α is a subspace of the Alexandroff cube $\{0, 1\}^{\mathfrak{m}}$ (1 is the isolated point of $\{0, 1\}$) [5, 2.3.26]. The factor N^{N+} will make T perfect and, consequently, metacompact, developable and T_1.

Let $G_0(i) = \{t \in T: |t(0)| \geq i\}$, $G_n(i) = \{t \in T: t(n) \geq i\}$ for $n \geq 1$ and $G_n(i, j) = \{t \in T: t \in G_n(i) \Rightarrow t(n+1) \geq j\}$ for $n \geq 0$.

It is easy to see that

(i) $G_0(i) = \{t \in T: |\{\alpha: t \in G_\alpha\}| \geq i\}$ for $i \geq 0$,

(ii) $G_n(i, j) = (T \backslash G_n(i)) \cup G_{n+1}(j)$ for $n, i, j \geq 0$

and, for each $n \geq 0$,

(iii) $\{G_n(i)\}_{i \geq 0}$ is decreasing, $G_n(0) = T$ and $\bigcap_{i \geq 0} G_n(i) = \emptyset$.

We consider T with the topology generated by assuming that $\mathcal{B} = \mathcal{G} \cup \{G_n(i): n, i \geq 0\} \cup \{G_n(i, j): n, i, j \geq 0\}$ is a subbase of T(1).

To see that T is a T_0-space, take two different points $t, t' \in T$. If $t(n) > t'(n)$ for an $n \geq 1$, then $G_n(t(n))$ contains t but does not contain t'. If $t(0) \neq t'(0)$ and $\alpha \in t(0) \backslash t'(0)$, then G_α contains t but does not contain t'.

From (ii) and (iii) it follows that $T \backslash G_n(i) = \bigcap_{j \geq 0} G_n(i, j)$ and $T \backslash G_n(i, j) = G_n(i) \cap T \backslash G_{n+1}(j) = G_n(i) \cap \bigcap_{k \geq 0} G_{n+1}(j, k)$. Thus $\mathcal{B} \backslash \mathcal{G}$ satisfies (see Remark 3)

(1) $\{t \in T: |\{\alpha: t \in G_\alpha\}| \geq i\} \in \mathcal{B} \backslash \mathcal{G}$ for $i \geq 0$,

(2) if $B \in \mathcal{B} \backslash \mathcal{G}$, then $T \backslash B = \bigcap_{j \geq 0} B(j)$ for some $B(j) \in \mathcal{B} \backslash \mathcal{G}$.

Consequently, the lemma of the first section implies that T is a metacompact developable T_1-space (the proof that T satisfies the assumptions of the lemma is the same as the proof for Z). Clearly, the weight of T is $\mathfrak{m} + \aleph_0$ and $\mathcal{G}$ is a point-finite collection of open subsets of T.

Let X be a perfect space and let $\mathcal{U}$ be a point-finite collection of open subsets of X such that $|\mathcal{U}| = \mathfrak{m}$. We can represent $\mathcal{U}$ as $\{U_\alpha\}_{\alpha \in \mathfrak{m}}$ in such a way that $\{\alpha: x \in U_\alpha\}$ is finite for $x \in X$.

We shall construct a mapping $f: X \to T$ satisfying $f^{-1}(G_\alpha) = U_\alpha$ by defining sets $V_n(i)$, open in X, which will be the inverse images of the sets $G_n(i)$. Our construction will be by induction on $n \geq 0$ and will be based on the fact that

(iv) $G_{n+1}(j) = \bigcup_{i \geq 0} (G_n(i) \cap G_n(i, j))$.

We are going to construct sets $V_n(i)$ such that

(i') $V_0(i) = \{x \in X: |\{\alpha: x \in U_\alpha\}| \geq i\}$ for $i \geq 0$

and, for $n \geq 0$,

(ii') $(X \backslash V_n(i)) \cup V_{n+1}(j)$ is open in X for $i, j \geq 0$,

(iii') $\{V_n(i)\}_{i \geq 0}$ is decreasing, $V_n(0) = X$ and $\bigcap_{i \geq 0} V_n(i) = \emptyset$.

We start with (i'). Assume that the sets $V_n(i)$ are given and satisfy (iii'). We shall construct sets $V_{n+1}(j)$ for $j \geq 0$, satisfying (ii') and (iii').

In order to apply a condition corresponding to (iv), we take, for each $i \geq 0$, a decreasing sequence $\{U_n(i, j)\}_{j \geq 0}$ of open subsets of X such that $U_n(i, 0) = X$ and $\bigcap_{i \geq 0} U_n(i, j) = X \backslash V_n(i)$. Let

(iv') $V_{n+1}(j) = \bigcup_{i \geq 0} (V_n(i) \cap U_n(i, j))$.

