On symmetric words in nilpotent groups

by

Olga Macedońska-Nosalska (Gliwice)

Abstract. An r-ary word $w(x_1, \ldots, x_r)$ is called *symmetric in a group* G if for any elements $g_1, \ldots, g_r$ in G, and any permutation σ of indexes: $w(g_1, \ldots, g_r) = w(g_{\sigma(1)}, \ldots, g_{\sigma(r)})$. Symmetric r-ary words build a group $S^{(r)}(G)$. We shall show that $S^{(r)}(G)$ and $S^{(r+1)}(G)$ are isomorphic for G nilpotent of the class n and $r \geqslant n-1$.

Introduction. Let X be the free group on the set of free generators $\{x_i, i \in N\}$, where N denotes the set of natural numbers, and let V be a verbal subgroup of X. A word w from X is called an *r-ary word* if the reduced form of w involves r generators, say $x_{i_1}, x_{i_2}, \ldots, x_{i_r}$. We then write $w = w(x_{i_1}, \ldots, x_{i_r})$. For any permutation σ of N we denote $w\sigma = w(x_{\sigma(i_1)}, \ldots, x_{\sigma(i_r)})$, and say that the words w and $w\sigma$ are equivalent under σ.

DEFINITION 1. A word $s(x_{i_1}, \ldots, x_{i_r})$ from X is called *symmetric* modulo V if for every permutation σ_0 of $\{i_1, \ldots, i_r\}$, $s\sigma_0 \equiv s \bmod V$.

We can see that the symmetry of $s(x_1, \ldots, x_r)$ implies the symmetry of all equivalent words $s\sigma = s(x_{i_1}, \ldots, x_{i_r})$. Indeed if σ_0 is a permutation of the set $\{i_1, \ldots, i_r\}$, then $\sigma\sigma_0\sigma^{-1}$ is a permutation of $R = \{1, 2, \ldots, r\}$ and because of the symmetry of s, $s(\sigma\sigma_0\sigma^{-1}) \equiv s \bmod V$. This implies $(s\sigma)\sigma_0 = s\sigma \bmod V$, which proves the symmetry of $s\sigma$.

According to [3] the set of all cosets sV, where $s = s(x_1, \ldots, x_r)$ is an r-ary word symmetric modulo V, written on x_i, $i \in R = \{1, 2, \ldots, r\}$ forms a group $S^{(r)}(V)$. We shall suppose V fixed and write $S^{(r)}$. It will be shown that if $r \neq q$, then $S^{(r)} \cap S^{(q)} = 1 \bmod V$. Let $sV \in S^{(r)}$, $tV \in S^{(q)}$, $q < r$, and for some $v \in V$ $s(x_1, \ldots, x_r) = t(x_1, \ldots, x_q) v(x_1, \ldots, x_r)$. Then for any y_i, $i \in R$, because of the symmetry of s we have

$$\begin{aligned} s(x_1, \ldots, x_r) &= t(x_1, \ldots, x_q) v(x_1, \ldots, x_r) \equiv s(x_1, \ldots, x_{r-1}, y_1) \\ &\equiv s(y_1, x_1, \ldots, x_{r-1}) = t(y_1, x_1, \ldots, x_{q-1}) v(y_1, x_1, \ldots, x_{r-1}) \\ &\equiv s(y_1, x_1, \ldots, x_{r-2}, y_2) \equiv s(y_1, y_2, x_1, \ldots, x_{r-2}) \\ &= t(y_1, y_2, x_1, \ldots, x_{q-2}) v(y_1, y_2, x_1, \ldots, x_{r-2}) \equiv \ldots \\ &\ldots \equiv s(y_1, y_2, \ldots, y_r) \bmod V. \end{aligned}$$

Put $y_i = 1$, $i \in R$, then $s(x_1, \ldots, x_r) = V$ which was required.

Since V is a fully invariant subgroup of X, the mapping

$$\delta_{r+1}\colon s(x_1, \ldots, x_r, x_{r+1}) \to s(x_1, \ldots, x_r, 1)$$

determines a homomorphism $\partial_r^{r+1}\colon S^{(r+1)} \to S^{(r)}$. In [3] it is shown that the homomorphism ∂_r^{r+1} is an isomorphism if $V = X^{(n)}$, and $r \geqslant n-1$. Here $X^{(1)} = X$, $X^{(k)} = [X^{(k-1)}, X]$. In [4] the same result is proved for $V \supseteq X^{(n)}$, $n \leqslant 4$, $r \geqslant n-1$, and the problem is formulated whether the mapping ∂_r^{r+1} is an isomorphism for every n, $V \supseteq X^{(n)}$, $r \geqslant n-1$.

