[J2] R. E. Jamison, *Some intersection and generation properties of convex sets*, Comp. Math. 35 (2) (1977), pp. 147–161.

[J3] — *Partition numbers for trees and ordered sets*, Pacific J. Math. 96 (1) (1981), pp. 115–140.

[K] V. Klee, *Unions of increasing and intersections of decreasing sequences of convex sets*, Israel J. Math. 12 (1972), pp. 70–78.

[KW] D. C. Kay and E. W. Womble, *Axiomatic convexity theory and relationships between the Carathéodory, Helly and Radon numbers*, Pacific J. Math. 38 (2) (1971), pp. 471–485.

[vM] J. van Mill, *Supercompactness and Wallman spaces*, M.C. tract 85, Amsterdam 1977.

[vMV1] — and M. van de Vel, *Subbases, convex sets, and hyperspaces*, Pacific J. Math. 92 (2) (1981), pp. 385–402.

[vMV2] — — *Convexity preserving maps in subbase convexity theory*, Proc. Kon. Ned. Acad. Wet. 81 (1) (1978), pp. 76–90.

[vMW] — and E. Wattel, *An external characterization of spaces which admit binary normal subbases*, Amer. J. Math. 100 (1978), pp. 987–994.

[N] S. B. Nadler, Jr., *Hyperspaces of Sets*, Marcel Dekker, New York 1978.

[V1] M. van de Vel, *Pseudo-boundaries and pseudo-interiors for topological convexities*, Dissertationes Math. 210 (1983), pp. 1–76.

[V2] — *Finite dimensional convexity structures I: general results*, Top. and Appl. 14 (1982), pp. 201–225.

[V3] — *A selection theorem for topological convexity structures*, to appear.

[V4] — *Dimension of convex hyperspaces*, to appear in Fund Math.

[Ve] A. Verbeek, *Superextensions of topological spaces*, M.C. tract 41, Amsterdam 1972.

[W] L. E. Ward, Jr., *A note on dentrites and trees*, Proc. Amer. Math. Soc. 5 (1954), pp. 992–994.

[Wh] G. T. Whyburn, *Cut points in general topological spaces*, Proc. Nat. Akad. Sc. 61 (1968), 380–387.

SUBFACULTEIT WISKUNDE
VRIJE UNIVERSITEIT

Accepté par la Rédaction le 13. 7. 1981

Hereditarily indecomposable continua with trivial shape

by

J. Krasinkiewicz (Warszawa) and **M. Smith** (Auburn, Ala.)

The result presented in this paper, in a slightly weaker form, was discovered by the second author. The proof was complicated. Later the first author found simpler proof and it was decided to write a joint paper with the simpler proof.

Morton Brown [1] has proved that the limit X of an inverse sequence of n-spheres S^n, $n \geqslant 2$, is not hereditarily indecomposable provided each bonding map is essential, i.e. not homotopic to a constant. In this situation we have: (1) X is the limit of an inverse sequence of locally connected unicoherent continua, and (2) $\check{H}^n(X) \neq 0$. We shall prove more: any h.i. continuum satisfying (1) must be acyclic (even tree-like).

A space X is said to be *contractible relatively another space* Y provided any mapping from X into Y is nullhomotopic. If a mapping f is nullhomotopic, we write $f \simeq 0$.

THEOREM. *If an hereditarily indecomposable continuum X is the limit of an inverse sequence of locally connected and unicoherent continua, then X is tree-like.*

Proof. Let $X = \varprojlim \{X_n, g_{nm}\}$, where X_n's are locally connected and unicoherent continua, and let $f\colon X \to Y$ be a mapping into a 1-dimensional polyhedron. We shall show that f is nullhomotopic. Since $Y \in \mathrm{ANR}$, there are an index $n \geqslant 1$ and a mapping $f_n\colon X_n \to Y$ such that $f \simeq f_n \circ g_n$, where g_n is the projection from X into X_n. Hence it suffices to show that $f_n \simeq 0$. By the Whyburn factorization theorem there exists a continuum Z, a monotone surjection $k\colon X_n \to Z$ and a 0-dimensional map $l\colon Z \to Y$ such that $f_n = l \circ k$. It follows that Z is a locally connected and unicoherent continuum. Since l is 0-dimensional and $\dim Y = 1$, by the Hurewicz theorem [4, p. 114, Th. 1] we infer that Z is a curve. It follows that Z is a dendrite [4, p. 442, Cor. 8], hence an absolute retract. This proves that $f_n \simeq 0$ because k (and also l) is nullhomotopic. Thus we have proved that X is contractible relatively any graph. By [3, Cor. 4] we infer that $\dim X \leqslant 1$. Applying the characterization of tree-like continua from [2] we conclude that X is tree-like. This completes the proof.

It follows from the theorem that h.i. continua with trivial shape must be tree-like. In this form the theorem was discovered by the second author. Continua with trivial shape may be characterized as those which are the limits of inverse sequences of absolute retracts.

References

[1] M. Brown, *On the inverse limit of euclidean n-spheres*, Trans. Amer. Math. Soc. 96 (1960), pp. 129–134.
[2] J. H. Case and R. E. Chamberlin, *Characterizations of tree-like continua*, Pacific J. Math. 10 (1960), pp. 73–84.
[3] J. Krasinkiewicz, *Mapping properties of hereditarily indecomposable continua*, Houston J. Math. (to appear).
[4] K. Kuratowski, *Topology*, vol. 2, New York–London–Warszawa 1968.

INSTITUTE OF MATHEMATICS
POLISH ACADEMY OF SCIENCES
00-950 Warszawa

AUBURN UNIVERSITY
Auburn, Alabama 36849

Accepté par la Rédaction le 17. 7. 1981

The L^t-theory of profinite abelian groups

by

Peter H. Schmitt (Heidelberg)

Abstract. The concept of an algebraically complete topological abelian (ACTA-) group was introduced by J. Flum and M. Ziegler in their monograph on the topological first-order language Ł ([5] below). We determine the structure of saturated ACTA-groups and give cardinal invariants for their Ł-equivalence. We show that the profinite abelian (PFA-) groups constitute a subclass of the ACTA-groups. We axiomatize the Ł-theory of PFA-groups and show its decidability.

The topological logic Ł, recently introduced by Sgro, turned out to be a surprisingly good analog of first-order logic in the context of topological structures. A detailed description of Ł will be presented in § 1 below.

In [5] Flum and Ziegler introduced the concept of an algebraically complete topological group. They proved that a topological abelian group is algebraically complete if and only if it is Ł-equivalent to a direct sum of abelian groups with discrete topologies. From this they inferred decidability of the Ł-theory of this class of groups. In § 2 we will determine the structure of saturated algebraically complete topological abelian groups and give cardinal invariants for Ł-equivalence. In § 3 we show that profinite abelian groups are in fact algebraically complete and we give axioms for the Ł-theory of this class of topological groups and prove its decidability. Our approach also yields a new proof of the decidability and axiomatizability results contained in [1].

We should like to thank Martin Ziegler for pointing out a mistake in the original proof of Corollary 3.6.

§ 1. Prerequisites

A. The topological logic Ł. We will present the first-order topological logic Ł in a form specifically adapted to the discussion of first-order properties of topological groups.

Let LG be the usual first-order language of group theory (written additively) and let LG^{II} be the extension of LG to the following weak second-order logic:

1. Syntax: Conventional second-order logic with second-order variables $X, Y, \ldots,$ second-order constants and the binary relation symbol $\in$. The class of formulas is closed under second-order quantification.