We have $U_n(i, j) \subset (X \backslash V_n(i)) \cup V_{n+1}(j)$. Thus

$$(X \backslash V_n(i)) \cup V_{n+1}(j) = U_n(i, j) \cup V_{n+1}(j)$$

and (ii') is satisfied.

To see that (iii') holds it is sufficient to check that $\bigcap_{j \geq 0} V_{n+1}(j) = \emptyset$. Assume that x is in this intersection. Then, for each $j \geq 0$, there is an i_j such that $x \in V_n(i_j) \cap U_n(i_j, j)$. Since $\bigcap_{i \geq 0} V_n(i) = \emptyset$, it follows that, for a certain $i \geq 0$, $i_j = i$ for infinitely many j and we obtain a contradiction with the definition of the sequence $\{U_n(i, j)\}_{j \geq 0}$.

Now we can define the mapping $f: X \to T$. We put $f(x)(0) = \{\alpha: x \in U_\alpha\}$ and $f(x)(n) = \max\{j: x \in V_n(j)\}$ for $n \geq 1$.

Clearly, $f^{-1}(G_\alpha) = U_\alpha$ and consequently $f^{-1}(G_0(i)) = V_0(i)$. Moreover, $f^{-1}(G_n(j)) = V_n(j)$ for $n \geq 1$. Thus (ii) and (ii') imply that the sets $f^{-1}(G_n(i, j))$ are open in X, which proves that f is a continuous function.

The construction of f given above shows that we have some freedom in choosing $f(x)$. In fact, Theorem 1 can be strengthened as follows (we use the notation introduced in the proof of Theorem 1):

THEOREM 2. *Let A be a closed subset of a perfect space X and $\{U_\alpha\}_{\alpha \in \mathfrak{m}}$ a collection of open subsets of X such that $\{\alpha: x \in U_\alpha\}$ is finite for $x \in X$. If $g: A \to T(\mathfrak{m})$ satisfies $g^{-1}(G_\alpha) = U_\alpha \cap A$ for $\alpha \in \mathfrak{m}$, then g has an extension $f: X \to T(\mathfrak{m})$ satisfying $f^{-1}(G_\alpha) = U_\alpha$ for $\alpha \in \mathfrak{m}$.*

Proof. We proceed as in the proof of Theorem 1. We construct the sets $V_n(i)$ by induction on $n \geq 0$ in such a way that (i'), (ii') and (iii') are satisfied. Moreover, since f should extend g, we also require

(v') $V_n(i) \cap A = g^{-1}(G_n(i))$.

We shall indicate modifications necessary to obtain (v'). We fix a

(1) In view of the results of the first section, it is more natural to define sets $G(\tau)$ for $\tau \in N^n$ with $n \geq 1$ by putting $G(\tau) = G_0(i)$ for $\tau = (i) \in N^1$ and $G(\tau, j) = \{t \in T: t \in G(\tau) \Rightarrow t(n) \geq j\}$ for $\tau \in N^n$. In fact, the sets $G(\tau)$ are open in T and, together with $\mathcal{G}$, form a subbase of T containing $\mathcal{B}$. However, in the proof of the continuity of $f: X \to T$, it is convinient to deal with a smaller subbase of T.

decreasing sequence $\{W(j)\}_{j\geq 0}$ of open subsets of X such that $W(0) = X$ and $A = \bigcap_{j\geq 0} W(j)$.

The sets $V_0(i)$ defined by (i′) satisfy (v′) because, for $\alpha \in \mathfrak{m}$, $U_\alpha \cap A = g^{-1}(G_\alpha)$. Assume that the sets $V_n(i)$ for $i \geq 0$ are given and satisfy (iii′) and (v′). We shall construct sets $V_{n+1}(j)$ for $j \geq 0$, satisfying (ii′), (iii′) and (v′).

We take, for $i \geq 0$, a decreasing sequence $\{U_n'(i, j)\}_{j\geq 0}$ of open subsets of X such that $U_n'(i, 0) = X$ and $X \setminus V_n(i) = \bigcap_{j\geq 0} U_n'(i, j)$. In order to obtain (v′), we replace $U_n'(i, j)$ with

$$U_n(i, j) = \big(U_n'(i, j) \cup W(j)\big) \cap \big(g^{-1}(G_n(i, j)) \cup X\setminus A\big).$$

Clearly, $U_n(i, j) \cap A = g^{-1}(G_n(i, j))$ and the inductive assumption (v′) implies that $\{U_n(i, j)\}_{j\geq 0}$ still has the properties of $\{U_n'(i, j)\}_{j\geq 0}$. Thus we can define the sets $V_{n+1}(j)$ satisfying (ii′) and (iii′) by using (iv′). Moreover, we have

$$V_{n+1}(j) \cap A = \bigcup_{i\geq 0} \big(V_n(i) \cap U_n(i, j) \cap A\big) = \bigcup_{i\geq 0} g^{-1}\big(G_n(i) \cap G_n(i, j)\big) = g^{-1}\big(G_{n+1}(j)\big).$$

Thus (v′) is satisfied too.

