Many thanks are due Prof. Richard M. Dudley for helpful conversations and comments, and E. Grzegorek, who improved the original statement of Lemma 6.

References

[1] H. Bauer, *Probability Theory and Elements of Measure Theory*, Holt, Reinhart and Winston, Inc., New York 1972.

[2] I. Berkes and W. Philipp, *Approximation theorems for independent and weakly dependent random vectors*, Annals of Prob. 7 (1) (1979), pp. 29–54.

[3] D. Blackwell, *On a class of probability spaces*, Proceedings of the 3rd Berkeley Symposium on Mathematical Statistics and Probability, Vol. 2, University of California Press, Berkeley 1976, pp. 1–6.

[4] — and C. Ryll-Nardzewski, *Non existence of everywhere proper conditional distributions*, Ann. Math. Stat. 34 (1963), pp. 223–225.

[5] L. Breiman, *Probability*, Addison-Wesley, Reading 1968.

[6] D. L. Cohn, *Measure Theory*, Birkhäuser, Boston 1980.

[7] R. B. Darst, *On universal measurability and perfect probability*, Ann. Math. Stat. 42 (1) (1971), pp. 352–354.

[8] P.R. Halmos, *Measure Theory*, Van Nostrand, Princeton 1950.

[9] — and J. von Neumann, *Operator methods in classical mechanics II*, Ann. of Math. 43 (1942), pp. 332–350.

[10] K. Kuratowski, *Topology*, Vol. I, New York–London–Warszawa 1966.

[11] J. Łoś and E. Marczewski, *Extensions of measure*, Fund. Math. 36 (1949), pp. 267–276.

[12] N. Lusin, *Sur un problème de M. Baire*, C. R. Acad. Sci. Paris 158 (1914), pp. 1258–1260.

[13] E. Marczewski, *The characteristic function of a sequence of sets and some of its applications*, Fund. Math. 31 (1938), pp. 207–223.

[14] — *On the isomorphism and the equivalence of classes and sequences of sets*, Fund. Math. 32 (1939), pp. 133–148.

[15] — and W. Sierpiński, *Remarque sur le problème de la mesure*, Fund. Math. 26 (1936), pp. 256–261.

[16] K. Musiał, *Existence of proper conditional probabilities*, Z. Wahrsch. Verw. Gebiete 22 (1972), pp. 8–12.

[17] H. L. Royden, *Real Analysis*, McGraw Hill, New York 1968.

[18] V. V. Sazonov, *On perfect measures*, Amer. Math. Soc. Trans. Series 2 (48) (1962), pp. 229–254.

[19] W. Sierpiński, *Sur une propriété des ensembles lineaires quelconques*, Fund. Math. 23 (1934), pp. 125–134.

[20] V. S. Varadarajan, *Measures on topological spaces*, Amer. Math. Soc. Trans. Series 2 (48) (1961), pp. 161–228.

[21] N. N. Vorob'ev, *Consistent families of measures and their extensions*, Theory Prob. Appl. 7 (1962), pp. 147–163.

Received 26 October 1981
in revised form 8 March 1982

Correction to my paper "Topological contraction principle" (Fundamenta Mathematicae 110 (1980), pp. 135–144)

by

Pedro Morales (Sherbrooke)

The implication (ii) ⇒ (iv) of Theorem 4.1 is false. The proof was taken from an erroneous argument of Meyers [15, p. 74]. Claiming a counter-example, Dr. F. Guénard first pointed out the error to me. Subsequently, the paper "A converse to the principle of contracting maps" by V. I. Opoitsev [Russian Math. Surveys 31 (1976), pp. 175–204] confirmed this and also afforded me a means of correction. We give a corrected version of Theorem 4.1.

Let $X = (X, \mathscr{U})$ be a uniform space and let $f\colon X \to X$. If u is a fixed point of f, we say that u is *stable* if, for every $U \in \mathscr{U}$, there exists $V = V(U) \in \mathscr{U}$ such that $f^n(V[u]) \subseteq U[u]$ for every $n = 1, 2, 3, \ldots$ This means that the set $\{f^n\colon n = 1, 2, 3, \ldots\}$ is equicontinuous at u. Using the arguments of the first and second paragraphs of the proof of Lemma 2.1 (where the hypothesis is merely that f is contractive), we show easily that, if f is contractive, then the set $\{f^n\colon n = 1, 2, 3, \ldots\}$ is uniformly equicontinuous, so every fixed point of f is stable. If u is a fixed point of f and $\lim_n f^n(x) = u$ for all $x \in X$, we say that u is *iteratively realizable*. The corrected version of Theorem 4.1 is the following:

4.1. THEOREM. *Let X be a compact Hausdorff space. For a continuous function $f\colon X \to X$ the following statements are equivalent:*

(i) *f is an occasionally small contraction.*

(ii) *f has one and only one fixed point which is stable and iteratively realizable.*

(iii) *The filter with base $\mathscr{B} = \{f^n(X)\colon n = 1, 2, 3, \ldots\}$ converges.*

Proof. (i) ⇒ (ii). This follows from Theorem 1.1 and the preceeding remark.

(ii) ⇒ (iii). Let u be the fixed point of f and let $\mathscr{F}$ be the filter on X generated by $\mathscr{B}$. Using a refinement of the argument of Opoitsev in his Lemma 2.2, p. 182, we will show that $\mathscr{F} \to u$.

Let $U \in \mathscr{U}$. Since u is iteratively realizable, for every $x \in X$, there exists a smallest positive integer $n(x)$ such that $f^k(x) \in U[u]$ for all $k > n(x)$. The conclusion will follow if we show that $\sup\{n(x): x \in X\}$ is finite. If not, then, by the compactness of X, there exists a sequence $\{x_j\}$ in X such that $n(x_j) > j$ and $x_j \to y$ for some $y \in X$.

Since u is stable, there exists $V \in \mathscr{U}$ such that $f^n(V[u]) \subseteq U[u]$ for all $n = 1, 2, 3, \ldots$ For every $x \in X$, we can choose a smallest positive integer $\tilde{n}(x)$ with $f^{\tilde{n}(x)}(x) \in V[u]$. So $f^k(x) \in U[u]$ for every $k > \tilde{n}(x)$, and therefore $n(x) \leqslant \tilde{n}(x)$. Thus $\tilde{n}(x_j) > j$ for all $j = 1, 2, 3, \ldots$

Since $f^{\tilde{n}(y)}(y) \in V[u]$ and $f^{\tilde{n}(y)}$ is continuous, there exists $W \in \mathscr{U}$ such that $f^{\tilde{n}(y)}(z) \in V[u]$ for all $z \in W[y]$. Thus $z \in W[y]$ implies $\tilde{n}(z) \leqslant \tilde{n}(y)$. But $x_j \in W[y]$ eventually, which is a contradiction.

(iii) $\Rightarrow$ (i). The same argument used in (iv) = (i) of Theorem 4.1.

Received 13 April 1981