We give here an affirmative solution of the above problem.

Definitions and lemmas. In X we introduce an endomorphism δ_k $(k > 0)$ given by $x_k \delta_k = 1$, $x_i \delta_k = x_i$ for $i \neq k$. Clearly

(1) $$\delta_i \delta_j = \delta_j \delta_i, \quad \delta_i^2 = \delta_i.$$

Any permutation σ of the set N induces an automorphism of X such that

(2) $$\delta_i \sigma = \sigma \delta_{\sigma(i)}, \quad \sigma \delta_i = \delta_{\sigma^{-1}(i)} \sigma.$$

We shall write $D_i = \operatorname{Ker} \delta_i$ and $D_R = \bigcap_{i \in R} D_i$.

Definition 2. A word $w(x_1, \ldots, x_r)$ is called *neutral* if $w\delta_i = 1$, $i \in R = \{1, 2, \ldots, r\}$.

The set of all r-ary neutral words on generators x_i, $i \in R$, obviously coincides with D_R.

Lemma 1. *If $w(x_1, \ldots, x_r)$ is a neutral r-ary word then $w \in X^{(r)}$.*

Proof. $w \in D_R \subseteq X^{(r)}$ follows from ([2], 33.38).

Lemma 2. *If $s(x_1, \ldots, x_r)$ is a neutral r-ary word, symmetric modulo $X^{(r+1)}$ then $s \in X^{(r+1)}$.*

Proof. Denote by S_r a group of permutations of R, and by A_r its alternating subgroup. We shall consider the word $s_0 = \prod_{\sigma \in S_r} c\sigma$, where $c = [x_1, x_2, x_3, \ldots, x_r]$ is the left-normed commutator, and the product is taken for some fixed order of factors. Obviously s_0 is a neutral r-ary word, symmetric modulo $X^{(2r)}$. If we denote by $\sigma_{(1,2)}$ the cycle $(1, 2)$ then $s_0 = \prod_{\sigma' \in A_r} (c\, c\sigma_{(1,2)})\sigma' \bmod X^{(r+1)}$. With the use of a commutator calculus modulo $X^{(r+1)}$ we have

$$\begin{aligned} c\, c\sigma_{(1,2)} &= [x_1, x_2, x_3, \ldots, x_r][[x_1, x_2]^{-1}, x_3, \ldots, x_r] \\ &= cc^{-1} \bmod X^{(r+1)} \in X^{(r+1)}. \end{aligned}$$

This implies $s_0 \in X^{(r+1)}$, which will be used later.

Now since $s(x_1, \ldots, x_r)$ is a neutral r-ary word, by Lemma 1 $s \in X^{(r)}$, and by ([2], 34.21) s is a product modulo $X^{(r+1)}$ of commutators $c\sigma$ for some $\sigma \in S_r$, say $s = \prod c\sigma \bmod X^{(r+1)}$. Now, since s is symmetric modulo $X^{(r+1)}$, for every $\sigma_0 \in S_r$, $\prod c\sigma \equiv \prod c(\sigma\sigma_0) \bmod X^{(r+1)}$. By Hall's basis theorem ([1], 11.2.4) we conclude that s is a power of s_0 and hence $s \in X^{(r+1)}$, which was required.

Now let $w \neq 1$ be an element of X. We write

(3) $$w(1-\delta_i) = w(w\delta_i)^{-1}.$$

Then by (1)

(4) $$(1-\delta_i)\delta_j = \delta_j(1-\delta_i), \quad (1-\delta_i)\delta_i = 1.$$

Definition 3. For a word $w(x_1, \ldots, x_r)$ and $k \leqslant r$, we define the *k-ary image* of w as $w_k = w(x_1, \ldots, x_k, 1, \ldots, 1) = w \prod_{i=k+1}^{i=r} \delta_i$. The *neutral part* of w we define as $\bar{w} = w \prod_{i=1}^{i=r} (1-\delta_i)$. The word $\bar{w}$ is neutral by ([2], 33.42).