By virtue of (v′), the mapping f defined as in the proof of Theorem 1 is an extension of g.

COROLLARY 2. *Let A be a closed subset of a perfect space X. If $g\colon A \to T(0)$, then g has an extension $f\colon X \to T(0)$.*

COROLLARY 3. *Any two disjoint closed subsets of a perfect space X can be separated by a mapping into $T(0)$* (see [6]).

COROLLARY 4. *The space $T(0)^{\aleph_0}$ is universal for all perfect T_1-spaces of weight $\aleph_0$* (see [6]).

COROLLARY 5. *A subset A of a perfect space X is closed if and only if $A = h^{-1}(0)$ for a mapping $h\colon X \to T(0)$, where $0 = (\emptyset, 0, 0, \ldots) \in T(0)$* (see [6]).

Proof. In order to obtain the non-trivial implication, we apply Theorem 2 for $\mathfrak{m} = 1$, $U_0 = X\setminus A$ and $g\colon A \to T(1)$ sending A to $(\emptyset, 0, 0, \ldots) \in T(1)$. We take $h = i\cdot f$, where f is an extension of g given by Theorem 2 and $i\colon T(1) \to T(0)$ is a natural embedding defined by $i(t) = (\emptyset, |t(0)|, t(1), t(2), \ldots)$.

We can strengthen the non-trivial part of Corollary 5 as follows:

COROLLARY 6. *If A is a closed subset of a perfect space X and $t \in T(0)$, then $A = h_t^{-1}(t)$ for a mapping $h_t\colon X \to T(0)$.*

Proof. Let $h\colon X \to T(0)$ satisfy $h^{-1}(0) = A$. For $t' \in T(0)$ define $p_t(t') = (\emptyset, t(1)+t'(1), t(2)+t'(2), \ldots)$. Clearly, $p_t\colon T(0) \to T(0)$ is continuous and $h_t = p_t \cdot h$ satisfies $h_t^{-1}(t) = A$.

Corollary 6 can also be obtained as a consequence of the following strengthening of Corollary 3:

COROLLARY 7. *Let A and B be disjoint closed subsets of a perfect space X. If $t, s \in T(0)$, then there exists a mapping $h\colon X \to T(0)$ such that $h^{-1}(t) = A$ and $h^{-1}(s) = B$.*

Proof. Let $m = t(1)+s(1)$. Define $t', s' \in T(m+1)$ as follows:

$t' = (\{1, \ldots, t(1)\}, t(2), t(3), \ldots)$ and $s' = (\{t(1)+1, \ldots, m\}, s(2), s(3), \ldots)$. Furthermore, put $U_0 = X \setminus (A \cup B)$, $U_k = X\setminus B$ for $k \in t'(0)$ and $U_k = X\setminus A$ for $k \in s'(0)$.

We apply Theorem 2 to the collection $\{U_k\}_{k=0,\ldots,m}$ and the mapping $g\colon (A\cup B) \to T(m+1)$ sending A to t' and B to s'.

Let f be an extension of g such that $f^{-1}(G_k) = U_k$ for $k = 0, \ldots, m$ and let $p\colon T(m+1) \to T(0)$ be given by $p(t'') = (\emptyset, |t''(0)|, t''(1), \ldots)$. It is easy to see that $h = p\cdot f$ satisfies $h^{-1}(t) = A$ and $h^{-1}(s) = B$.

COROLLARY 8. *Let $H(\mathfrak{m}) = \{t \in T(\mathfrak{m})\colon t = (\emptyset, 0, 0, \ldots)$ or $|t(0)| = 1\}$. The space $H(\mathfrak{m})^{\aleph_0}$ is universal for all perfect T_1-spaces with a σ-disjoint base of cardinality $\mathfrak{m}$* (see [5, 4.4.9]).

3. Final remarks.

Remark 1. Corollary 7 implies that no two points of $T(0)$ can be separated by disjoint open sets. Spaces $T(\mathfrak{m})$ have the same property. It is easy to observe that any finite intersection of elements of a subbase $\mathscr{P}$ of $T(\mathfrak{m})$ contains a non-empty set of the form $\bigcap_{k=1}^{i} G_{\alpha_k} \cap \bigcap_{k=1}^{n} H_k(j_k)$. From [4], it follows that there is no universal space for developable Hausdorff spaces of the weight of the continuum.