For a set $M = \{i_1, i_2, \ldots, i_k\} \subseteq N$ we shall always suppose $i_1 < i_2 < \ldots < i_k$. We denote now $K = \{1, 2, \ldots, k\}$ and introduce a permutation σ_M of the set $M \cup K$ such that $\sigma_M(j) = i_j$, $j \leqslant k$. Then $\sigma_M\colon K \to M$. If $M \subseteq R = \{1, 2, \ldots, r\}$, then σ_M can be considered as a permutation of R, since $M \cup K \subseteq R$. In case $k = r$, σ_M is obviously the identical permutation. If $s(x_1, \ldots, x_k)$ is a k-ary word then $s\sigma_M$ is an equivalent word written on generators with indices form M.

Definition 4. We shall define a special *order* $(*)$ for subsets in N. There exists on-to-one correspondence between subsets $M = \{i_1, i_2, \ldots, i_k\}$, $i_1 < i_2 < \ldots < i_k$, of N and formal sequences $\langle i_1, i_2, \ldots, i_k, \omega, \omega, \ldots \rangle$. We assume $\omega > i$ for every $i \in N$. The lexicographical order for the sequences induces the order for subsets in N. We shall refer to it as to the order $(*)$.

Lemma 3. *Every symmetric word $s(x_1, \ldots, x_r)$ is a product (modulo V) of the neutral parts $\bar{s}_k$ of its k-images, $k \leqslant r$, and the equivalent words. More precisely $s = \prod \bar{s}_k \sigma_M$ is a product of 2^r factors corresponding to the subsets $M \subseteq R$, for $k = |M|$, taken in order $(*)$.*

Proof. We introduce an algorithm that allows us to write any word $s(x_1, \ldots, x_r)$ as a product $\prod u_M$ of 2^r factors corresponding to the subsets $M \subseteq R$ in order $(*)$, where $u_M = s \prod_{i \in M} (1-\delta_i) \prod_{j \in R \setminus M} \delta_j$. According to (3) for any word s and any $i \in N$

(5) $$s = s(1-\delta_i)\, s\delta_i.$$

So for $i = 1$, $s = s(1-\delta_1)\, s\delta_1$. Now apply (5) for $i = 2$ to each factor separately. We get

$$s = s(1-\delta_1)(1-\delta_2)\, s(1-\delta_1)\delta_2\, s\delta_1(1-\delta_2)\, s\delta_1\delta_2.$$

Apply (5) for $i = 3$ to each factor separately and so on. The result will be achieved in r steps with the use of (4). See ([2], 33.44).

We can see that for the case $r = 2$ the factors correspond to the subsets

$$\{1, 2\} < \{1\} < \{2\} < \{\emptyset\}.$$

Suppose that for the $(r-1)$-st step the factors correspond to the subsets

$$M_1 < M_2 < \ldots < M_{2^{r-1}} < \{\emptyset\},$$

then by applying (5) for δ_r we get a product of 2^r factors corresponding to the sequence of subsets

$$\{M_1, r\} < M_1 < \{M_2, r\} < M_2 < \ldots < \{M_{2^{r-1}}, r\} < M_{2^{r-1}} < \{r\} < \{\emptyset\},$$

which was required.

To prove the lemma we shall show that if s is a symmetric word, then $u_M = \bar{s}_k \sigma_M$. Indeed, since $\sigma_M\colon R \to R$, $s \equiv s\sigma_M \bmod V$. Then by using (2) and (4)

$$u_M = s\sigma_M \prod_{i\in M}(1-\delta_i) \prod_{j\in R\setminus M}\delta_j = s\prod_{i\in M\sigma_M^{-1}}(1-\delta_i)\prod_{j\in R\setminus M\sigma_M^{-1}}\delta_j\sigma_M$$
$$= s\prod_{i\in K}(1-\delta_i)\prod_{j\in R\setminus K}\delta_j\sigma_M = \Big(s\prod_{i=k+1}^{r}\delta_j\Big)\prod_{i=1}^{k}(1-\delta_i)\sigma_M = \bar{s}_k\sigma_M$$

as required.

For $M = \{i_1, i_2, \ldots, i_k\}$ and $K = \{1, 2, \ldots, k\}$ we have defined $\sigma_M\colon K \to M$. If take $M_0 = \{1, 2, \ldots, i_0-1, i_0+1, \ldots, r+1\}$, and $R = \{1, 2, \ldots, r\}$ then $\sigma_{M_0}\colon R \to M_0$ is a cycle $\sigma_{M_0} = (i_0, i_0+1, \ldots, r, r+1)$ which is a permutation of $J = \{1, 2, \ldots, r+1\}$. Here in the case $i_0 = r+1$ σ_{M_0} is the identical permutation.

LEMMA 4. *For a symmetric word $w(x_1, \ldots, x_{r+1})$ denote $w\delta_{r+1} = s$, then*

(6) $$w\delta_{i_0} \equiv s\sigma_{M_0} \bmod V \quad \textit{for} \quad i_0 \leqslant r+1.$$

(7) $$\bar{w}_k = \bar{s}_k \quad \textit{for} \quad k \leqslant r.$$

For any neutral word $w(x_1, x_2, \ldots, x_k)$

(8) $$w\sigma_M\delta_{i_0} = \begin{cases} w\sigma_M & \textit{if} \quad i_0 \notin M, \\ 1 & \textit{if} \quad i_0 \in M. \end{cases}$$

Proof. Because of the symmetry of w and (2) $w\delta_{i_0} = w\sigma_{M_0}\delta_{i_0} = w\delta_{r+1}\sigma_{M_0} = s\sigma_{M_0}$ which gives (6). By Definition 3 and (4)

$$\bar{w}_k = w\prod_{i=k+1}^{r+1}\delta_i\prod_{j=1}^{k}(1-\delta_j) = (w\delta_{r+1})\prod_{i=k+1}^{r}\delta_i\prod_{j=1}^{k}(1-\delta_j) = \bar{s}_k,$$

gives (7). Now by (2) $\sigma_M\delta_{i_0} = \delta_{\sigma_M^{-1}(i_0)}\sigma_M$. The index $\sigma_M^{-1}(i_0)$ belongs to K if and only if $i_0 \in M$. Since w is neutral, we have (8). The lemma is proved.

Theorems on the homomorphism $\partial_r^{r+1}\colon S^{(r+1)} \to S^{(r)}$.

THEOREM 1. *If $V \supseteq X^{(n)}$ then ∂_r^{r+1} is an epimorphism for $r \geqslant n-1$.*

Proof. A coset $sV \in S^{(r)}$ has a contraimage under ∂_r^{r+1} if and only if there exists an $(r+1)$-ary symmetric word w such that $wV \in S^{(r+1)}$ and $w\delta_{r+1} = s$. By Lemma 3, $w = \prod \bar{w}_k\sigma_M$, where $k = |M|$ and M runs over all non-empty subsets of the set $J = \{1, 2, \ldots, r+1\}$ in order (*). In this order the first factor of the product corresponds to $M = J$ and coincides with the neutral part of w, namely $\bar{w} = w\prod_{i\in J}(1-\delta_i)$. Then $w = \bar{w}\prod \bar{w}_k\sigma_M$ where $M \subset J$, i.e. $k < r+1$. We shall denote $w_0 = \prod \bar{w}_k\sigma_M$, $k < r+1$. Since by Lemma 1 $\bar{w} \in X^{(r+1)} \subseteq V$, we have $w \equiv w_0 \bmod V$ and w_0 is completely defined by the word s because of (7).

Let $s(x_1, \ldots, x_r)$ be a symmetric word. We construct $w_0 = \prod \bar{s}_k\sigma_M$ where M runs over the proper subsets of $J = \{1, 2, \ldots, r+1\}$ in order (*). We shall show that w_0V is a contraimage of sV under ∂_r^{r+1}. We shall first check the equality

(9) $$w_0\delta_{i_0} = s\sigma_{M_0} \bmod V \quad \text{for} \quad i_0 \leqslant r+1.$$