Remark 2. The subspace $\{\emptyset\} \times \{0, 1\}^{N+}$ of $T(0)$ has the topology of the Cantor cube.

Remark 3. Call a subset A of X a D-closed subset if A is an element of a collection $\mathscr{A}$ of closed subsets of X such that $A' \in \mathscr{A}$ implies $X\setminus A' = \bigcup \{A(j)\colon j \in N\}$ for some $A(j) \in \mathscr{A}$. The complements of D-closed sets are D-open sets (see [1]). It is easy to check that all the results of this paper can be generalized by replacing the assumption that X is perfect by the weaker assumptions that certain open subsets of X are D-open and closed subsets of X are D-closed. In particular, any mapping g of a D-closed subset of an arbitrary space X into $T(0)$ can be extended to a mapping $f\colon X \to T(0)$. Moreover, D-closed (D-open) subsets of X can be characterized as inverse images of closed (open) subsets of $T(0)$ under mappings of X into $T(0)$. Obviously, $T(0)$ in the above characterization can be replaced by a perfect space Z depending on X and the D-closed (D-open) subset of X.

Remark 4. A space X is said to be a D-normal space [2] if any two disjoint closed subsets of X can be separated by disjoint closed G_δ-sets(2). It is easy to check that closed G_δ-subsets of D-normal spaces are D-closed.

(2) It can be shown that a space X is D-normal if and only if any two disjoint closed subsets of X can be separated by disjoint subsets of which the first is open and the second a G_δ-set in X.

Thus, the generalization of Corollary 3 (7) for D-closed sets shows that any two disjoint closed subsets of a D-normal space X can be separated by a mapping $f\colon X \to T(0)$ (see [6]) (sending each of these sets into an arbitrarily chosen point of $T(0)$). We do not known whether a mapping $g\colon A \to T(0)$, where A is a closed subset of a D-normal space X, can always be extended to $f\colon X \to T(0)$. Obviously, it can be extended if A is a closed G_δ-subset of X.

Remark 5. From Corollary 2 it follows that any T_1-space with a σ-discrete network of cardinality not greater than the continuum has a one-to-one mapping onto a subspace of $T(0)^{\aleph_0}$.

Remark 6. One can modify the construction of $T(\mathfrak{m})$ in order to obtain an orthocompact developable T_1-space $T'(\mathfrak{m})$ with a locally finite collection $\mathscr{F}$ of closed subsets such that any perfect space X with a locally finite collection $\mathscr{E}$ of cardinality $\mathfrak{m}$ consisting of closed subsets of X can be mapped into $T'(\mathfrak{m})$ by a mapping f satisfying $\mathscr{E} = \{f^{-1}(F)\colon F \in \mathscr{F}\}$.

The space $T'(\mathfrak{m})$ has the same underlying set as $T(\mathfrak{m})$ but its subbase consists of sets $G'_a = \{t \in T\colon t(0) \subset a\}$ for $a \in \mathrm{Fin}(\mathfrak{m})$ and of sets $G'_0(i) = \{t \in T'\colon |t(0)| \leqslant i\}$, $G'_n(i) = \{t \in T'\colon t(n) \geqslant i\}$ for $n \geqslant 1$ and $G'_n(i, j) = \{t \in T'\colon t \in G'_n(i) \Rightarrow t(n+1) \geqslant j\}$ for $n \geqslant 0$.

The collection $\mathscr{F}$ is equal to $\{F_\alpha\colon \alpha \in \mathfrak{m}\}$, where $F_\alpha = \{t \in T\colon \alpha \in t(0)\}$ (see [3, Theorem 2.1.A and Lemma 2.2.A]). As a consequence, we infer that any T_1-space with a σ-discrete network of cardinality $\mathfrak{m}$ has one-to-one mapping onto a subspace of $T'(\mathfrak{m})^{\aleph_0}$ (3).

Added in proof. Another construction of Heldermann's space with the proofs of Corollaries 2–4 is given by H. Brandenburg, *An extension theorem for D-normal spaces*, Topology and Appl. 15 (1983), pp. 223–229.

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(3) If $\mathscr{F}$ is discrete, then $T'(\mathfrak{m})$ can be replaced by its subspace $H'(\mathfrak{m}) = \{t \in T'(\mathfrak{m})\colon t = (\{\alpha\}, 0, 0, \ldots)$ or $|t(0)| = 0\}$ (see [6]).