By (8) we have $w_0\delta_{i_0} = \prod_{M\subset J}\bar{s}_k\sigma_M\delta_{i_0} = \prod_{M\subseteq M_0}\bar{s}_k\sigma_M$ with factors in order (*). We now consider $s\sigma_{M_0}$. Notice that $\sigma_{M_0}\colon R \to M_0$ gives a one-to-one correspondence for subsets of M_0 and R, preserving the order (*). So if M runs over the subsets of M_0, then $M' = M\sigma_{M_0}^{-1}$ runs over the subsets of R and $s = \prod_{M'\subseteq R}\bar{s}_k\sigma_{M'}$. Since $\sigma_{M'}\colon K \to M' \subseteq R$ and $\sigma_{M_0}\colon R \to M_0$ we have $K \xrightarrow{\sigma_{M'}} M\sigma_{M_0}^{-1} \xrightarrow{\sigma_{M_0}} M\sigma_{M_0}^{-1}\sigma_{M_0} = M$, which means that $\sigma_{M'}\sigma_{M_0}$ coincides with σ_M on K and hence $s\sigma_{M_0} = \prod_{M'\subseteq R}\bar{s}_k\sigma_{M'}\sigma_{M_0} = \prod_{M\subseteq M_0}\bar{s}_k\sigma_M$. So (9) is proved. We notice that $\sigma_{M_0} = (i_0, i_0+1, \ldots, r, r+1) = (i_0, i_0+1, \ldots, r)(i_0, r+1) = \sigma\sigma_{(i_0,r+1)}$. The cycle σ is a permutation of R and because of the symmetry of s equality (9) can be written as

(10) $$w_0\delta_k = s\sigma_{(k,r+1)}, \quad k \leqslant r+1.$$

We have shown that w_0 is an $(r+1)$-ary word and by (10) $w_0\delta_{r+1} = s$. Now we have to check the symmetry of w_0 modulo V. Since every permutation is a product of cycles it is enough to show $w_0\sigma_{(i,j)} \equiv w_0 \bmod V$, for $i < j \leqslant r+1$. We denote $v = w_0\sigma_{(i,j)}w_0^{-1}$ and show first that for every $k \leqslant r+1$ $v\delta_k \in V$. For this purpose we shall consider seven cases:

1. $i = k \neq j$; a. $j = r+1$, b. $j \neq r+1$.
2. $i \neq k = j$; a. $j = r+1$, b. $j \neq r+1$.
3. $i \neq k \neq j$; a. $k = r+1$, b. $j = r+1$, c. $j \neq r+1 \neq k$.

By (2) and (10) we have

1. $v\delta_k = w_0\sigma_{(k,j)}\delta_k w_0^{-1}\delta_k = w_0\delta_j\sigma_{(k,j)}w_0^{-1}\delta_k = s\sigma_{(j,r+1)}\sigma_{(k,j)}s^{-1}\sigma_{(k,r+1)}$.

In case 1a. $j = r+1$, $v\delta_k = s\sigma_{(k,r+1)}s^{-1}\sigma_{(k,r+1)} = 1$.

In case 1b. $j \neq r+1$,

$$v\delta_k = s\sigma_{(j,r+1)}\sigma_{(k,j)}s^{-1}\sigma_{(k,r+1)}$$
$$= s\sigma_{(k,j)}\sigma_{(k,r+1)}s^{-1}\sigma_{(k,r+1)} = (s\sigma_{(k,j)}s^{-1})\sigma_{(k,r+1)} \in V,$$

because of the symmetry of s, since $k, j < r+1$.

2. $v\delta_k = w_0\sigma_{(i,k)}\delta_k w_0^{-1}\delta_k = w_0\delta_i\sigma_{(i,k)}w_0^{-1}\delta_k = s\sigma_{(i,r+1)}\sigma_{(i,k)}s^{-1}\sigma_{(k,r+1)}$.

In case 2a. $k = j = r+1$, $v\delta_k = ss^{-1} = 1$.

In case 2b. $k = j \neq r+1$,

$$v\delta_k = s\sigma_{(i,r+1)}\sigma_{(i,k)}s^{-1}\sigma_{(k,r+1)}$$
$$= s\sigma_{(i,k)}\sigma_{(k,r+1)}s^{-1}\sigma_{(k,r+1)} = (s\sigma_{(i,k)}s^{-1})\sigma_{k,r+1)} \in V.$$

3. $v\delta_k = w_0\sigma_{(i,j)}\delta_k w_0^{-1}\delta_k = w_0\delta_k\sigma_{(i,j)}w_0^{-1}\delta_k = s\sigma_{(k,r+1)}\sigma_{(i,j)}s^{-1}\sigma_{(k,r+1)}$.

In case 3a. $k = r+1$, $v\delta_k = s\sigma_{(i,j)}s^{-1} \in V$.

In case 3b. $j = r+1$,

$$v\delta_k = s\sigma_{(k,r+1)}\sigma_{(i,r+1)}s^{-1}\sigma_{(k,r+1)}$$
$$= s\sigma_{(i,k)}\sigma_{(k,r+1)}s^{-1}\sigma_{(k,r+1)} = (s\sigma_{(i,k)}s^{-1})\sigma_{(k,r+1)} \in V.$$

In case 3c. $j \neq r+1 \neq k$, $v\delta_k = (s\sigma_{(i,j)}s^{-1})\sigma_{(k,r+1)} \in V$. We have shown that $v\delta_k \in V$, $k \leqslant r+1$. By Definition 2 this implies the neutrality of v modulo V and by Lemma 1, $v \in X^{(r+1)}V \subseteq V$, since $X^{(r+1)} \subseteq X^{(n)} \subseteq V$. Now $v \in V$ implies the required symmetry of w_0 modulo V. So w_0V is a contraimage of sV under ∂_r^{r+1} and the proof is complete.

THEOREM 2. *If $V \supseteq X^{(n)}$, then ∂_r^{r+1} is an isomorphism for $r \geqslant n-1$.*

Proof. If $wV \in \operatorname{Ker}\partial_r^{r+1}$ then $w\delta_{r+1} = 1 \bmod V$ and by (6) $w\delta_i = 1 \bmod V$ for $i \leqslant r+1$. Hence w is a neutral $(r+1)$-ary word modulo V and, by Lemma 1, $w \in X^{(r+1)}V \subseteq V$. This means that ∂_r^{r+1} is a monomorphism, which completes the proof because of Theorem 1.

THEOREM 3. *If $V = X^{(n)}$, then ∂_r^{r+1} is a monomorphism for $r \geqslant n-2$, and is not a monomorphism for $r < n-2$.*

Proof. For $r \geqslant n-1$ the statement follows from Theorem 2. If $r = n-2$ and $wV \in \operatorname{Ker}\partial_r^{r+1}$, then w is an $(r+1)$-ary word neutral modulo $V = X^{(r+2)}$, and by Lemma 2 $w \in X^{(r+2)} = V$. This means that $\operatorname{Ker}\partial_r^{r+1}$ is trivial.

Let $r < n-2$. Denote by $d = [x_2, x_1, x_1, \ldots, x_1, x_3, x_4, \ldots, x_{r+1}]$ a left-normed commutator of the weight k, with x_1 repeated at least twice. If x_1 is repeated twice then $k = r+2 < n$. We shall also suppose $\frac{1}{2}n \leqslant k < n$ repeating x_1 if necessary. The word $d_0 = \prod_{\sigma \in S_{r+1}} d\sigma$ is obviously an $(r+1)$-ary neutral word symmetric modulo $X^{(2k)} \subseteq X^{(n)} = V$, hence $d_0 V \in \operatorname{Ker}\partial_r^{r+1}$. We have to check that $d_0 \notin V$. By the use of the commutator calculus (see [1]) modulo $X^{(k+3)}$ and the identity which follows from ([1], 10.2.1.4): $[[x, y], z] = [x, [y, z]][[x, z], y] \bmod X^{(4)}$, we can write d_0 as a product of basic commutators (see [1]) modulo $X^{(k+3)}$. The process is based on the typical step: $t = [x_i, x_j, \ldots, x_k, x_l]$ is a basic commutator, $i > j < \ldots < k < l$, and $l > m$, then $[t, x_m] = [[x_i, x_j, \ldots, x_k], [x_l, x_m]]\,[x_i, x_j, \ldots, x_k, x_m, x_l]$. It can be shown by induction that $d\sigma$ written as a product of basic commutators involves d if and only if $\sigma(1) = 1$, $\sigma(2) = 2$. Then d_0 written as a product of basic commutators modulo $X^{(r+3)}$ contains d to the power $(r-1)!$, hence $d_0 \notin X^{(r+3)}$ and $d_0 \notin V$, which completes the proof.

References

[1] M. Hall, *The theory of groups* (Russian translation), Moskwa 1962.

[2] H. Neumann, *Varieties of groups* (Russian translation), Moskwa 1969.

[3] E. Płonka, *On symmetric words in free nilpotent groups*, Bull. Acad. Polon. Sci. 18 (8) (1970), pp. 427–429.

[4] — *Symmetric words in nilpotent groups of class* $\leqslant 3$, Fund. Math. 97 (1977), pp. 95–103.

INSTYTUT MATEMATYKI, POLITECHNIKA ŚLĄSKA
ul. Zwycięstwa 42, Gliwice 44–100

Accepté par la Rédaction le 12. 11. 1981